

Fractional Quantum Hall Effect of Composite Fermions

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In a GaAs/AlGaAs quantum well of density $1 \times 10^{11} \text{ cm}^{-2}$ we observed a fractional quantum Hall effect (FQHE) at $\nu = 4/11$ and $5/13$, and weaker states at $\nu = 6/17$, $4/13$, $5/17$, and $7/11$. These sequences of fractions do not fit into the standard series of integral quantum Hall effects of composite fermions (CF) at $\nu = p/(2mp \pm 1)$. They rather can be regarded as the FQHE of CFs attesting to residual interactions between these composite particles. In tilted magnetic fields the $\nu = 4/11$ state remains unchanged, strongly suggesting it to be spin polarized. The weak $\nu = 7/11$ state vanishes quickly with tilt.

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The composite fermion (CF) model [1–6] has been very successful in providing a rationale for the observed sequences of principal fractional quantum Hall effect (FQHE) states at Landau level fillings $\nu = p/(2mp \pm 1)$ ($p, m = 1, 2, 3, \dots$) around major even-denominator fractions, $\nu = 1/2m$. In this model, the dominating electron-electron interaction is very effectively incorporated into the carriers by transforming them into new particles, 2^m CFs, by virtue of the attachment of an even number, $2m$, of magnetic flux quanta. As a consequence, CFs can be treated as independent particles in an effective magnetic field, B_{eff} , which is reduced from the external field, B , by the density of the attached magnetic flux. As B_{eff} deviates from zero, Landau levels of CFs develop, giving rise to an integral quantum Hall effect (IQHE) of these flux-transformed, noninteracting composite particles. This IQHE of CFs in the effective magnetic field becomes equivalent to the FQHE of the original, highly interacting electrons exposed to the full external field. Experiments in the FQHE regime closely follow the sequence of proposed states according to this model. Prime examples for the applicability of this model are the sequences of FQHE states at filling factors $\nu = p/(2p \pm 1)$ around $\nu = 1/2$ (made from 2^m CFs) and $\nu = p/(4p \pm 1)$ around $\nu = 1/4$ (made from 4^m CFs) which match, when appropriately shifted, the sequence of IQHE states around $B = 0$.

The question arises as to the ultimate validity of the assumption of vanishing interaction between CFs. Much of the modeling of CF physics requires a mean field approach, whose exact applicability to the conditions at hand is doubtful. Residual interactions between CFs may simply lead to small corrections of the CF properties or, more interestingly, they themselves may create novel electronic states. Experimentally, distortions of the shape of R_{xx} maxima between neighboring FQHE states (or IQHE states of CFs) have hinted in the past towards residual CF-CF interactions [7]. In this paper we present

extensive experimental evidence for the considerable strength of these interactions by observing the appearance of FQHE states at filling factors $\nu = 4/11$, $5/13$, $7/11$, $4/13$, $6/17$, and $5/17$, located between the minima of the primary FQHE sequences. Regarding the primary sequences as the IQHE of CFs, the new states can be viewed as the FQHE of CFs, brought about by CF-CF interactions. From angular dependent measurements we can deduce the spin polarization of the strong $\nu = 4/11$ state and find it to be spin polarized in our specimen. Its much weaker, electron-hole symmetric state at $\nu = 7/11$ rapidly disappears under tilt leaving its spin polarization uncertain.

The sample consists of a 500 \AA wide modulation-doped GaAs/AlGaAs quantum well and has a size of about $5 \text{ mm} \times 5 \text{ mm}$. The well is δ -doped with silicon from both sides at a distance of $\sim 2200 \text{ \AA}$. Electrical contacts to the two-dimensional electron system (2DES) are accomplished by rapid thermal annealing of indium beads along the edge. An electron density of $n \sim 1.0 \times 10^{11} \text{ cm}^{-2}$ and a mobility of $\mu \sim 10 \times 10^6 \text{ cm}^2/\text{Vs}$ were achieved after illumination of the sample at low temperatures by a red light-emitting diode. A self-consistent calculation shows that at this density only one electrical subband is occupied, consistent with the low-field Shubnikov–de Haas data. Conventional low frequency ($\sim 7 \text{ Hz}$) lock-in amplifier techniques were employed to measure the magnetoresistance R_{xx} and Hall resistance R_{xy} .

Figure 1 shows an experimental trace of R_{xx} in the regime of $2/3 > \nu > 2/7$, taken at $T \sim 35 \text{ mK}$. The very low and very high field data are omitted to emphasize the field range central to this study. Several outstanding features are apparent in Fig. 1: (1) Very high denominator FQHE states at $\nu = 10/19$ and $\nu = 10/21$ appear around $\nu = 1/2$, attesting to the exceptionally high quality of the specimen. (2) Slight undulations, previously observed between $\nu = 1/3$ and $2/5$ [7], are

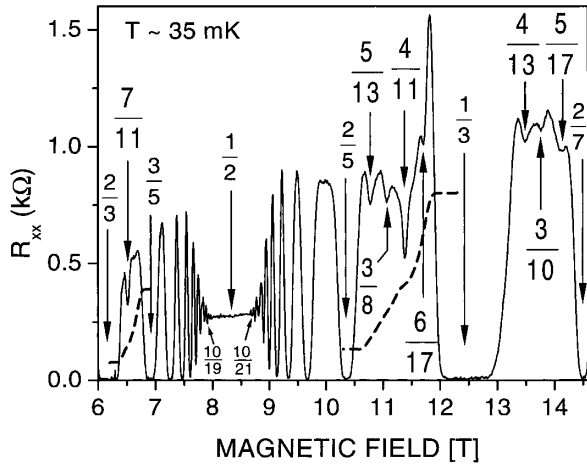


FIG. 1. R_{xx} in the regime $2/3 > \nu > 2/7$ at $T \sim 35$ mK. Major fractions are marked by arrows. Dashed traces are the Hall resistance R_{xy} around $\nu = 7/11$ and $\nu = 4/11$.

resolved into clear R_{xx} minima at $\nu = 5/13, 3/8, 4/11$, and $6/17$. The minimum at $\nu = 4/11$ is particularly strong. (3) R_{xx} minima are even observed at fractions $\nu = 4/13, 3/10$, and $5/17$ between $\nu = 1/3$ and $2/7$. Between $\nu = 2/3$ and $3/5$, a minimum appears at $\nu = 7/11$ [8] and a weaker feature around $\nu = 5/8$. (4) A clear slope change is apparent in R_{xy} (dashed lines) at $\nu = 4/11$ and $7/11$, although, as in the early stages of many developing fractions, the quantization value remains poorly defined.

The sum of new fractions, appearing between traditional FQHE states, can be understood naively as the FQHE of CFs. In terms of 2^m CF Landau level filling factor, ν_{2^m} , the states at $\nu = 4/11, 5/13, 6/17$ reside between the consecutive minima of the $\nu_2 = 1$ and $\nu_2 = 2$ IQHE of 2 CFs as counted from $B_{\text{eff}} = 0$ at $\nu = 1/2$. In the same spirit, the $\nu = 4/13$ and $5/17$ reside between the $\nu_4 = 1$ and $\nu_4 = 2$ IQHE of 4 CFs as counted from $B_{\text{eff}} = 0$ at $\nu = 1/4$. At $\nu = 4/11$ the lowest 2 CF Landau level, $\nu_2 = 1$, is completely filled, whereas the second 2 CF Landau level is filled to $1/3$ of its 2 CF capacity. Hence $\nu = 4/11$, in terms of electrons, corresponds to $\nu_2 = 1 + 1/3$ in terms of 2 CFs, which, therefore, can be regarded as the $1 + 1/3$ FQHE of 2 CFs. The correspondence of the other fractions is as follows: $\nu = 5/13 \rightarrow \nu_2 = 1 + 2/3$, $\nu = 6/17 \rightarrow \nu_2 = 1 + 1/5$, $\nu = 3/8 \rightarrow \nu_2 = 1 + 1/2$, $\nu = 4/13 \rightarrow \nu_4 = 1 + 2/3$, $\nu = 5/17 \rightarrow \nu_4 = 1 + 1/3$, $\nu = 3/10 \rightarrow \nu_4 = 1 + 1/2$, $\nu = 7/11 \rightarrow \nu_2 = 1 + 1/3$, $\nu = 5/8 \rightarrow \nu_2 = 1 + 1/2$. There are also hints in the data for further features between $\nu = 2/5$ and $\nu = 3/7$ as well as between $\nu = 2/9$ and $\nu = 1/5$. Furthermore, we have observed similar deformation in R_{xx} traces of other, higher density samples (not shown) between $\nu = 3/5$ and $\nu = 4/7$, between $\nu = 2/3$ and $\nu = 5/7$, and between $\nu = 1 + 1/3$ and $\nu = 1 + 2/5$, as well as between $\nu = 1 + 2/7$ and $\nu = 1 + 1/3$ [9], hinting towards a continuation of FQHE states of CFs to higher CF Landau levels.

The relative strength of these features resembles the relative strength of the electron FQHE and the progression of their discovery [10,11]. A self-similarity seems to be at work in which the pattern of FQHE features, initially observed in electrons, is now observed in CFs and may progress further to higher-order CFs in yet lower disorder samples. Of course, one may already regard the sequence of FQHE state at $\nu = p/(4p \pm 1)$ as the FQHE of 2 CFs, since their lowest 2 CF Landau level is only partially occupied. However, the situation is equally well, and more naturally, described as the IQHE of 4 CF, emanating from $\nu = 1/4$ [4]. Instead of two flux quanta, each electron is now carrying four flux quanta.

For the new sequences the hypothetical flux attachment process would be much more intricate. For example, the $\nu = 4/11$ state is created by the following mental sequence. Two flux quanta attach themselves to each electron forming 2 CFs, which, at $\nu = 1/2$, form a Fermi sea with $B_{\text{eff}} = 0$. At $\nu = 1/3$, the lowest 2 CF Landau level created by the now finite B_{eff} has reached a degeneracy sufficient to accept all 2 CFs and the $\nu_2 = 1$ IQHE of 2 CF occurs. As B_{eff} is reduced to $1/2$ of its strength, all 2 CFs fill exactly two 2 CF Landau levels and the $\nu_2 = 2$ IQHE of 2 CFs occurs (equivalent to $\nu = 2/5$). At $\nu = 3/8$, equivalent to $\nu_2 = 3/2$ the lowest 2 CF Landau level is totally occupied, whereas the second 2 CF Landau level is occupied only to $1/2$ of its 2 CF capacity, assuming total spin polarization. At this stage, two flux quanta attach themselves to the 2 CFs in the higher 2 CF Landau level. The lower one, being completely full, can be ignored. In total, $1/3$ of all CFs have become 4 CFs, whereas $2/3$ remained 2 CFs [12]. This is a rather intricate situation, in which every three electrons carry eight flux quanta. It exactly cancels out the external B field at $\nu = 3/8$ and $B_{\text{eff}} = 0$, again [4]. As B moves toward $\nu = 4/11$ the rising B_{eff} creates 4 CF Landau levels with a degeneracy sufficient to accept all 4 CFs into the lowest 4 CF Landau level and a $\nu_4 = 1$ IQHE of 4 CFs occurs. This would be the $\nu = 4/11$ FQHE state.

The other observed fractions can be derived in an equivalent fashion. The $\nu = 6/17$ requires $1/5$ 6 CFs and $4/5$ 2 CFs. The states around $\nu = 3/10$ requires $2/3$ 2 CFs and $1/3$ 4 CFs. In all cases we have assumed complete spin polarization, as one may expect at such high fields. If this were indeed the case, then the $\nu = 3/8$ (equivalent to $\nu_2 = 3/2$) would, in fact, be equivalent to the $\nu = 5/2$ electron state, since it occupies the second 2 CF Landau level and not the upper spin state of the lowest Landau level as is the case for electrons. This may explain the observation of a minimum at $\nu = 3/8$. At $\nu = 5/2$ electrons show a FQHE, believed to originate from paired 2 CFs. At $\nu = 3/8$ the 4 CFs of the Fermi liquid may pair and form a paired state of 4 CFs [13]. Furthermore, the suppression in R_{xx} observed around $\nu = 5/12$ between $\nu = 2/5$ and $\nu = 3/7$, equivalent to $\nu = 5/2$, would, in fact, be equivalent to the anisotropic electron state at $\nu =$

9/2 [14,15], since all spins are polarized due to the large external B field.

Beyond the qualitative demonstration of this apparent self-similarity in the FQHE sequences we have also performed quantitative studies on the stronger of the fractional states. Figure 2 summarizes the T dependences for the states at $\nu = 4/11$ and $7/11$ states. Three temperature traces are shown in Fig. 2(a) and four in Fig. 2(b). Unlike the well-developed FQHE states, R_{xx} at exactly $\nu = 4/11$ barely changes with temperature. On the other hand, the strength of the whole $\nu = 4/11$ feature decreases markedly with increasing temperature. Such a T dependence is reminiscent of the initial T dependence of many FQHE states. In analogy to earlier procedures [16,17], we deduce the gap energy of the $\nu = 4/11$ state of approximately 30–50 mK from the strength of its minimum. The enormous change of shape of the data around $\nu = 7/11$ leads to an erratic T dependence and no effective energy scale can be deduced.

In Fig. 3 we address the spin polarization of the strongest state. At $B = 11$ T the Zeeman energy of electrons in GaAs is ~ 3 K. These energies are vastly larger than the characteristic energies (several mK) extracted from Fig. 2(a). Already at this point it appears highly unlikely that $\nu = 4/11$ is spin unpolarized. The ground state energy and Zeeman energy would have to balance each other closely in order to show such small gap energies for the states. To further strengthen this conjecture we measured R_{xx} , *in situ*, as a function of tilted magnetic field as shown in Fig. 3 for the $\nu = 4/11$ minima and several representative angles, θ [18]. For an ideal 2DES the correlation energy remains unchanged under tilt, while the Zeeman energy increases as $1/\cos(\theta)$, rising to ~ 4.5 K at $\theta \sim 42^\circ$ for the $\nu = 4/11$ state. The huge differential increase of ~ 1.5 K over the Zeeman energy at $\theta = 0^\circ$ should move any possible close balance of correlation energy and Zeeman energy at $\theta = 0^\circ$, vastly in favor of the latter at $\theta \sim 42^\circ$ and lead to a transition in the spin polarization. Any such macroscopic change of the

spin property between $\theta = 0^\circ$ and $\theta \sim 42^\circ$ would be visible as a vanishing or strongly reduced strength of the $\nu = 4/11$ state. The data of Fig. 3, for tilts up to $\theta = 42.2^\circ$, show practically no variation in the strength nor the shape of the $\nu = 4/11$ state, from which we infer that no spin transition occurs. Therefore, the $\nu = 4/11$ state in our sample must be spin polarized for all angles. Even the feature at $\nu = 5/13$ shows no angular dependence. Although this fraction is not very well developed, this lack of variation suggests this state to be also spin polarized.

We also measured the angular dependence of the $\nu = 7/11$ state at base temperature (not shown). In contrast to the $\nu = 4/11$ state, the features of the $\nu = 7/11$ state change considerably under tilt and the minimum seems to have disappeared entirely by $\theta = 29.5^\circ$. This points to a spin transition in the $\nu = 7/11$ state and hence a spin unpolarized or, at least, partially polarized state at $\theta = 0^\circ$. However, as in previous tilt data on the $\nu = 5/2$ state one cannot rule out that it is an orbital effect that is destroying the $\nu = 7/11$ gap. We therefore refrain from assigning any spin polarization to the $\nu = 7/11$ state at this time.

The drawing of analogies, such as those presented above, about the continuations of the CF picture to higher orders in a self-similar scheme is rather simple. However, theoretical calculations as to the stability of such higher-order FQHE states are very difficult, since they require many particles to treat the inherent correlations realistically. Consequently, the theoretical situation regarding such states remains in flux. The early hierarchical model of the FQHE [19,20] obviously allows for the existence of all of our observed states, since it covers all odd-denominator fractions [21]. However, several of the newly discovered states are expected not to be stable [22]. A paper by Wójs and Quinn studies quasiparticle interactions in the FQHE regime and compares different hierarchies [23]. It finds the $\nu = 6/17$ state to be possibly stable, but the $\nu = 4/11$ and $\nu = 4/13$ states to be definitely unstable within their pseudopotential classification

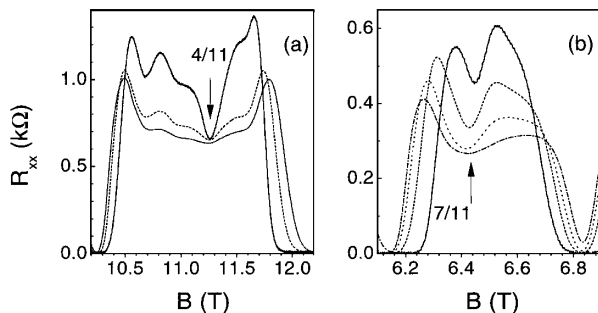


FIG. 2. (a) T dependence of R_{xx} around $\nu = 4/11$. Three temperature traces are shown: solid line, 35 mK; dashed line, 70 mK; dotted line, 95 mK. (b) T dependence of R_{xx} around $\nu = 7/11$. Four temperature traces are shown: solid line, 40 mK; dashed line, 70 mK; dotted line, 105 mK; dash-dotted line, 185 mK.

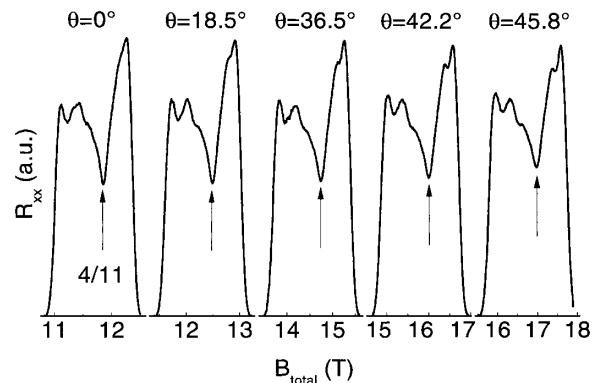


FIG. 3. R_{xx} between $2/5 > \nu > 1/3$ at five selected tilt angles. A total of 18 tilt angles were actually measured. Position of the $\nu = 4/11$ state is marked by arrow.

approach and an eight-electron exact diagonalization for zero-thickness layers. These studies rely on total spin polarization and the apparent absence of an incompressible state at $\nu = 4/11$ persuaded Park and Jain [12] to investigate partially polarized states at $\nu = 4/11$, for which they find, indeed, a small energy gap. However, both investigations seem to conflict with our experimental result of the existence of a spin polarized state at $\nu = 4/11$. A very recent preprint by Mandal and Jain [24] tests for the existence of higher-order CF states via a new numerical scheme, which neglects CF Landau level mixing, but can handle as many as 24 electrons. Surprisingly, this approach generates no condensed state at any of the higher-order states we observe. The study concludes that “the physical mechanism, in which some of the CFs turn into higher-order CFs and condense into new Landau levels to exhibit a QHE, does not appear to be relevant for fully polarized electrons.” This would negate a simple, self-similar model for CFs as we have proposed on the basis of the newly observed sequences of FQHE states. On the one hand, this conflict may arise from some fundamentally important aspect of the interaction, which has been omitted in the numerics. On the other hand, it may arise from the neglect of more subtle experimental realities such as finite thickness of the 2DES or mixing between Landau levels. Yet it appears unlikely that the stability of all sequences of observed higher-order states would be a finite-thickness effect. In fact, we have observed the strong $\nu = 4/11$ state and weaker reflection of the other states also in a triangular well and in square wells of thicknesses from 30 to 60 nm. This suggests a large independence of the stability of such states from the confining potential.

At this stage, the origin of the minima observed in R_{xx} at $\nu = 4/11, 5/13, 6/17, 4/13, 5/17,$ and $7/11$ is unresolved. Numerical calculations seem to exclude the existence of incompressible states at such filling factors for spin polarized systems, at least for zero-thickness systems. Yet angular dependent measurement on the $\nu = 4/11$ state and the existence of any such state, in spite of large Zeeman energies compared to their characteristic energies, suggests all of them to be fully spin polarized. This clearly points to a lack of our understanding of these new features. Furthermore, the simple-minded and intuitive continuation of the CF flux attachment scheme to partially filled CF Landau levels and the analogies that can be drawn from the sequential observation of the principal FQHE sequence to the observation of the new fractions seem to work so well that one is tempted to accept their fundamental appropriateness. The same naive analogy thinking would also provide a rationale for the existence of minima at $\nu = 3/8$ and $\nu = 3/10$, since those would be equivalent to the $\nu = 5/2$ FQHE state. Even the relative strength of the $\nu = 6/17$ state finds its similarity in the state at the $\nu = 11/5$. The aggregate of our experimental observations highly suggests a continuation of the CF model to higher orders, or, equivalently,

the existence of a FQHE of CFs. Such a self-similarity in the sequence of FQHE states is too appealing to be discarded yet.

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