Taking Advantage of Multiple Scattering to Communicate with Time-Reversal Antennas

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We present an experimental demonstration showing that, contrary to first intuition, the more scattering a mesoscopic medium is, the more information can be conveyed through it. We used a multiple input—multiple output configuration: a multichannel ultrasonic time-reversal antenna is used to transmit random series of bits simultaneously to different receivers which were only a few wavelengths apart. Whereas the transmission is free of error when multiple scattering occurs in the propagation medium, the error rate is huge in a homogeneous medium.

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The ever-growing need for faster wireless communication in urban areas has brought to light interesting connections between issues of communication engineering (how can one optimize the data transfer rate between N antennas and M receivers?) and mesoscopic wave physics. Indeed, both subjects have to do with the correlations (in space, time, and frequency) of a diffuse wave field propagating in a random medium.

When waves propagate through a disordered medium, the key element is the ability of a communication system to exploit independent "channels" of propagation [1]; very roughly, the more heterogeneous and scattering the medium is, the more degrees of freedom there are to communicate through it. In the case of radio signals, for instance, multiple paths arise because of scattering and multiple reverberation on the buildings or indoors; similar multipath phenomena are also frequently encountered in underwater acoustics. For radio signals, scattering permits one to exploit different polarizations of the electromagnetic waves to convey more information than in free space: Andrews *et al.* [2] showed that the data transfer rate in a scattering environment could be increased 6 times, as long as the six components of the electromagnetic field are uncorrelated with each other, which therefore yields six independent channels. This approach is based on the *polarization decorrelation*. Another approach is to exploit *spatial decorrelation*: with N antennas transmitting to M receivers, with $N > M$, Shannon's capacity (i.e., the maximum amount of information conveyed without any error) can be N times higher than with a single-transmitter and single-receiver scheme [1,3,4].

We demonstrate here the efficiency of another approach based not only on spatial decorrelation but also on frequency decorrelation due to the randomness of the medium. Indeed, unlike what happens in a homogeneous medium (free space), the response of a highly scattering medium can change dramatically even if the frequency is changed by a small amount $\delta\omega$. This opens up another way to exploit the variety of communication channels in a scattering medium using time reversal. To that end, the bandwidth $\Delta\omega$ has to be as large as possible compared to the correlation frequency $\delta\omega$. Moreover, the waveforms that are sent or received must be completely controllable both in amplitude and phase all along the bandwidth, in order to perform a coherent treatment. For the time being, in the gigahertz domain, electromagnetic antennas cannot achieve this. But our experimental demonstration uses ultrasonic waves, for which both conditions are met.

The propagation medium we considered in the experiments is deliberately disordered and highly scattering: it is a forest of parallel steel rods with density 18.75 cm^{-2} and diameter 0.8 mm ($\sim$ 1.7 times the wavelength). The typical distance between two scattering events is measured by the mean free path ℓ. The sample thickness is $L = 40$ mm, much larger than ℓ which was found to be 4 mm [5]. As a consequence, when a short ultrasonic pulse (typically 1 μs) is sent, the transmitted waveforms received on the other side of the slab last several hundreds of times the initial pulse duration (Fig. 1).

The experiment takes place in a water tank, and we try to communicate to five different receivers with a 23-element array (Fig. 1) at a 3.2-MHz central frequency.

One simple way to address simultaneously different receivers is time-reversal (TR) focusing [6]. In a first step, the five receivers fire a 1-μs pulse one by one; five sets of $N = 23$ signals $h_{ij}(t)$ are recorded on the array and digitized. If the array sends back the time-reversed signals $h_{3j}(-t)$ $1 \leq j \leq N$, then the wave will focus back to the receiver No. 3 and recover its initial duration, as if it were traveling backwards in time. Since the system is linear, it is also possible to send back any combination of the $h_{ij}(-t)$; for instance, in order to transmit a series of positive and negative impulses such as $\{+1 - 1 - 1 + 1 - 1\}$, one has to send back $h_{1j}(-t) - h_{2j}(-t) - h_{3j}(-t) + h_{4j}(-t) - h_{5j}(-t)$, $1 \leq j \leq N$. Then five short pulses will arrive simultaneously to the five receivers (Fig. 2). If one wants to transmit five different series of K pulses with various signs, the signal to be sent back by the jth element is $\sum_{k=1}^{K}\sum_{i=1}^{5} a_{ik}h_{ij}(-t + kT)$, with a_{ik} the amplitude of the kth pulse to be transmitted to the ith receiver and T the delay between two pulses.

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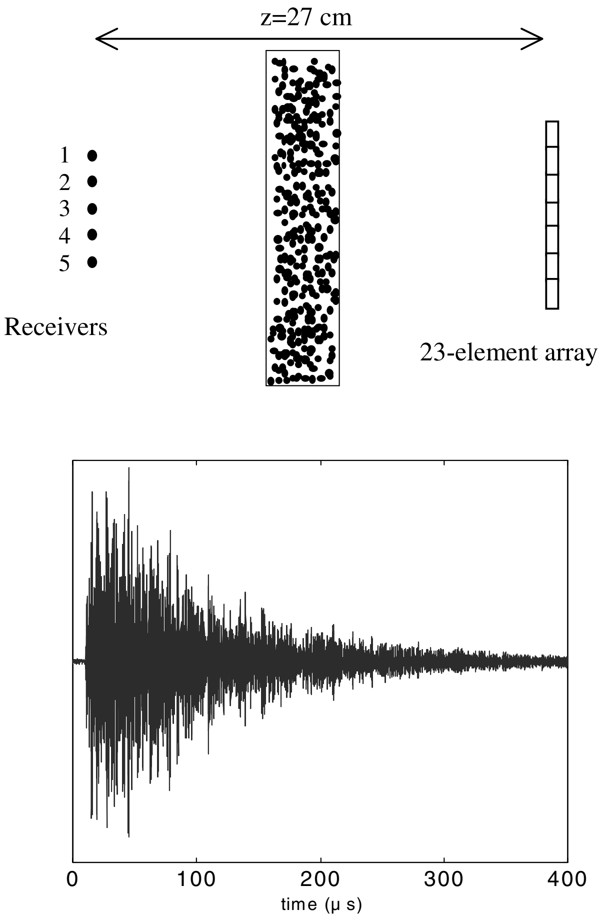

FIG. 1. Top: Experimental setup. An ultrasonic array (central frequency 3.2 MHz, pitch 0.4 mm, total aperture 9.2 mm) communicates to five points simultaneously through a multiple scattering medium. The distance between neighboring receivers is 2 mm. Bottom: typical waveform scattered by the medium when a 1-μs pulse is sent through it by the array to the receivers.

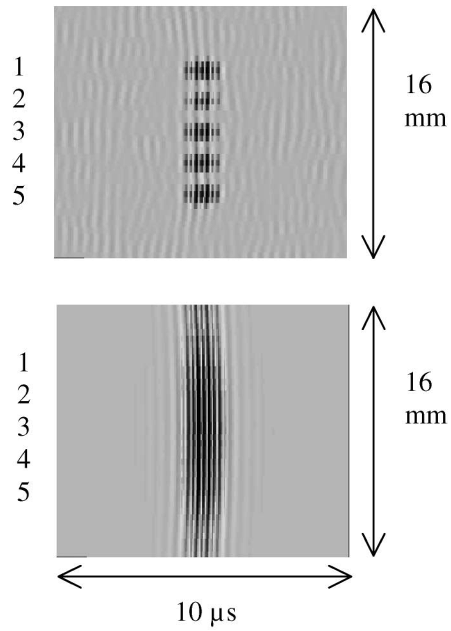

FIG. 2. The five sets of twenty-three impulse responses $h_{ij}(t)$ $1 \leq i \leq 5$, $1 \leq j \leq 23$ have been recorded and time reversed. The linear combination is retransmitted by the array, and the resulting waves travel backwards to the receivers. Through a multiple scattering medium (top) we observe five short pulses very well focused on each receiver. Through water, the pulses overlap, hence a strong cross talk between the receivers.

We used this approach to transmit simultaneously five random and uncorrelated series of $K = 2000$ bits each to the five receivers, with $N = 23$ transmitters. The result was that all bits were correctly transmitted through the scattering medium, whereas the error rate was 28% through water. The reason for these errors is cross talk between the receivers. Indeed, in a homogeneous medium such as water, the resolution of the array is diffraction limited: the array size is $D = 9.2$ mm, the wavelength $\lambda \sim 0.47$ mm, and the distance $z = 27$ cm, so $\lambda z/D \sim 13.8$ mm (Fig. 3). Since the receivers are only 2 mm apart, the messages overlap. On the contrary, it was shown [7,8] that due to multiple scattering, a finite-size TR array manages to focus a pulse back to the source with a spatial resolution that beats by far the diffraction limit in the homogeneous medium: it is limited only by the correlation length of the scattered field and no longer by the array aperture; as the wave is completely scrambled by multiple scattering and loses its coherence, the correlation length of the scattered waves is of the order of the wavelength and so is the spatial resolution of the focal spot. As to the peak-to-noise ratio of the focal spots, it depends obviously on the number of receivers since each of them receives its bitstream, plus noise due to the bitstreams going to all the others and on the number of transmitting elements. It also highly depends on the bandwidth, as we now argue.

To simplify the discussion, let us consider a situation where only one transmitter is used, instead of an array, to focus on some receiver. In this case, TR focusing can still be achieved (Fig. 3). Here the importance of *frequency decorrelation* must be emphasized [7]. Imagine a single element trying to focus on the same receiver but in a narrow frequency bandwidth; the phase-conjugated wave has no reason at all to be focused on this receiver since the element sends back only a sinusoidal spherical wave through the medium. But if the frequency bandwidth $\Delta\omega$ is much larger than the correlation frequency $\delta\omega$, then the spectral components of the scattered field at two frequencies apart by more than $\delta\omega$ are decorrelated and there are roughly $\Delta\omega/\delta\omega$ decorrelated frequencies in the scattered signals. When we time reverse (i.e., phase conjugate *coherently* all along the bandwidth, and not just at one frequency) all these components, they add up in phase at the receiver position, because all the phases have been

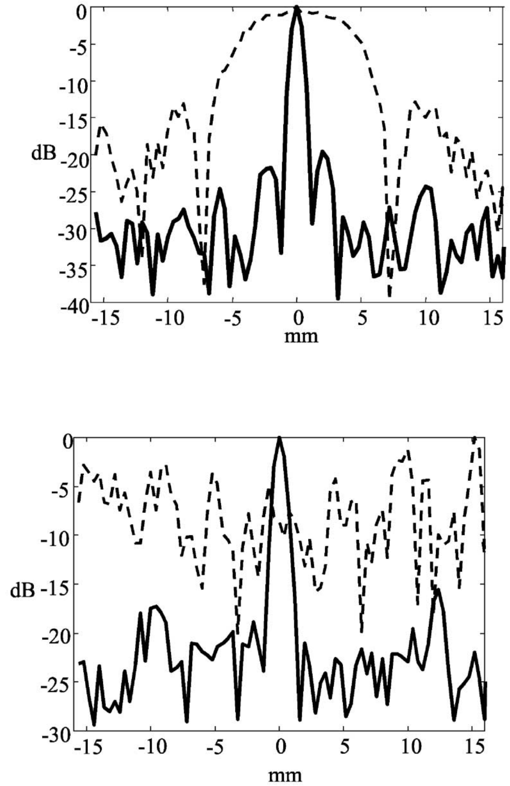

FIG. 3. Directivity of the time-reversed waves around the desired focal point. Top: Comparison between the multiple scattering medium (solid line) and water (dashed line), for a 23-element array; the -6 dB widths are 1.1 mm and 10.3 mm. Bottom: Comparison between quasimonochromatic (3.2 MHz, dashed line) and wideband (solid line) focusing through the multiple scattering medium, for a single element.

set back to 0 all along the bandwidth. Thus, the amplitude at this position increases as $\Delta\omega/\delta\omega$, whereas outside the receiver position, the various frequency components add up incoherently and their sum rises as $\sqrt{\Delta\omega/\delta\omega}$. On the whole, the peak-to-noise ratio increases as $\sqrt{\Delta\omega/\delta\omega}$ as the bandwidth is enlarged. Through the forest of rods, we found $\delta\omega \sim 10$ kHz; the total bandwidth at half maximum is 1.5 MHz, the frequency ratio is therefore $\sim$150, thus an improvement of more than 20 dB compared to a monochromatic phase conjugation technique.

Therefore, the data transfer rate of a TR antenna highly depends both on the ratio $\Delta\omega/\delta\omega$, giving the number of uncorrelated frequencies, and on the number of independent focal spots it can generate in the receivers plane at one frequency. In a homogeneous medium [9] this number equals the rank of the propagation matrix that appears in Shannon's capacity, which we now discuss.

In the case of a one-to-one communication scheme, Shannon [10] proved the following theorem in 1948: if a receiver receives a complex-valued signal with a zero-mean Gaussian distribution (variance S) plus a complex noise with a zero-mean Gaussian distribution (variance N), then the maximum number of error-free bits that can be decoded is $\log_2(1 + S/N)$. This number is expressed in bits per second and per Hertz, indicating that it grows linearly with $\Delta\omega T$, with $\Delta\omega$ the frequency bandwidth and T the duration of the transmission. Shannon's formula was recently generalized [1] to the case of N transmitters and M receivers: if the signals transmitted by each element are uncorrelated, Shannon's capacity is given by $C = \log_2(\det[I + \rho\, {}^tH^*H])$ bits/s/Hz, with ρ the signal-to-noise ratio, I the identity matrix, and H the $N \times M$ propagation matrix. H is the Fourier transform of the point-to-point impulse responses $h_{ij}(t)$. Interestingly, ${}^tH^*H$ is the time-reversal operator [9] which would describe a TR sequence between N transmitters and M receivers. Indeed, imagine that the N transmitters send N waveforms $e_i(t)$ ($i = 1, \ldots, N$), which can be described in the Fourier domain by a set of vectors $\mathbf{E}$ with N components for each frequency. The M-component received "vectors" can be written as a matrix product $\mathbf{HE}$. When the signals are time reversed (i.e., phase conjugated in the Fourier domain) and retransmitted, the resulting vector is ${}^t\mathbf{H}\mathbf{H}^*\mathbf{E}^*$. At this step, it is worth performing a singular value decomposition of H to get a more physical insight into the capacity: H can be written as $UD{}^tV^*$, where U and V are unitary matrices and D is a diagonal matrix whose elements are the singular values λ_i. Thus, ${}^tH^*H = VD^2\,{}^tV^*$ which means that the eigenvalues of the time-reversal operator are the squared singular values of the propagation matrix. Then C can be rewritten as $\log_2(\det[I + \rho VD^2\,{}^tV^*]) = \sum_{i=1}^{\min(M,N)} \log_2(1 + \rho\lambda_i^2)$. This formula implies that each significant eigenvalue of the time-reversal operator adds an independent "channel" of propagation which contributes to increase the capacity.

With ultrasonic devices, the propagation matrix H can be easily measured. To that end, we have used two 40-element 0.4 mm-pitch arrays at a distance $z = 27$ cm and measured the 1600 interelement impulse responses $h_{ij}(t)$ through water and through the multiple scattering sample described above. After a Fourier transform of $h_{ij}(t)$, H is known for a whole set of frequencies. For each frequency, we applied a singular value decomposition. The singular values of H are represented on Fig. 4; through the scattering medium, the number of singular values is much higher than through water and H has a higher rank. Physically, this increase is related to the possibility of talking to different receivers by focusing on them with a TR array. The number of significant singular values (or "degrees of freedom") is roughly the number of independent receivers we can talk to through the medium in a given region of space and at a given frequency. However, Shannon's formula is essentially monochromatic. In a wide-band coherent technique such as TR, the whole bandwidth must be taken into account. Then the total number of degrees of freedom is roughly the number of significant singular values at the center frequency multiplied by frequency ratio $\Delta\omega/\delta\omega$.

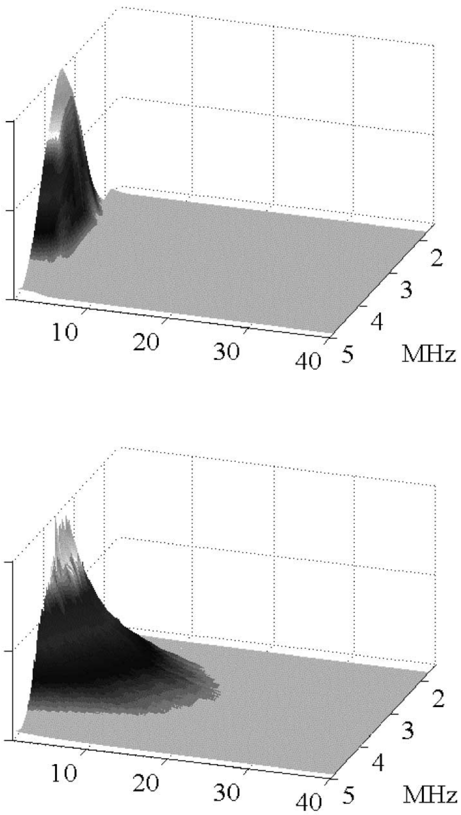

FIG. 4. Singular values of the propagation operator H through water (top) and through the multiple scattering medium (bottom). At the central frequency (3.2 MHz), there are 34 singular values for the scattering medium and only 6 through water (with a -32 dB threshold relatively to the first singular value).

Our experimental results demonstrate that high-order scattering in a disordered medium [11] can help by increasing the information transfer rate, especially if the time-reversal technique is used to naturally focus the different bitstreams onto the receivers. The first key parameter in that experiment is the number of independent focal spots that can be created by the transmitting array in the receiving plane, which is also the number of different receivers one can address simultaneously. At a given frequency, this number is directly related to the number of significant singular values of the propagation matrix or the number of eigenvalues of the time-reversal operator, which are basically the same. In the ultrasonic range, the time-reversal operator which is studied in mesoscopic physics and whose trace gives the conductance [12] can be easily measured. The second key parameter is the number of uncorrelated frequencies within the bandwidth, which governs the peak-to-noise ratio on each receiver; in the medium we studied, we have $\Delta\omega/\delta\omega \sim 150$. Could these ideas be applied to radio signals? Having complete control of the field amplitude and phase over a very large frequency band is possible in everyday laboratory life for ultrasonic waves as well as in ocean acoustics [13], but for the time being, applying wide-band coherent techniques such as time reversal to electromagnetic waves remains a technical challenge.

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