but, of course, since the coupling is only electromagnetic, it will not have an important effect in π-π scattering, in accordance with the above observations. One also expects a $2\pi\gamma$ decay mode of the ζ, but estimates in reference 7 indicate that the 2π decay mode will be the dominant one. Presumably, the absence of ζ production in experiments with higher bombarding energies[3,14] is due to the fact that the OPE process tends to dominate the competition between the processes of Figs. 1 and 2 once the energy becomes high enough that the 750-MeV resonance becomes important.

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π-N SCATTERING LENGTH AND SINGULARITIES IN THE COMPLEX J PLANE*

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It has been proposed[1-5] that all baryons and mesons are associated with Regge poles which move in the complex angular momentum plane as a function of energy and that these poles should control the asymptotic behavior of scattering amplitudes in crossed channels. Experimental evidence is consistent with the validity of these ideas,[6,7] although there is, as yet, no proof that there are no cuts in addition to poles in the complex angular momentum plane of the relativistic S matrix. The existence of some domain in complex energy and J plane in which there can be poles and no cuts has been established by Bardakci[8] using the Mandelstam representation. Barut and Zwanziger[9] have given plausible arguments to suggest that in $\mathrm{Re}J > 1$ there are no cuts.

On the other hand, Amati, Fubini, and Stanghellini[10] have recently suggested the possible existence of cuts in addition to Regge poles, and thrown doubt on the possibility of finding experimentally any Regge poles except the top level Pomeranchuk pole.[1,2] They use the strip approximation, however, which does not include inelastic unitarity, and so the result is inconclusive. Chew, Frautschi, and Mandelstam[3] arrived at the quite different conclusion that the strip approximation is to blame for the inconsistency in asymptotic behavior and that exchange of multiparticle systems must be includ ed if a consistent solution is to be achieved.

It is the purpose of this Letter to propose a sun rule which enables one to investigate experimentally whether there are only Regge poles or wheth er there exist cuts in the region of interest. We consider the πN noncharge exchange forward scat tering amplitude

$$f^{(+)}(\nu) \equiv (1/4\pi)[A^{(+)}(\nu) + \nu B^{(+)}(\nu)],$$

whose asymptotic behavior will be controlled by the Pomeranchuk pole. Here we have used the same notation introduced by Chew, Goldberger, Low, and Nambu[11]; ν is the laboratory energy in the πN channel and also is the cosine of the scattering angle in the crossed channel at $t=0$. We separate[3] $f^{(+)}(\nu)$ into the Pomeranchuk-Regge term $F_P(\nu)$ which gives a divergent behavior as $\nu \to \infty$ and $f^{(+)\prime}(\nu)$ which vanishes at infinity unless there is some other singularity in the J plane for the vacuum quantum numbers with real part lying between 0 and 1 for $t=0$. Thus

$$f^{(+)}(\nu)=F_P(\nu)+f^{(+)\prime}(\nu), \tag{2}$$

where

$$F_P(\nu)=-\beta_P \lim_{\alpha_P\to 1}\frac{P_{\alpha_P}(-\nu)+P_{\alpha_P}(\nu)}{\sin\pi\alpha_P}$$

$$=\frac{\beta_P\nu}{\pi}\ln\frac{1+\nu}{1-\nu}-\frac{2\beta_P}{\pi}. \tag{3}$$

Then the dispersion relation for $f^{(+)\prime}(\nu)$ can be written:

$$f^{(+)\prime}(\nu)=\frac{1}{\pi}\int_1^\infty d\nu'\,\mathrm{Im}f^{(+)\prime}(\nu')\left[\frac{1}{\nu'-\nu}+\frac{1}{\nu'+\nu}\right]$$

$$+\frac{1}{4\pi}\frac{g_r^{\,2}}{2M}\left[\frac{1}{\nu_B-\nu}+\frac{1}{\nu_B+\nu}\right]\nu_B. \tag{4}$$

Here

$$\mathrm{Im}f^{(+)\prime}(\nu)=\frac{(\nu'^2-1)^{1/2}}{4\pi}\sigma_{\mathrm{tot}}^{(+)}(\nu')-\beta_P\nu',$$

$$\sigma_{\mathrm{tot}}^{(+)}(\nu')=\tfrac{1}{2}[\sigma_{\mathrm{tot}}^{\pi^+p}(\nu')+\sigma_{\mathrm{tot}}^{\pi^-p}(\nu')],$$

$$\beta_P=\frac{\sigma_{\mathrm{tot}}^{(+)}(\infty)}{4\pi} \tag{5}$$

(see reference 7); and

$$\nu_B=-\mu^2/2M. \tag{6}$$

When ν' becomes infinite $\mathrm{Im}f^{(+)\prime}(\nu')$ approaches zero, and the convergence of the integral in (4) is assured. By making use of (2), (3), (4), (5), and (6), separating out the logarithmic term coming from the low-energy integral and cancelling it from both sides, we get

$$\mathrm{Re}f^{(+)}(\nu)$$

$$=-\frac{f^2}{M}\frac{1}{\nu^2-\nu_B^{\,2}}+\frac{P}{2\pi^2}\int_1^\infty d\nu'\left[\frac{\nu'(\nu'^2-1)^{1/2}}{\nu'^2-\nu^2}\sigma_{\mathrm{tot}}^{(+)}(\nu')\right.$$

$$\left.-\frac{\nu'}{(\nu'^2-1)^{1/2}}\sigma^{(+)}(\infty)\right]. \tag{7}$$

Putting $\nu=1$ and changing the integration variable to $k'^2=\nu'^2-1$, we get the following simple sum rule for the s-wave (+) amplitude scattering length:

$$\left(1+\frac{1}{M}\right)a^{(+)}$$

$$=-\frac{f^2}{M}\frac{1}{1-1/4M^2}+\frac{1}{2\pi^2}\int_0^\infty dk'[\sigma_{\mathrm{tot}}^{(+)}(k')-\sigma_{\mathrm{tot}}^{(+)}(\infty)]. \tag{8}$$

This sum rule should hold if no J singularities extend above $J=0$ at $t=0$, except for the Pomeranchuk-Regge pole.

In order to check the validity of the above relation the following total cross-section data were used: the πp total cross-section data tabulated by Sokolov et al.[12] and Barashenkov et al.[13] up to 1.6 BeV/c, the data by the Moyer group[14] between 1.6 BeV/c and 4.5 BeV/c, and the data by Dardel et al.[15] between 4.5 BeV/c and 20 BeV/c. Above 20 BeV/c we have used the expression given by the Regge pole hypothesis with values of parameters obtained by Udgaonkar. The numerical value of $(1+1/M)a^{(+)}$ is 0.0015 ± 0.0041,[16] that of

$$-\frac{f^2}{M}\frac{1}{1-1/4M^2}=-0.012\pm0.001,$$

and the integral of Eq. (8) becomes 2.22 ± 0.21 in the case of $\sigma^{(+)}(\infty)=24.1$ mb, $\alpha_{\mathrm{ABC}}=-0.3$ and $\beta_{\mathrm{ABC}}=25.7$ (in pion mass unit).[17] The above relation therefore does not hold, and other choices for $\sigma^{(+)}(\infty),\beta_{\mathrm{ABC}},\alpha_{\mathrm{ABC}}(<0)$ do not change this conclusion. There must exist another singularity in $0<\mathrm{Re}J<1$ at zero energy.

The first possibility is that the ABC trajectory[6] passes above[18] $J=0$ at $t=0$. This would imply a "ghost" $J=0$ state similar to the Pomeranchuk-Regge ghost,[7] and also the possibility that the ABC trajectory might pass through $\mathrm{Re}\alpha=2$, giving rise to spin 2 resonance with the vacuum quantum numbers, similar to the Pomeranchuk spin 2 resonance.[2,7,19]

The second possibility is that there might be some other singularity, for example, a branch point between 0 and 1.

We propose now an exact sum rule which enables one to check experimentally whether there exist any singularities between 0 and 1 except the Pomeranchuk and ABC poles. Suppose that we separate $f^{(+)}(\nu)$ into the Pomeranchuk-Regge term $F_P(\nu)$, the ABC-Regge term $F_{\mathrm{ABC}}(\nu)$, defined by

$$-\beta_{\mathrm{ABC}}\frac{P_{\alpha_{\mathrm{ABC}}}(-\nu)+P_{\alpha_{\mathrm{ABC}}}(\nu)}{\sin\pi\alpha_{\mathrm{ABC}}},$$

and $f^{(+)\prime}(\nu)$, which should vanish at infinity if we have no singularity between 0 and 1 except the

Pomeranchuk and ABC poles. Along the same line as before we obtain

$$\left(1+\frac{1}{M}\right)a^{(+)}=\frac{f^2}{M}\frac{1}{1-1/4M^2}-\frac{2\beta_{\mathrm{ABC}}}{\sin\pi\alpha_{\mathrm{ABC}}}P_{\alpha_{\mathrm{ABC}}}(0)$$

$$+\frac{1}{2\pi^2}\int_1^\infty d\nu'\left[\frac{\nu'}{k'}\left\{\sigma_{\mathrm{tot}}^{(+)}(\nu')-\sigma^{(+)}(\infty)\right\}-4\pi\beta_{\mathrm{ABC}}\frac{P_{\alpha_{\mathrm{ABC}}}(\nu')}{\nu'}\right]. \qquad (9)$$

Since

$$P_\alpha(0)=-\frac{\sin\pi\alpha}{2\pi^{3/2}}\Gamma\left(\frac{\alpha+1}{2}\right)\Gamma\left(-\frac{\alpha}{2}\right),$$

and $4\pi\beta_{\mathrm{ABC}}[P_{\alpha_{\mathrm{ABC}}}(\nu')/\nu']$ behaves like

$$\frac{4\pi^{1/2}2^{\alpha_{\mathrm{ABC}}}\Gamma(\alpha_{\mathrm{ABC}}+\frac{1}{2})}{\Gamma(\alpha_{\mathrm{ABC}}+1)}\beta_{\mathrm{ABC}}\nu'^{(\alpha_{\mathrm{ABC}}-1)}\equiv\bar{\beta}_{\mathrm{ABC}}\nu'^{(\alpha_{\mathrm{ABC}}-1)}$$

at high energy, the following form is more convenient for practical purposes:

$$\left(1+\frac{1}{M}\right)a^{(+)}=-\frac{f^2}{M}\frac{1}{1-1/4M^2}+\frac{\Gamma(\alpha_{\mathrm{ABC}}+1)\Gamma(\alpha_{\mathrm{ABC}}/2+\frac{1}{2})\Gamma(-\alpha_{\mathrm{ABC}}/2)}{4\pi^2 2^{\alpha_{\mathrm{ABC}}}\Gamma(\alpha_{\mathrm{ABC}}+\frac{1}{2})}\bar{\beta}_{\mathrm{ABC}}$$

$$+\frac{1}{2\pi^2}\int_0^\infty dk'\left[\sigma_{\mathrm{tot}}^{(+)}(k')-\sigma^{(+)}(\infty)-\frac{\pi^{1/2}\Gamma(\alpha_{\mathrm{ABC}}+1)}{2^{\alpha_{\mathrm{ABC}}}\Gamma(\alpha_{\mathrm{ABC}}+\frac{1}{2})}\bar{\beta}_{\mathrm{ABC}}\frac{P_{\alpha_{\mathrm{ABC}}}((k'^2+1)^{1/2})}{(k'^2+1)^{1/2}}\right]. \qquad (10)$$

It should be noted that the contributions from both the second term on the right-hand side and the last term in the integral are always negative if $1>\alpha_{\mathrm{ABC}}>0$ (it is also assumed that $\beta_{\mathrm{ABC}}>0$ which follows from experiment[6]). Thus it becomes possible that relation (10) can hold.[20]

We hope that more extensive and accurate data on the total cross sections at high energies will soon be available so that an experimental check of relation (10) can be made. If the relation (10) is then seen not to hold it would be strong evidence for the existence of cuts in the J plane.

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DIRECT TEST FOR THE STRONG EQUIVALENCE PRINCIPLE*

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The strong equivalence principle asserts that in a freely falling, nonrotating laboratory, not only do all free particles move with constant velocities—this is the weak equivalence principle—but all the laws of physics are the same in that laboratory, independent of its position in space and time.[1,2] While it is the strong form of the equivalence principle which leads to general relativity, it is often stated that only the weak form is supported by the Eötvös-Dicke experiments[3] and that one can only raise indirect arguments for believing that the strong equivalence principle also holds.[1] The purpose of this note is to show that a simple modification of these experiments, namely, using test bodies with aligned nuclei, would represent a very severe direct test for the strong equivalence principle.

First, we note that in an accelerated coordinate system, all bodies do not fall at the same rate, because inertial forces are velocity dependent. This is well known for the Coriolis forces that appear in a rotating coordinate system, and this is also true for a coordinate system in uniform linear acceleration. Indeed, in such a system, the metric is[4]

$$g_{00} = -[c + (gz/c)]^2, \quad \text{other } g_{\alpha\beta} = \delta_{\alpha\beta}, \tag{1}$$

where g is the constant acceleration rate. The geodesic equation

$$(dv^\gamma/dt) + (\Gamma_{\alpha\beta}{}^\gamma - \Gamma_{\alpha\beta}{}^0 v^\gamma) v^\alpha v^\beta = 0 \tag{2}$$

(where $v^\alpha = dx^\alpha/dt$, $v^0 = 1$) then gives

$$F_z = m dv_z/dt = -mg[1 + (gz/c^2)][1 + 2(v_z{}^2/g_{00})], \tag{3}$$

and shows that the d'Alembert forces are velocity dependent. (Note that this result is just a mathematical consequence of the transformation law to an accelerated coordinate system and is quite independent of the validity of general relativity.)

Thus, if we consider a hot and a cold body in a uniformly accelerated coordinate system, then the inertial forces on the particles of the hot body will be, on the average, smaller than those acting on the particles of the cold body. Nevertheless, both bodies will "fall" at the same rate, as may easily be seen by considering the coordinate transformation which restores the original inertial frame in which they are at rest. This "paradox" is easily resolved if we note that the internal forces within each body have also to be transformed from the inertial frame to the accelerated one, and their law of transformation is such that the change in the internal forces exactly compensates the difference between the in-