Decoherence-Induced Continuous Pointer States

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We investigate the reduced dynamics in the Markovian approximation of an infinite quantum spin system linearly coupled to a phonon field at positive temperature. The achieved diagonalization leads to a selection of the continuous family of pointer states corresponding to a configuration space of the one-dimensional Ising model. Such a family provides a mathematical description of an apparatus with continuous readings.

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Decoherence is a process of continuous measurement-like interactions between a quantum system and its environment which results in limiting the validity of the superposition principle in the Hilbert space of the system [1]. It accepts the wave function description of the combined state of the system and its environment but contends that it is practically impossible to distinguish it from the corresponding statistical mixture. In other words, the environment destroys the vast majority of the superpositions quickly, and, in the case of macroscopic objects, almost instantaneously. It was shown that decoherence is a universal short time phenomenon independent of the character of the system and the reservoir [2].

In recent years decoherence has been widely discussed and accepted as a mechanism responsible for the emergence of classicality in quantum open systems [3–5]. A particular aspect of decoherence is the selection of the preferred basis of pointer states [6] which occurs when the reduced density matrix of the system becomes approximately diagonal in time much shorter than the relaxation time. In practical situations this results in the disappearance of nondiagonal elements in the reduced density matrix. Hence, by definition, pointer states do not evolve at all, while all other states deteriorate in time to classical probability distributions over the one-dimensional projections corresponding to these states. However, it should be pointed out that the algebra generated by these projections is always of a discrete type, and, as was shown in [7], the discreteness is unavoidable as long as we consider quantum systems with a finite number of degrees of freedom. Such a situation is clearly unsatisfactory since there are quantum measurements with continuous outcomes. In this Letter we demonstrate by a completely solvable model that "openness" of a macroscopic measuring device, regarded as a quantum system in the thermodynamic limit, yields continuous pointer states. By continuous pointer states we understand an uncountable family of commuting and dynamically invariant projections which contains no minimal projections, and such that any observable of the apparatus evolves towards the Abelian algebra generated by these projections.

The model is the following (we shall work in the Heisenberg picture). The apparatus is a semi-infinite linear array of spin-$\frac{1}{2}$ particles, fixed at positions $n = 1, 2, \ldots$. Such a model of the apparatus was considered also by Bell [8] in connection with the wave packet reduction. Since we neglect the position variables of the spin particles the algebra $\mathcal{M}$ of (bounded) observables is given by the σ-weak closure of $\pi(\otimes_1^\infty M_{2\times 2})$, where π is a (faithful) Gelfand-Naimark-Segal representation with respect to a tracial state tr on the Glimm algebra $\otimes_1^\infty M_{2\times 2}$, and $M_{2\times 2}$ is the algebra generated by Pauli matrices. Since there is no free evolution of the apparatus, $H_A = 0$. The reservoir is chosen to consist of noninteracting phonons of an infinitely extended one-dimensional harmonic crystal at the inverse temperature $\beta = \frac{1}{kT}$. The Hilbert space $\mathcal{H}$ representing pure states of a single phonon is (in the momentum representation) $\mathcal{H} = L^2(\mathbf{R}, dk)$. A phonon energy operator is given by the dispersion relation $\omega(k) = |k|$ ($\hbar = 1, c = 1$). It follows that the Hilbert space of the reservoir is $\mathcal{F} \otimes \mathcal{F}$, where $\mathcal{F}$ is the symmetric Fock space over $\mathcal{H}$. A phonon field $\phi(f) = \frac{1}{\sqrt{2}}[a^*(f) + a(f)]$, where $a^*(f)$ and $a(f)$ are given by the Araki-Woods representation [9]:

$$a^*(f) = a_F^*((1+\rho)^{1/2}f) \otimes I + I \otimes a_F(\rho^{1/2}\bar{f}), \quad (1)$$

$$a(f) = a_F((1+\rho)^{1/2}f) \otimes I + I \otimes a_F^*(\rho^{1/2}\bar{f}). \quad (2)$$

Here $a_F^*(a_F)$ denotes, respectively, creation (annihilation) operators in the Fock space, and ρ is the thermal equilibrium distribution related to the phonons energy according to the Planck law

$$\rho(k) = \frac{1}{e^{\beta\omega(k)} - 1}. \quad (3)$$

Since the phonons are noninteracting, their dynamics is completely determined by the energy operator

$$H_E = H_0 \otimes I - I \otimes H_0, \quad (4)$$

where $H_0 = d\Gamma(\omega) = \int \omega(k) a_F^*(k) a_F(k) dk$ describes dynamics of the reservoir at zero temperature. The reference state of the reservoir is taken to be a gauge-invariant quasifree thermal state given by

$$\omega_E[a^*(f)a(g)] = \int \rho(k)\bar{g}(k)f(k)dk. \quad (5)$$

Clearly, ω_E is invariant with respect to the free dynamics of the environment.

The Hamiltonian H of the joint system consists of the reservoir term H_E and an interacting part H_I. We assume that the coupling is linear (as in the spin-boson model), i.e., $H_I = \lambda Q \otimes \phi(g)$, where

$$Q = \pi\left(\sum_{n=1}^{\infty} \frac{1}{2^n} \sigma_n^3\right), \quad (6)$$

σ_n^3 is the third Pauli matrix in the nth site, and $\lambda > 0$ is a coupling constant. In real interactions one should also include bilinear terms in the coupling. However, even this simplified model turns out to yield an efficient loss of coherence of the vast majority of the apparatus observables. The factor $\frac{1}{2^n}$ in Eq. (6) reflects the property that interaction between spin particles and the reservoir decreases as $n \to \infty$. Its form was chosen to simplify further calculations. The test function $g(k) = |k|^{1/2}\chi(k)$, where $\chi(k)$ is an even and real valued function such that (i) χ is differentiable with bounded derivative, (ii) for large $|k|$, $|\chi(k)| \leq (C/k^{2+\epsilon})$, $C > 0$, $\epsilon > 0$, and $\chi(0) = 1$. The behavior of the test function g at the origin and the asymptotic bound (ii) are taken to ensure that H is essentially self-adjoint. Properties (i) and (ii) will also secure that the thermal correlation function of the field operator is integrable (see below). The cutoff function $\chi(k)$ may be of a Gaussian, exponential, or algebraic type. Since

$$H_I = \frac{\lambda}{\sqrt{2}} Q \int dk g(k)(a_k^* + a_k), \quad (7)$$

the spectral density of the environmental coupling

$$J(\omega) \sim \int dk g(k)^2 \delta[\omega - \omega(k)] \quad (8)$$

is linear for small values of ω. Hence environmental dissipation modeled by Eq. (7) corresponds to the so-called Ohmic case [10].

The reduced dynamics of an apparatus observable X is given by

$$T_t(X) = \Pi^{\omega_E}[e^{itH}(X \otimes I)e^{-itH}], \quad (9)$$

where Π^{ω_E} is a conditional expectation (the dual operation to the partial trace) with respect to the reference state ω_E of the reservoir. We derive an explicit formula for T_t in the Markovian approximation which proved to be successful also in other models [11,12]. Because the thermal correlation function

$$\langle \phi_t(g)\phi(g)\rangle = \omega_E[e^{itH_E}\phi(g)e^{-itH_E}\phi(g)] = \omega_E[\phi(e^{it\omega}g)\phi(g)] \quad (10)$$

is integrable, we use the so-called singular coupling limit [13,14] which states that $T_t = e^{tL}$ is a quantum Markov semigroup with the generator L given by a master equation in the standard (Gorini-Kossakowski-Sudershan-Lindblad) form

$$L(X) = ib[X, Q^2] + \lambda a(QXQ - \{Q^2, X\}). \quad (11)$$

Parameters $a > 0$ and $b \in \mathbf{R}$ are determined by

$$\int_0^{\infty} \langle \phi_t(g)\phi(g)\rangle dt = \frac{a}{2} + ib. \quad (12)$$

By direct calculations

$$\langle \phi_t(g)\phi(g)\rangle = \sqrt{2\pi}F(f_1)(t) + \frac{\sqrt{2\pi}}{2}F(f_2)(t) + \frac{\sqrt{2\pi}}{2}F(f_3)(t), \quad (13)$$

where

$$f_1(k) = \frac{|k|\chi^2(k)}{e^{\beta|k|} - 1}, \quad (14a)$$

$$f_2(k) = |k|\chi^2(k), \qquad f_3(k) = k\chi^2(k), \quad (14b)$$

and F stands for the Fourier transform. Hence, by the inverse Fourier formula,

$$a = 2\pi f_1(0) + \pi f_2(0) = \frac{2\pi}{\beta}, \quad (15)$$

and

$$b = \frac{\sqrt{2\pi}}{2}\int_0^{\infty}\left(\frac{d}{dt}F(\chi^2)(t)\right)dt = -\int_0^{\infty}\chi^2(k)dk. \quad (16)$$

The master Eq. (11) consists of two terms. The first one is a Hamiltonian term $H_A' = bQ^2$, and the second is a dissipative operator

$$L_D(X) = \frac{2\pi\lambda}{\beta}\left(QXQ - \frac{1}{2}\{Q^2, X\}\right). \quad (17)$$

Because these two parts commute so, by the Trotter product formula,

$$T_t(X) = e^{itH_A'}(e^{tL_D}X)e^{-itH_A'}. \quad (18)$$

We now describe effects of dissipation. Because $\mathcal{M}$ is a limit of local algebras $M_{2^n\times 2^n} = \otimes_1^n M_{2\times 2}$, and e^{tL_D} preserves each local algebra so we may assume that $X = (x_{ij}) \in M_{2^n\times 2^n}$. Then

$$L_D(X)_{ij} = -\frac{\pi\lambda}{\beta}\frac{(j-i)^2}{4^{n-1}}x_{ij}, \quad (19)$$

$i, j \in \{1, \ldots, 2^n\}$, and so

$$e^{tL_D}(X)_{ij} = x_{ij}\exp\left(-\gamma t\frac{(j-i)^2}{4^{n-1}}\right), \quad (20)$$

where $\gamma = \pi k\lambda T$. It follows from Eq. (20) that the loss of coherence is faster for coefficients which are more distant to the diagonal, and it increases with reservoir temperature similarly as in the model of a harmonic oscillator linearly coupled to an infinite bath of harmonic oscillators [12], and for a spinless quantum particle minimally coupled to the radiation field [15]. In the thermodynamic limit $n \to \infty$ dissipation leads to an approximate diagonalization of apparatus observables in any finite time interval. However, the $t \to \infty$ limit leads to the following result. Suppose $\mathcal{A}$ is a von Neumann algebra generated by $\pi(\sigma_n^3)$, $n \in \mathbf{N}$. Then $\mathcal{A}$ is a maximal commutative subalgebra in $\mathcal{M}$, and let $P:\mathcal{M} \to \mathcal{A}$ be a von Neumann projection onto it. Since $[H'_A, \sigma_n^3] = 0$, it follows from Eqs. (18) and (20) that all observables from $\mathcal{A}$ are T_t invariant.

THEOREM: *For any statistical state (density matrix) Λ of the spin system and any spin observable X*

$$\lim_{t\to\infty} \langle T_t(X)\rangle_\Lambda = \langle P(X)\rangle_\Lambda, \tag{21}$$

where $\langle X\rangle_\Lambda = \mathrm{tr}(\Lambda X)$ is the expectation value of X in state Λ.

By Eq. (21) all expectation values of $\pi(\sigma_n^k)$, where $k = 1, 2$, and $n \in \mathbf{N}$, tend to zero, and so the $t \to \infty$ limit yields a complete diagonalization of any spin observable.

Finally, we describe the algebra $\mathcal{A}$. Since $\mathcal{A}$ is commutative, it is an algebra of functions on some configuration space Ω. In the sequel we identify an operator $X \in \mathcal{A}$ with the corresponding function $X(\eta)$, $\eta \in \Omega$. Let P_n^+ and P_n^- be spectral projections of σ_n^3, i.e., $\sigma_n^3 = P_n^+ - P_n^-$. An infinite product $P_1^\sharp P_2^\sharp \cdots$, where $\sharp$ stands for $+$ or $-$, defines a state on the subalgebra of continuous functions in $\mathcal{A}$, and so corresponds to a point in the configuration space. Thus $\Omega = \{(i_1, i_2, \ldots): i_n = \pm\}$ or, in other words, each point of Ω describes a configuration of up and down spins located at $n = 1, 2, \ldots$. If μ_0 is a probability measure on $\{-1, 1\}$ which assigns the value one-half to both $\uparrow$ and $\downarrow$ spin positions, and if $\mu = \otimes_1^\infty \mu_0$ is the corresponding product probability measure on Ω, then for any $X \in \mathcal{A}$

$$\mathrm{tr}(X) = \int X(\eta) d\mu(\eta), \tag{22}$$

and so the induced pointer states form an uncountable family. More precisely, for any $s \in [0, 1]$ there exists a projection $e \in \mathcal{A}$ such that $\mathrm{tr}(e) = s$. Thus, since normalization to the unit interval is not essential, the decoherence induced pointer states of the presented model indeed correspond to a pointer with continuous readings. Let us point out, however, that although the induced algebra is continuous and commutative our model has not the complexity needed to describe the position variable of quantum theory. Nevertheless, it suggests the existence of the so-called collective variable and in this sense it can be seen as the first step to the description of such continuous variables.

It is worth noting that continuous families of projections have been selected by a decoherence process also in other models. For example, using the so-called predictability sieve coherent states of a harmonic oscillator coupled to a heat bath (quantum Brownian motion) were shown to be the most stable ones [16]. In a different framework it was shown that coherent states on the Lobatchevski space offer minimal entropy production for the underlying quantum stochastic process [17]. However, coherent states cannot be thought of as continuous pointer states. First, although they offer maximum predictability and are least affected by the environment, they do evolve in time, and, second, they are not orthogonal. Hence, the algebra they generate is neither immune to the interaction with the reservoir, nor commutative.

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