Spin-Charge Separation in Ultracold Quantum Gases

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(Received 8 July 2002; published 15 January 2003; publisher error corrected 17 January 2003)

We investigate the physical properties of quasi-1D quantum gases of fermionic atoms confined in harmonic traps. Using the fact that for a homogeneous gas the low-energy properties are exactly described by a Luttinger model, we analyze the nature and manifestations of spin-charge separation, where in the case of atoms "spin" and "charge" refer to two internal atomic states and the atomic mass density, respectively. We discuss the necessary physical conditions and experimental limitations confronting possible experimental implementations.

DOI: 10.1103/PhysRevLett.90.020401 PACS numbers: 03.75.Kk, 03.75.Hh, 71.10.Pm, 71.27.+a

One dimensional (1D) quantum liquids are very rich and interesting systems. In spite of their apparent conceptual simplicity, both the ground state and the excitations exhibit strong correlation effects and posses a number of exotic properties, ranging from spin-charge separation to fractional statistics (see [1–3] and references therein). Progress in creating, manipulating and studying ultra-cold quantum gases with controlled and adjustable interactions [4,5], and, in particular, the recent development of 1D magnetic and optical wave guides opens the door for a new and clean physical realization of such 1D systems with the tools of atomic physics and quantum optics. While most of the recent theoretical and experimental work has focused on 1D Bose gases (as a Tonks gas or a quasicondensate) [6,7] progress in cooling Fermi gases into the quantum degenerate regime [8] point to the possibility of realizing a Luttinger liquid (LL) [2] with cold fermionic atoms. One of the key predictions of the Tomonaga-Luttinger model (LM) for interacting fermions is spin-charge separation [2]. It is a feature of interacting spin-1/2 particles and manifests itself in complete separation in the dynamics of spin and density waves. Both branches of the excitations are soundlike and characterized by different propagation velocities. This phenomenon is one of the hallmarks of a Luttinger liquid, however it has never been demonstrated in a clean way in an actual condensed matter system (see, e.g., [9]). It is the purpose of this Letter to analyze in detail the conditions of realizing an (inhomogeneous) LL with a gas of cold fermionic atoms in 1D harmonic trap geometries, and, in particular, to study the possibilities of seeing spin-charge separation in the spectroscopy and wave packet dynamics of laser excited 1D Fermi gases [10]. We note that in the case of atoms spin and charge refers to two internal atomic states and the atomic mass density, respectively. We prefer in the following, however, to adopt the traditional condensed matter language, and refer to this phenomenon as "spin-charge separation."

The simplest example of LL made of a gas of cold atoms consists of fermionic atoms with two internal states representing a spin-1/2 system. We assume the atoms harmonically trapped and cooled below the Fermi-degeneracy temperature $k_B T_F \sim N\hbar\omega$, where N is the number of particles and ω is the frequency of the longitudinal confinement. The condition for a quasi-1D system is tight transverse trapping in an external potential with the frequency $\omega_\perp$ exceeding the characteristic energy scale of the longitudinal motion. Because of the quantum degeneracy the longitudinal motion has all the energy levels up to the Fermi energy $\epsilon_F \sim k_B T_F$ filled. Thus we require the total number of particles to be restricted by $N \ll \omega_\perp/\omega$, which for realistic traps is of the order of a few hundred or thousand. Because of Pauli principle, at low temperature, only s-wave collisions between atoms in different internal states are allowed. Therefore, all the relevant interactions are characterized by a single parameter, the scattering length a corresponding to intercomponent interaction. The effective 1D interaction can thus be represented as a zero-range potential of the strength $g = 2\pi\hbar^2 a/m l_\perp^2$, where $l_\perp$ is the width of the ground state in the transverse direction ($a \ll l_\perp$) [7] and m is the mass of the gas particle. The interaction strength in a LL is then characterized by the dimensionless parameter $\xi = g/\pi\hbar v_F$, where $m v_F^2/2 = k_B T_F$ defines the Fermi velocity (at trap center). Remarkably, in a trapped gas $\xi \sim (a/l_\perp)(\omega_\perp/\omega N)^{1/2}$ and thus can be tuned externally either by changing the transverse confinement, or by changing the scattering length by magnetic field. The ratio of the transverse and the longitudinal frequencies is quite large, thus we can easily reach the strong coupling limit $\xi \sim 1$, even in a dilute gas.

To study experimentally spin-charge separation, we assume that a short far off-resonant laser pulse is focused at the center of the harmonic trap with a two-component atomic LL (see Fig. 1(a)). Depending on the laser parameters (e.g., light polarization) density and spin wave packets can be excited. They have, in a LL, different propagation speeds (see Fig. 1(b)). This can be probed at a later time with a second short laser pulse. The goal of the following derivations is thus to (i) derive

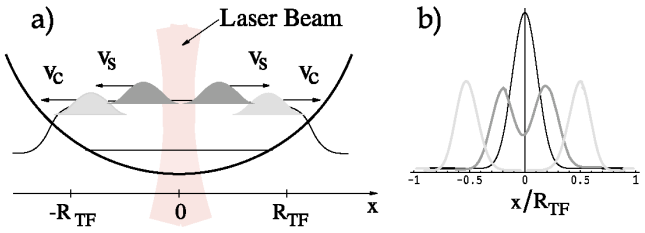

FIG. 1 (color online). (a) A two component quasi-1D Fermi gas in a harmonic potential. At time $t = 0$ a short laser pulse focused near the center of the trap excites a density or a spin wave packet. (b) Wave packet dynamics for different times as a function of position: spin-charge separation manifests itself in a spatial separation of the spin (dark-gray line) and density (pale-gray line) wave packets (shown at half a trap oscillation period $\omega t = \pi/2$), which can be probed by a second short laser pulse at a later time. The parameters correspond to $N = 10^3$ ^{6}Li atoms (with coupling parameter $\xi = 1$, see text).

the frequencies of the spin and charge modes of atomic LL confined in a harmonic trap, and (ii) to discuss the wavepacket dynamics as superposition of these modes.

According to [11] interacting spin-1/2 fermions have a soundlike spectrum. As was shown by Haldane [1] an effective Hamiltonian for gapless 1D quantum liquids which completely describes the behavior at wavelengths much larger than the interparticle spacing in the case of spin-1/2 fermions reads:

$$H = \sum_{\nu=\rho,s} \int dx \frac{\hbar v_\nu}{2} \left[K_\nu \Pi_\nu^2 + \frac{1}{K_\nu} (\partial_x \phi_\nu)^2 \right]. \quad (1)$$

The gradients of the phases $\partial_x \phi_\rho$ and $\partial_x \phi_s$ are proportional to the density and to the spin density fluctuations, respectively. The canonical momenta Π_ν conjugate to the phases ϕ_ν, i.e., $[\phi_\nu(x), \Pi_\mu(x')] = i\delta_{\nu\mu}\delta(x - x')$, are related to the density ($\nu = \rho$) and the spin density currents ($\nu = s$) $j_\nu = v_\nu K_\nu \Pi_\nu$. The parameters K_ν and v_ν completely characterize the low-energy physics. A distinctive feature of the LM Hamiltonian Eq. (1) is the complete separation of the spin and the charge degrees of freedom. In a spatially homogeneous gas the spin and the charge waves propagate at the velocities v_c and v_s, respectively. The K_ν's are related to the low-energy behavior of the correlation functions. For a noninteracting gas $v_\nu = v_F$ and $K_\nu = 1$. For exactly solvable models such as the 1D lattice model, the parameters can be directly expressed in terms of the microscopic parameters of the theory [12]. In a spin rotationally invariant Fermi gas the quantity $K_s = 1$, so that the only independent parameters are K_ρ, and $v_{s,\rho}$ [1,2].

The ground state density of the system can be found within the local density approximation (Thomas-Fermi) [13]

$$\frac{dE(n)}{dn} = \mu - V(x). \quad (2)$$

Here $E(n)$ is the internal energy per unit length of the gas as a function of its (total) density, μ is the chemical potential, $V(x) = m\omega^2 x^2/2$ is the longitudinal external potential, and ω is the frequency of the longitudinal confinement. This equation is just the expression of the fact that the energy cost of adding a particle to the system equals to the chemical potential corrected by the local value of the external potential. Generally speaking, the external potentials acting on the two different spin components can be different.

The local density approach to model a trapped gas assumes that the size of the atom cloud $R \gg k_F^{-1}$, i.e., the size of the gas sample is much larger than the interparticle separation, consistent with $N \gg 1$. The variation of K_ν and v_ν is assumed to originate only from the spatial dependence of the gas density, $K_\nu[n] \to K_\nu[n(x)]$ and $v_\nu[n] \to v_\nu[n(x)]$. We will also assume that the number of particles in the two different internal levels is the same.

The noninteracting case ($g = 0$), has been extensively studied using different approaches [13,14]. The internal energy of the noninteracting gas is just the density of the kinetic energy (the so-called quantum pressure) $E(n) = \hbar^2\pi^2 n^3(x)/24m$. Substituting these expressions into Eq. (2) we find

$$n_{TF}(x) = n_0 \sqrt{1 - \frac{x^2}{R_{TF}^2}}, \quad (3)$$

for $|x| < R_{TF}$, and 0 otherwise. Here $n_0 \equiv n(x = 0) = (8\ \mu m/\hbar^2\pi^2)^{1/2}$ is the density in the center and $R_{TF} = (2\mu/m\omega^2)^{1/2}$ is the Thomas-Fermi size of the cloud [13]. From the requirement that the integrated density equals the particle number we have the condition $\mu = \hbar\omega N/2$.

From the Hamiltonian Eq. (1) and the canonical commutation relations one can derive the equation of motion for the excitations of the gas:

$$\dot{\phi}_\nu = K_\nu(x) v_\nu(x) \Pi_\nu, \qquad \dot{\Pi}_\nu = \frac{\partial}{\partial x} \frac{v_\nu(x)}{K_\nu(x)} \frac{\partial}{\partial x} \phi_\nu. \quad (4)$$

For an ideal gas, from Eq. (4), one gets the solution in the form $\phi_{\nu n} = A_\nu \sin(\omega_{\nu n}/\omega \arccos\tilde{x}) + B_\nu \cos(\omega_{\nu n}/\omega \arccos\tilde{x})$ where $\tilde{x} = x/R_{TF}$. The discrete spectrum of eigenfrequencies is found by analyzing the boundary conditions: $\omega_{\nu n} = \omega(n + 1)$ [14] both for the spin and the density modes. The first mode ($n = 0$) with the eigenfunction $\phi_\nu \sim \sqrt{1 - \tilde{x}^2}$ corresponds to harmonic oscillations of the center of mass of the total density and the total spin (dipole modes).

Before going on, we note that in a finite system there is additional energy scale: the level spacing ($\hbar\omega$). In order for the interaction effects to manifest themselves in a way similar to a bulk system, we need the interaction to be stronger than the level spacing, $n(x)g \gg \hbar\omega$.

In a homogeneous gas the Luttinger parameters in the Hamiltonian (1) to the lowest order in $\xi = g/\pi\hbar v_F \ll 1$ can be found using perturbation theory: $K_s = 1$, $v_s = v_F(1 - \xi/2)$, $K_\rho = 1 - \xi/2$, and $v_\rho = v_F(1 + \xi/2)$ [15]. The energy density of the ground state can be obtained by averaging the interparticle interaction over the ground state, $E_0 = \hbar^2\pi^2 n^3/24m + gn^2/4$. In the spirit of the

local density approximation we substitute a spatially dependent density $n(x)$ in the expressions for homogeneous gas. Then, using Eq. (2), we find that, in the first order in $\xi = g/\pi\hbar v_F(0)$, the density of the gas uniformly decreases by $\delta n(x) = -2gm/\hbar^2\pi^2$, i.e., the interaction reduces the density, as expected. This simple conclusion holds everywhere as long as $g \ll \hbar^2 n(x)/m$, i.e.,

$$(R_{\mathrm{TF}} - x)/R_{\mathrm{TF}} \gtrsim (gm/\hbar^2 n_0)^2 \sim O(\xi^2)\,. \tag{5}$$

The velocities of the spin and the density waves are

$$v_{s,\rho}(x) = \frac{\pi\hbar n_{\mathrm{TF}}(x)}{2m}\left[1 - \frac{A_{s,\rho} gm}{\pi^2\hbar^2 n_{\mathrm{TF}}(x)}\right], \tag{6}$$

where $A_s = 3$ and $A_\rho = 1$. Using the expansion in powers of ξ of the Luttinger parameters, the density profile obtained by Eq. (2) and the equation of motion Eq. (4) we find that the frequency of the density dipole mode does not depend on the interaction (as it should be according to Kohn's theorem), while the spin dipole mode shift is given by an integral logarithmically diverging at the border of the gas cloud. The divergence occurs due to localization of the excitations of a free gas close to the gas cloud border [14] and arises in any potential, which is a power law in x. Using the condition (5) to cut off the divergence, we find

$$\delta\omega_{s1} = -\omega\frac{3gm}{\pi^2\hbar^2 n_{\mathrm{TF}}(0)}\log\frac{\pi\hbar^2 n_{\mathrm{TF}}(0)}{2gm}\,. \tag{7}$$

This shift is negative and can be observed by comparing the spin and the density oscillations of the gas cloud. Note that in a harmonic trap the perturbation theory requirement is stronger than in a homogeneous LL: we have to require $\xi\log(1/\xi) \ll 1$ instead of simply $\xi \ll 1$.

For higher modes the application of the perturbation theory in Eqs. (4) turns inconvenient and the frequencies of the excitations can be analyzed within the WKB approximation. The accuracy of the WKB eigenfrequency estimation is $\sim 1/\pi^2 n^2$ [16], n being the quantum number, whereas the expected corrections are of order $g/\hbar v_F$. Therefore for sufficiently high n the eigenfrequencies can be reliably obtained from the WKB quantization condition,

$$\int_{-x_0}^{x_0} p(x)dx = \hbar\pi(n + \alpha)\,, \tag{8}$$

where $p(x)$ is the WKB momentum corresponding to a given energy, x_0 is the classical turning point, and the constant $\alpha = 1$ is fixed by comparing the WKB results and the exact solutions of Eqs. (4) for a weakly interacting gas. Substituting the dispersion relation $\epsilon = v_{\rho,s}(x)p(x)$ with the velocities given by Eq. (6) into Eq. (8), we obtain the same sort of logarithmically diverging integrals as those for in the perturbation theory above. By regularizing them using condition (5) we find that

$$\epsilon_{\rho,s} = \hbar\omega(n+1)\left[1 - \frac{2gmA_{\rho,s}}{\pi^2\hbar^2 n_{\mathrm{TF}}(0)}\log\left(\frac{\hbar^2 n_{\mathrm{TF}}(0)\pi}{mg}\right)\right]. \tag{9}$$

This simple WKB calculation confirms the interaction dependent split of the spin and the density oscillation frequencies.

In the limit of very large interaction strength ($g \gg \pi\hbar v_F$) the repulsion between the atoms of the two different species is very strong. The analytical results we will present are based on the exact solutions [11]. The density profile coincides with the density of an ideal single component gas [12]. This is still given by Eq. (3), but now with $n_0 = n_\infty = (2\mu m/\hbar^2\pi^2)^{1/2}$, $\mu = \hbar N\omega$, and $R_{\mathrm{TF}} = R_\infty = (2\mu/m\omega^2)^{1/2}$. This distribution is less dense and thus broader than that for a weakly interacting gas. The density wave speed is equal to the Fermi velocity $v_F = \hbar\pi n_\infty/m$ and, after integration in Eq. (8), we find that spectrum of the density waves is the same as in the noninteracting case above. In turn, the relation between the energy and the WKB momentum for the spin wave is given by

$$\epsilon_s(p) = \frac{B\hbar n^2(x)p(x)}{m^2 g}\,, \tag{10}$$

where the coefficient $B \sim 13\ (\pm 2)$ was found numerically. Once again, using the quantization condition (8) and cutting off the logarithmically divergent integral at the point $n \ll g$, we find that

$$\epsilon_{ns} = \hbar\omega(n + \alpha)\frac{\hbar^2 B n_\infty}{gm\log(gm/\hbar^2 n_\infty)}\,, \tag{11}$$

where $\alpha \sim 1$. As it is clear from the latter expression, the interaction profoundly changes the properties of the spin mode. In the limit of the strong interaction the level spacing decreases and is much smaller than that between the density waves (ω).

In order to confirm our analytical results, we performed a numerical calculation valid for arbitrary interaction strength. Using the exact solution [11] for calculation of the Luttinger constants and the Thomas-Fermi approximation (2), we determined the WKB level spacings for the spin and the charge modes. The results are presented in Fig. 2 as a plot of the excitations level spacing vs the dimensionless interaction strength $\xi = g/\pi\hbar v_F$ calculated at the center of the trap.

It is worth mentioning that our results do not rely on specific assumptions about backscattering processes. The phenomenological values of the K and v of our LL model are found from the exact solution of the microscopic model. It is also known, from renormalization group analysis [2], that the backscattering contribution is marginally irrelevant, which is also confirmed by our numerical calculation. The combined effects of backscattering and the finite size of the system were considered in [17].

As outlined in Fig. 1, wave packets of the spin and density excitations can be generated by short off-resonant and state selective laser pulses focused to a spot size ℓ with $R \gg \ell \gg k_F^{-1}$, where R is the size of the atom cloud and k_F^{-1} the interparticle distance. This procedure is

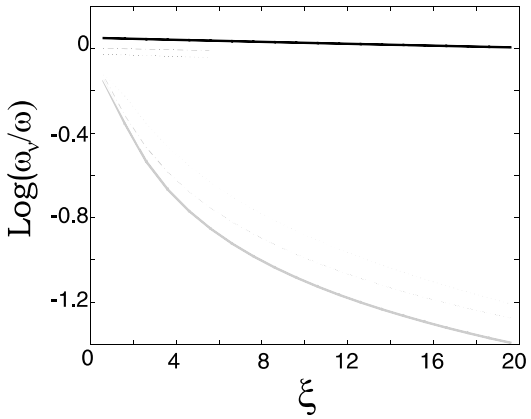

FIG. 2. Level spacing (in units of ω) of the spin (gray curve) and the charge (black curve) modes for a Fermi gas. The level spacing is shown as a function of $\xi = g/\pi\hbar v_F(0)$.

analogous to the MIT setup originally used to study propagation of sound waves in elongated condensates [18]. Figure 1 represents a numerical solution of Eq. (4) and is an example of the wave packet dynamics for the states $|F = 1/2, M_F = \pm 1/2\rangle$ of ^{6}Li with $\xi = 1$. The scattering length is magnetically tunable, positive for $B \geq 550$ G and exhibits a Feshbach resonance at about 800 G [19]. For instance for $a \sim 20$ Å one has $\xi = 1$ with $N = 500$ particles at trap frequencies $\omega = 1$ Hz, $\omega_\perp = 250$ kHz. Near the Feshbach resonance one can use $N = 1000$ atoms at a trap frequency of $\omega_\perp = 100$ kHz.

A promising option is to trap fermionic atoms in optical lattices [20]. Using *bosonic* atoms with very large onsite repulsion is even more attractive, since cold trapped bosons are readily available in many laboratories. The low-energy sector of the system is similar to a spin-1/2 chain, which can be "fermionized" [21] and made formally equivalent to a fermionic LL.

The Hamiltonian (1) represents only the first term in long wave length expansion $q/k_F \ll 1$. The higher order terms originate from, e.g., nonlinearity of the fermionic spectrum and account for the interaction of the excitations with each other. The first corrections are third powers in Π and $\partial\phi$, and hence lead to scattering of the excitations [1]. The damping of the oscillations can be found using Fermi golden rule. A simple estimate [22] shows that the relaxation rate of an excitation with momentum q and energy ω_q is $\Gamma_q \sim (k_B T/\epsilon_F)^2 \omega_q \ll \omega_q$. This means that in a degenerate quasi-1D Fermi gas the excitations are only weakly damped and can be observed for several trap periods.

In conclusion, we performed an analysis of a double component Fermi gas confined in a 1D harmonic trap. Based on the LM we have investigated the nature of the excitations and analyzed an experiment where spin-charge separation can be observed "directly" in experiments addressing the spectral properties of the lowest excitations with laser light.

Discussions with J. I. Cirac, J. von Delft, D. Jaksch, and U. Schollwöck are gratefully acknowledged. Work at Innsbruck supported in part by the A.S.F., EU Networks, and the Institute for Quantum Information. W. Z. and P. Z. thank ITP, UCSB, for hospitality. P. Z. thanks also the MPQ, Garching, for hospitality and support from the Alexander von Humboldt Foundation.

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