Instantaneous Measurement of Nonlocal Variables

Lev Vaidman
[1]School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel-Aviv University, Tel-Aviv 69978, Israel

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It is shown, under the assumption of the possibility to perform an arbitrary local operation, that all nonlocal variables related to two or more separate sites can be measured instantaneously, except for a finite time required for bringing to one location the classical records from these sites which yield the result of the measurement. It is a verification measurement: it yields reliably the eigenvalues of the nonlocal variables, but it does not prepare the eigenstates of the system.

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Seventy years ago Landau and Peierls [1] claimed that the instantaneous measurability of nonlocal variables (i.e., variables which related to more than one small region of space) contradicts relativistic causality. Twenty years ago, Aharonov and Albert [2] showed that some nonlocal variables (e.g., the Bell operator; see below) can be measured instantaneously and that this does not contradict causality. They also showed explicitly how the possibility of performing instantaneous von Neumann measurements of some other nonlocal variables does contradict causality. The question "What are the *observables* of relativistic quantum theory?" remains topical even today [3].

A variable can obtain the status of an observable if it can be measured. However, the standard (von Neumann) definition of quantum measurement is too restrictive for defining a physical observable: the von Neumann definition requires that eigenstates of the measured variable are not changed due to the measurement process. The existence of a *verification* measurement which yields the eigenvalue of a variable with certainty, if prior to the measurement the quantum system was in the corresponding eigenstate, is enough for giving the status of an observable for such a variable, even if the measurement does not leave the system in this eigenstate as the von Neumann measurement does. (If, initially, the system is in a superposition or mixture of the eigenstates of the observable, then the verification measurement yields one of the corresponding eigenvalues according to the quantum probability law.)

The meaning of "instantaneous measurement" is that in a particular Lorentz frame, at time t, observers perform local actions for a duration of time which can be as short as we wish. At the end of the measurement interactions, the information about the outcome of the measurement is classically recorded in the results of local (irreversible) measurements. In order to infer which eigenvalue of the nonlocal variable the system had originally, or to generate the correctly distributed probabilistic outcome, these classical results are later combined at a point within the future light cones of all the observers.

Note the difference with the case of *exchange measurements* [4] which can also be performed for all nonlocal variables. In the exchange measurement, local operations of swapping lead to swapping between the quantum state of the composite system and the quantum state of the local separated parts of the measuring device. In order to find out which eigenvalue the system had originally, it is required coherent maintaining of all these parts until they enter the forward light cone of all of the original subsystems one wishes to measure where final local joint measurement is performed. After instantaneous swapping, the outcome of the measurement is not written in the form of classical information and, in fact, the outcome of the quantum measurement does not exist yet: at this stage the exchange measurement can be reversed and the system can be brought back to its original (in general unknown) state.

In this Letter, I show that apart from variables related to the spreadout fermionic wave function, *all* nonlocal variables have the status of observables in the framework of relativistic quantum mechanics, i.e., all variables related to two or more separate sites are measurable instantaneously using verification measurements. This includes variables with entangled eigenstates and nonlocal variables with product eigenstates [5].

Verification measurements have been considered before. It has been shown [6] that verification measurements of some nonlocal variables erase local information and, therefore, cannot be ideal von Neumann measurements. Recently, Groisman and Reznik [7] showed that there are instantaneous verification measurements for all spin variables of a system of two separated spin-$\frac{1}{2}$ particles. Consider, for example, a nonlocal variable of two spin-$\frac{1}{2}$ particles located in separate locations A and B, whose eigenstates are the following product states:

$$\begin{aligned} |\Psi_1\rangle &= |\uparrow_z\rangle_A|\uparrow_z\rangle_B, \qquad |\Psi_2\rangle = |\uparrow_z\rangle_A|\downarrow_z\rangle_B, \\ |\Psi_3\rangle &= |\downarrow_z\rangle_A|\uparrow_x\rangle_B, \qquad |\Psi_4\rangle = |\downarrow_z\rangle_A|\downarrow_x\rangle_B. \end{aligned} \tag{1}$$

An instantaneous ideal von Neumann measurement of this variable does contradict causality. Assume that at time t such an ideal measurement is performed. Then

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we can send a superluminal signal from A to B in the following way. We prepare in advance the system in the state $|\Psi_1\rangle$ and agree that Bob at site B measures the spin z component of his particle shortly after time t. Now, in order to send a superluminal signal, Alice at site A can at a very short time before time t flip her spin. If she does so, then after the nonlocal measurement at time t, the system will end up either in state $|\Psi_3\rangle$ on in state $|\Psi_4\rangle$. In both cases Bob has a nonvanishing probability to find his spin "down" in the $\hat{z}$ direction, while this probability is zero if Alice decides not to flip her spin.

The method for the verification measurement I present here uses the teleportation technique [8]. The first step is the teleportation of the state of the spin from B (Bob's site) to A (Alice's site). Bob and Alice do not perform the full teleportation (which invariably requires a finite period of time), but only the Bell measurement at Bob's site which might last, in principle, as short a time as we wish. (I will continue to use the term "teleportation" just for this first step of the original proposal [8].)

In the teleportation procedure for a spin-$\frac{1}{2}$ particle we start with a prearranged EPR (Bohm) pair of spin-$\frac{1}{2}$ particles one of which is located at Bob's site and another at Alice's site, $|\Psi_-\rangle_{AB} = (1/\sqrt{2})(|\uparrow\rangle_A|\downarrow\rangle_B - |\downarrow\rangle_A|\uparrow\rangle_B)$. The procedure is based on the identity

$$|\Psi\rangle_1|\Psi_-\rangle_{2,3} = \tfrac{1}{2}(|\Psi_-\rangle_{1,2}|\Psi\rangle_3 + |\Psi_+\rangle_{1,2}|\tilde{\Psi}^{(z)}\rangle_3 + |\Phi_-\rangle_{1,2}|\tilde{\Psi}^{(x)}\rangle_3 + |\Phi_+\rangle_{1,2}|\tilde{\Psi}^{(y)}\rangle_3), \quad (2)$$

where $|\Psi_\mp\rangle = (1/\sqrt{2})(|\uparrow\rangle|\downarrow\rangle \mp |\downarrow\rangle|\uparrow\rangle)$, $|\Phi_\mp\rangle = (1/\sqrt{2}) \times (|\uparrow\rangle|\uparrow\rangle \mp |\downarrow\rangle|\downarrow\rangle)$ are eigenstates of the Bell operator and $|\tilde{\Psi}^{(z)}\rangle$ signifies the state $|\Psi\rangle$ rotated by π around the $\hat{z}$ axis, etc. Thus, the Bell operator measurement performed on the two particles in Bob's site "collapses" (or effectively collapses) to one of the branches of the superposition, the right-hand side of (2), and, therefore, teleports the state $|\Psi\rangle$ of Bob's particle to Alice except for a possible rotation by π (known to Bob) around one of the axes.

The second step is taken by Alice. She can perform it at time t without waiting for Bob. She measures the spin of her particle in the z direction. If the result is "up," she measures the spin of the particle teleported from Bob in the z direction, and if her spin is "down," she measures the spin of Bob's particle in the x direction.

This completes the measurement except for combining local results together for finding out the result of the nonlocal measurement. Indeed, the eigenstates of the spin in the z direction and in the x direction are teleported without leaving their lines. Thus, Bob's knowledge about possible flip together with Alice's results distinguish unambiguously between the four eigenstates (1).

The method I presented above can be modified for measurement of other nonlocal variables of two spin-$\frac{1}{2}$ particles. However, I turn now to another, universal, method which is applicable to any nonlocal variable $O(q_A, q_B, \ldots)$, where q_A belongs to region A, etc. I do not try to optimize the method or consider any realistic proposal: my task is to show that, given unlimited resources of entanglement and arbitrary local interactions, any nonlocal variable is measurable.

I start with the case of a general variable of a composite system with two parts. First (for simplicity), Alice and Bob perform unitary operations which swap the states of their systems with the states of sets of K spin-$\frac{1}{2}$ particles. In this way Alice and Bob will need the teleportation procedure for spin-$\frac{1}{2}$ particles only. Teleportation of the states of all K individual spins leads to teleportation of the state of the set, be it entangled or not.

The general protocol is illustrated in Fig. 1. The resources include numerous teleportation channels arranged in a particular way: two channels for the first round of back and forth teleportations, then $4^K - 1$ clusters; each includes two channels for the second round of back and forth teleportations and $4^{2K} - 1$ subclusters. Each subcluster, in turn, includes two channels for the third round of teleportation and $4^{2K} - 1$ similar sub-sub-clusters, etc. The protocol consists of the following steps:

(i) Bob teleports his system (K spin-$\frac{1}{2}$ particles) to Alice and records the outcome of the Bell measurements n.

As before, "teleports" means that Bob performs the Bell measurements but does not send the outcome to Alice. The number of possible outcomes is $N = 4^K$. We signify them by $n = 1, 2, \ldots N$, with $n = 1$ corresponding to singlets in all Bell measurements, i.e., to teleportation without distortion.

(ii) Alice performs a unitary operation U on the composite system of her and the teleported spins which, under the assumption of nondistorted teleportation, transforms the eigenstates of the nonlocal variable (which now actually are fully located in Alice's site) to product states in which each spin is either up or down along the z direction.

(iii) Alice teleports the complete composite system ($2K$ spin-$\frac{1}{2}$ particles) to Bob.

Note that if the system is in one of the product states in the spin z basis, then it will remain in this basis.

(iv) If $n = 1$ Bob measures the teleported system in the spin z basis.

In this case (the probability for which is $\frac{1}{N}$), Bob gets the composite system in one of the spin z product states and his measurements in the spin z basis complete the measurement of the nonlocal variable.

If $n \neq 1$ Bob teleports the system back to Alice in the teleportation channel of cluster n. He records the outcome of the Bell measurements m_1 which can have values from 1 to $M = 4^{2K}$.

Since in this case Alice's operations do not bring the eigenstates of the nonlocal variable to the spin z basis, Bob teleports the system back to Alice "telling" her the outcome of his previous Bell measurements via the channel he uses for the teleportation.

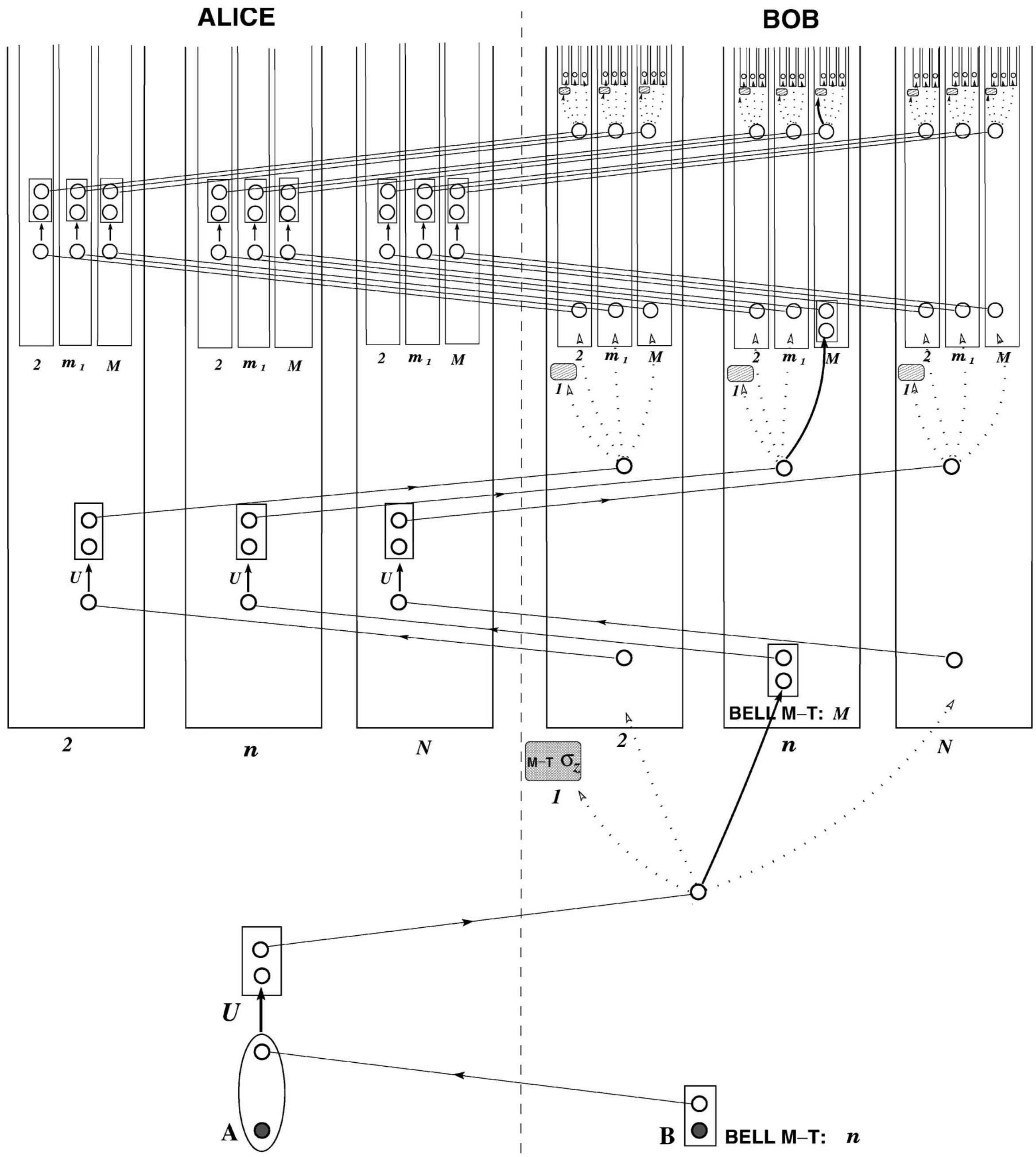

FIG. 1. The scheme of the measurement of a nonlocal variable of a two-part system. In the example shown, the results of the Bell measurement in Bob's site were $n, M, 1$. Thus, the nonlocal measurement has been essentially completed after three teleportation rounds.

(v) Alice performs unitary operations on each system in $N - 1$ teleportation channels of the second round which, under the assumption of no distortion in these teleportations, transforms the eigenstates of the nonlocal variable to product spin z eigenstates.

Alice's operations include corrections required due to her and Bob's teleportations and her unitary transformation of the first round.

(vi) Alice teleports all $N - 1$ systems back to Bob.

(vii) If $m_1 = 1$ Bob measures the system teleported from Alice in cluster n in the spin z basis.

Again, in that case, the spin measurements complete the measurement of the nonlocal variable, since their results together with the outcomes of Alice's and Bob's Bell measurements specify uniquely the eigenvalue of the nonlocal variable.

If $m_1 \neq 1$ Bob teleports the system back to Alice in the teleportation channel of subcluster m_1 of cluster n. He records the outcome of the Bell measurements m_2.

(viii) Alice performs unitary operations on each system in $(N - 1)(M - 1)$ teleportation channels of the third round. The operation on each system is such that if Bob, indeed, teleported the system in this channel, and if his last teleportation happened to be without distortion, then the eigenstates of the nonlocal variable are transformed into product spin z states.

Alice's operations include corrections required due to her and Bob's teleportations and her unitary transformations of the first and second rounds.

(ix) Alice teleports all $(N-1)(M-1)$ systems back to Bob.

(x) If $m_2 = 1$ Bob measures the system teleported from Alice in subcluster m_1 of cluster n in the spin z basis.

If $m_2 \neq 1$ Bob teleports the system back to Alice in the teleportation channel of sub-sub-cluster m_2 of subcluster m_1 of cluster n. He records the outcome of the Bell measurements m_3.

Alice and Bob continue this procedure. The nonlocal measurement is completed when, for the first time, Bob performs a teleportation without distortion. Since, conceptually, there is no limitation for the number of teleportation rounds, and each round (starting from the second) has the same probability for success, $\frac{1}{M}$, the measurement of the nonlocal variable can be performed with probability arbitrarily close to 1. Given the desired probability of the successful nonlocal measurement, Alice and Bob decide about the number of rounds of teleportations. The number of entangled pairs required for each round grows exponentially with the number of rounds. While Bob uses only one teleportation channel in each round and stops after his first teleportation without distortion, Alice has to perform all teleportations in all channels.

The generalization to a system with more than two parts is more or less straightforward. Let us sketch it for a three-part system. First, Bob and Carol teleport their parts to Alice. Alice performs a unitary transformation which, under the assumption of undisturbed teleportations of both Bob and Carol, transforms the eigenstates of the nonlocal variable to product states in the spin z basis. Then she teleports the complete system to Bob. Bob teleports it to Carol in a particular channel n_B depending on the results of the Bell measurement of his first teleportation. Carol teleports all the systems from the teleportation channels from Bob back to Alice. In particular, the system from channel n_B she teleports in the channel (n_B, n_C) depending on her Bell measurement result n_C. The system corresponding to $(n_B, n_C) = (1, 1)$ is not teleported but measured by Carol in the spin z basis. Alice knows the transformation performed on the system which arrives in her channels (n_B, n_C) except for corrections due to the last teleportations of Bob and Carol. She assumes that there was no distortion in those and teleports all the systems back to Bob after the unitary operation which transforms the eigenstates of the variable to product states in the spin z basis. Alice, Bob, and Carol continue the procedure until the desired probability of successful measurement is achieved.

The required resources, such as the number of teleportation channels and required number of operations, are very large, but this does not concern us here. We have shown that there are no relativistic constraints preventing instantaneous measurement of any variable of a quantum system with spatially separated parts, answering the above long-standing question. This question is relevant for quantum cryptography and quantum computation performed with distributed systems. The practical advantage of the method presented in this Letter is that it relies on prior entanglement and does not require coherent transportation of quantum systems.

Can this result be generalized to a quantum system which itself is in a superposition of being in different places? The key to this question is the generality of the assumption of the possibility to perform any local operation. If a quantum state of a particle which is in a nonlocal superposition can be locally transformed to (an entangled) state of local quantum systems, then any variable of the particle is measurable through the measurement of the corresponding composite system. However, while for bosons it is clear that there are such local operations (transformation of the photon state to the entangled state of atoms has been achieved in the laboratory [9]); for fermion states the situation is different [10]. If the transformation of a superposition of a fermion state to local variables is possible, then these local separated in space variables should fulfill anticommutation relations. This is the reason to expect superselection rules which prevent such transformations.

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