Table I. η decay for various quantum numbers.

Quantum numbers	$\lvert\Delta\vec{T}\rvert$ rule	T_{final}	Relationship to τ decay
$T=0,\ 0^{-+}$	1	1	Charged mode similar to τ' (or τ^0)
$T=0,\ 0^{--}$	0, 2	0, 2	None
$T=1,\ 1^{--}$	0, 2	1, 3	None
$T=1,\ 2^{++}$	1	0, 2	None

structure of the decay amplitudes for K and η is determined by the final-state interactions. If either of the amplitudes contains important intrinsic P-wave structure, the terms in the respective spectra which should be compared are not the linear but rather the quadratic terms calculated, for example, by Barton and Kacser.[11] For these terms the correspondence still holds.

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*Work performed under the auspices of U. S. Atomic Energy Commission.

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[4]This result, as well as the results of the present paper, are correct only to the extent that the 10% mass difference between K and η may be ignored. (See the relevant discussion in BR.) Also the πK and $\pi\eta$ interactions should be negligible over the decay spectrum; this is substantially correct since the relevant thresholds are far removed from the physical region.

[5]G. Feinberg and A. Pais (to be published) have noted independently the utility of exploiting this property.

[6]It should perhaps be noted that C invariance alone gives the final isotopic spin to be 1 or 3; the latter is eliminated if terms of order α^2 are ignored.

[7]R. H. Dalitz, Phys. Rev. 94, 1046 (1954).

[8]M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nuclear Sci. 7, 407 (1957).

[9]Quadratic terms will, of course, affect the branching ratio. The effect of these terms is being investigated by K. C. Wali (private communication from R. F. Sawyer).

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[11]G. Barton and E. Kacser, Phys. Rev. Letters 8, 228, 353(E) (1962).

EXPERIMENTAL TESTS OF THE $\Delta I=\frac{1}{2}$ RULE, AND THE $\Delta S=+\Delta Q$ RULE IN THREE-BODY DECAYS OF NEUTRAL K MESONS*

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We present the results of an experiment using the three-body decays of netural K mesons to test three rules that have been suggested to hold in strange-particle decay:

1. The rule[1-3] $\Delta I=\frac{1}{2}(NL)$, for nonleptonic (NL) decays. This rule is in agreement with many observations.[4]

2. The rule[5] $\Delta I=\frac{1}{2}(L)$, for strangeness-changing leptonic (L) decays. Here ΔI refers to the isotopic spin changes of the strongly-interacting particles.

3. The rule[6] $\Delta S=+\Delta Q$, correlating the changes in strangeness (S) and charge of the strongly-interacting particles in S-changing L decay.

The rules $\Delta I=\frac{1}{2}(L)$ and $\Delta S=+\Delta Q$ are not independent. Three amplitudes $a(\frac{1}{2},\frac{1}{2})$, $a(\frac{3}{2},\frac{1}{2})$, and $a(\frac{3}{2},\frac{3}{2})$ suffice to describe L decays of charged and neutral K mesons. Here, for instance, $(\frac{3}{2},\frac{1}{2})$ means $\Delta I=\frac{3}{2}$ and $\Delta I_z=\frac{1}{2}$, where ΔI is the isotopic spin difference between the initial and final state, for the strongly interacting particles, and can be thought of as carried by a Wentzel "spurion."[3] The amplitudes are assumed to be independent of the sign of ΔI_z. Assuming CP invariance, using standard spurion technique,[3] and letting L stand for either an electron or a muon, one has, after cancelling a common factor, the decay rates

$$\Gamma(K^0\to L^+)=\Gamma(\overline{K}^0\to L^-)=|a|^2\equiv|-2a(\tfrac{1}{2},\tfrac{1}{2})+a(\tfrac{3}{2},\tfrac{1}{2})|^2,$$

$$\Gamma(\overline{K}^0\to L^+)=\Gamma(K^0\to L^-)=|\overline{a}|^2\equiv|6^{1/2}a(\tfrac{3}{2},\tfrac{3}{2})|^2,$$

and

$$\Gamma(K^+\to L^+\pi^0\nu)\equiv\Gamma_+(L^+)=|\sqrt{2}\,a(\tfrac{1}{2},\tfrac{1}{2})+\sqrt{2}\,a(\tfrac{3}{2},\tfrac{1}{2})|^2.$$

For the K_1^0 and K_2^0 rates we have

$$\Gamma_1(L^+) = \Gamma_1(L^-) = \tfrac{1}{2}|a + \bar{a}|^2$$

and

$$\Gamma_2(L^+) = \Gamma_2(L^-) = \tfrac{1}{2}|a - \bar{a}|^2.$$

The rule $\Delta S = +\Delta Q$ forbids $K^0 \rightarrow L^-$ and $\bar{K}^0 \rightarrow L^+$ and is equivalent to $a(\frac{3}{2}, \frac{3}{2}) = 0$. It thus predicts

$$\Gamma_1(L) = \Gamma_2(L). \tag{1}$$

The rule $\Delta I = \frac{1}{2}(L)$ corresponds to setting both $a(\frac{3}{2}, \frac{3}{2}) = 0$ and $a(\frac{3}{2}, \frac{1}{2}) = 0$. Then, in addition to Eq. (1), one has

$$\Gamma_2(L^\pm) = 2\Gamma_+(L^+), \tag{2}$$

where

$$\Gamma_2(L^\pm) = \Gamma_2(L^+) + \Gamma_2(L^-) = |a - \bar{a}|^2.$$

Thus, if we sum over e and μ decay modes, and use the known K^+ lifetime (footnote a, Table I) and branching ratios,[7,8] the rule $\Delta I = \frac{1}{2}(L)$ predicts

$$\Gamma_2(L^\pm) = (16.5 \pm 1.18) \times 10^6 \text{ sec}^{-1}. \tag{3}$$

In an earlier experiment, Crawford et al.[9] obtained on the basis of 8 events $\Gamma_2(L^\pm) = (20.4^{+7.2}_{-5.6}) \times 10^6 \text{ sec}^{-1}$, compatible with the prediction of Eq. (3). Their result was obtained, however, under the assumption of Eq. (1), that is, under the assumption of the rule $\Delta S = +\Delta Q$. Recently the rule $\Delta S = +\Delta Q$ has had its first test, in an experiment by Ely et al.,[10] and apparently failed.

In the present experiment we test all three rules, using K^0 produced in the Alvarez 72-inch hydrogen bubble chamber through the reaction $\pi^- + p \rightarrow \Lambda + K^0$ ($p_{\pi^-} = 1.03$ BeV/c). We study those cases where the K subsequently decays into one of the modes $e^\pm\pi^\mp\nu$, $\mu^\pm\pi^\mp\nu$, or $\pi^\pm\pi^\mp\pi^0$, in association with a visible Λ decay, $\Lambda \rightarrow p + \pi^-$.

Let $N(\Lambda)$ be the number of visible decays $\Lambda \rightarrow p + \pi^-$ (whether or not the K^0 undergoes visible decay). Let t be the time after K^0 production, measured in the K^0 rest frame, and $\epsilon(t)$ be the probability for observing a three-body K^0 decay, given a Λ decay. Then the number of decays, either L^+ or L^-, (L^+, L^-), in time interval dt is given by

$$dN(L^+, L^-)$$
$$= \tfrac{1}{2}N(\Lambda)\epsilon(t)dt\{\tfrac{1}{2}|a + \bar{a}|^2 \exp(-t/\tau_1) + \tfrac{1}{2}|a - \bar{a}|^2 \pm (|a|^2 - |\bar{a}|^2)\cos(\Delta m t)\exp(-t/2\tau_1)\}, \tag{4}$$

Table I. Summary of predictions and experimental results.

	$\Gamma_+(L^+)$ (10^6 sec^{-1})	$\Gamma_2(L^\pm)$ (10^6 sec^{-1})	$\frac{\Gamma_1(L^\pm)}{\Gamma_2(L^\pm)}$	$\Gamma_+(++-)$ (10^6 sec^{-1})	$\Gamma_+(+00)$ (10^6 sec^{-1})	$\frac{\Gamma_+(+00)}{\Gamma_+(++-)}$	$\Gamma_2(+-0)$ (10^6 sec^{-1})
Other experiments	8.25 ± 0.59[a]	$20.4^{+7.2}_{-5.6}$[b]; 8.5 ± 2.8[c]	$11.9^{+7.5}_{-5.6}$[e]	4.65 ± 0.15[f]	1.39 ± 0.11[f]	0.298 ± 0.025[f]	...
$\Delta S = +\Delta Q$	...	...	1	...	...	...	...
$\Delta I = \frac{1}{2}$	...	16.50 ± 1.18[d]	1	...	...	0.311[g]	2.87 ± 0.23[h]
This experiment	...	9.31 ± 2.49	$6.6^{+6.0}_{-4.0}$	...	...	...	1.44 ± 0.43[i]

[a]We take the K^+ lifetime to be $(1.224 \pm 0.013) \times 10^{-8}$ sec as an average of the results of L. W. Alvarez, F. S. Crawford, M. L. Good, and M. L. Stevenson, in Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957 (Interscience Publishers, Inc., New York, 1957); and V. Fitch and R. Motley, Phys. Rev. 101, 496 (1956); and 105, 265 (1957). We average the K^+ branching ratio results of Alexander et al., and of Roe et al., reference 7, to obtain $\mu^+\pi^0\nu = (5.07 \pm 0.54)\%$ and $e^+\pi^0\nu = (5.01 \pm 0.47)\%$, and add these to obtain the total L^+ fraction. We point out that although these two experiments are in good agreement with each other, they are in poor agreement with the earlier branching-ratio results of Birge et al., reference 8, who obtain a total L^+ fraction of $(6.0 \pm 1.6)\%$, i.e., nearly a factor of two less than in the two later experiments.

[b]Reference 9 as published, with assumption $\Gamma_1 = \Gamma_2$.

[c]Reference 9 reanalyzed, with assumption $\Gamma_1 = 9\Gamma_2$. (See text.)

[d]See Eq. (2) and footnote a.

[e]Reference 10.

[f]We average the K^+ branching-ratio results of references 7 and 8, and also S. Taylor, G. Harris, J. Orear, J. Lee, and P. Baumel, Phys. Rev. 114, 359 (1959).

[g]The prediction is $\frac{1}{4}(1.244)$ (see reference 20).

[h]See Eq. (11), reference 20, and footnote f.

[i]See the discussion preceding Eq. (15).

where Δm is the K_1^0-K_2^0 mass difference, and τ_1 is the K_1^0 mean life.[11] We here omit a factor $\exp(-t/\tau_2)$, since in our experiment it differs from unity by at most about 1% according to the previously measured value of τ_2.[12] Adding both signs of charge in (4) we obtain for the total L rate,

$$dN(L^\pm) = \tfrac{1}{2}N(\Lambda)\epsilon(t)dt\{\Gamma_1(L^\pm)\exp(-t/\tau_1) + \Gamma_2(L^\pm)\}, \quad (5)$$

which is independent of Δm.

We now describe our experimental procedures.

(a) We use all events with $\Lambda \to p + \pi^-$, whether or not a visible K_1^0 or K_2^0 decay has been found. We accept only events from $\pi^- + p \to \Lambda + K^0$, i.e., none from $\Sigma^0 K^0$ production are used.

(b) A second scan is performed along the direction of flight of the K^0, as predicted from the production and decay of the Λ.

(c) With the double-vee events we first try to fit a normal decay (and production) $K_1^0 \to \pi^+ + \pi^-$, using the least-squares kinematic fitting program KICK.[13] If that fails we try a one-constraint (1-C) fit[13] to each of the six three-body-decay hypotheses. (The K momentum is well known from the ΛK production and the Λ decay.)

(d) The Λ and K production and decay satisfy certain fiducial criteria, and both decays must occur at least 0.5 cm from the production point. In addition, a K decay is accepted only for $0.2 \le t \le 20.0$ (in units of 10^{-10} sec).

(e) Appropriate cutoffs are introduced in order to eliminate fake three-body-decay events due to normal K_1^0 decay into $\pi^+\pi^-$, followed by a Coulomb scatter or $\pi \to \mu$ decay; or into $\pi^0\pi^0$, followed by Dalitz decay[14] of the π^0 into $e^+e^-\gamma$. These and several less frequent types of fake have, we believe, been eliminated.

No cutoffs are needed to eliminate fake K_2^0 decays (due to K_1^0) for $t > 4\tau_1$, and none is applied in the determination of Γ_2. In determining Γ_1/Γ_2 one "certain" K_2^0 ($t = 11.3$) is cut off.

After applying the fiducial criteria a sample of about 5000 decays is reduced to $N(\Lambda) = 2703$. Associated with these are 27 three-body decays between $t = 0.2$ and 20.0. Four of the 27 events are $\pi^+\pi^-\pi^0$, all with $t > 4\tau_1$, and are therefore due to K_2^0 decay. The remaining 23 events are L decays.

Since the assignment of decay mode is based on kinematics alone, there is often an ambiguity as to which of the four L modes is to be assigned to a given event. (There is no ambiguity between L modes and $\pi^+\pi^-\pi^0$.) The χ^2 probability distribution for the "best interpretation" on each event agrees excellently with that expected for 1-C. In the few cases where the identification is certain, because of a stopping particle, the best interpretation (by χ^2) is the correct one.

Because of the ambiguity in L assignment and especially because of the small number of events, we limit ourselves to testing the predictions of Eqs. (1) and (3) without attempting to separate μ and e, and without regard to charge ambiguity.

We consider first the prediction (3) of $\Delta I = \frac{1}{2}(L)$. From the 14 L decays with $3.44 \le t \le 20.0$ we find from (5), with $N(\Lambda) = 2703$ and $\epsilon(\text{av}) = 0.67$, the result[15]

$$\Gamma_2(L^\pm) = (9.31 \pm 2.49) \times 10^6 \text{ sec}^{-1}. \quad (6)$$

Our result (6) differs by 2.6 standard deviations (std. dev.) from the prediction (3). We calculate one chance in 53 for a statistical fluctuation at least this large,[16] and therefore believe that our data are incompatible with the rule $\Delta I = \frac{1}{2}(L)$ for leptonic K decays.

Of the 14 K events used to obtain (6), two fit $K^0 \to \pi^+\pi^-\gamma$ (and also L). If these were indeed $\pi\pi\gamma$ our result (6) would be reduced by a factor 12/14 and would then be in even worse agreement with $\Delta I = \frac{1}{2}(L)$.

We thus find that at least one of the amplitudes $a(\frac{3}{2}, \frac{1}{2})$ or $a(\frac{3}{2}, \frac{3}{2})$ is not zero.

We next test the prediction (3) of the rule $\Delta S = +\Delta Q$. A total of 22 L decays satisfy the cutoff criteria. Their time distribution is given in Fig. 1(b). To make a quantitative comparison we construct a normalized likelihood function corresponding to Eq. (5). We find

$$\Gamma_1(L^\pm)/\Gamma_2(L^\pm) = 6.6^{+6.0}_{-4.0} \quad (7)$$

in rather poor agreement with prediction (1). The calculated chance that $\Gamma_1(L) = \Gamma_2(L)$ and that our results are then due to a statistical fluctuation is 6.5%.[17,18]

Our result[7] can be compared with the result of Ely <u>et al.</u>,[10] who find

$$\Gamma_1(e^\pm)/\Gamma_2(e^\pm) = 11.9^{+7.5}_{-5.6}. \quad (8)$$

We thus support the result of Ely <u>et al.</u>, indicating that the rule $\Delta S = +\Delta Q$ is not valid.

Since Eq. (5) is invariant under the interchange of a and $\bar{a}$, our result (7) corresponds to[19]

$$\bar{a}/a \text{ or } a/\bar{a} = +0.44^{+0.12}_{-0.20}. \quad (9)$$

In order to resolve the ambiguity in (9) it is necessary to include the information as to the lepton's

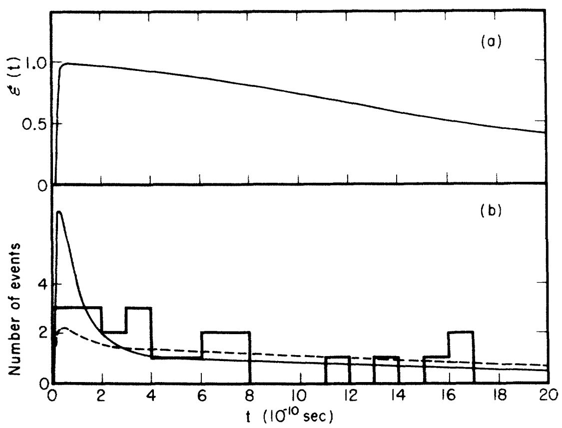

FIG. 1. (a) Geometrical efficiency $\epsilon(t)$ for detecting K decay. The decrease at short t is due to the cutoffs at 0.5 cm, and at 0.2×10^{-10} sec. At large t, $\epsilon(t)$ is the fraction of K's having potential time $T>t$. (b) Time distribution of leptonic decays. The smooth curves are predicted differential counting rates in counts per 10^{-10} sec normalized to 22 counts, for the cases $\Gamma_1=\Gamma_2$ (dashed curve) and $\Gamma_1=9\Gamma_2$ (smooth curve). The best fit has $\Gamma_1=6.6\Gamma_2$, and is not plotted.

charge, and use Eq. (4). Only the events in the first one or two K_1^0 mean lives are useful, because of the uncertainty in Δm. Because of the small size of our sample, and the charge ambiguity often present, we are not able to resolve the ambiguity. Corresponding to (8), Ely et al. obtain[19]

$$\bar{a}/a=+0.55^{+0.08}_{-0.12}. \qquad (10)$$

We now compare our results with those of Crawford et al.[9] In that experiment, it was necessary to use a value for Γ_1/Γ_2 in order to obtain $\Gamma_2(L^\pm)$, since, because of the small (10-in.) chamber, it was not possible to get rid of the K_1^0 decays by going to large t. From the t distribution, they obtained $\Gamma_1/\Gamma_2=3.5^{+3.9}_{-2.7}$. But after that they assumed $\Gamma_1=\Gamma_2$. They then obtained $\Gamma_2(L^\pm)=(20.4^{+7.2}_{-5.6})\times10^6$ sec^{-1}. One of us has reanalyzed that experiment, using $\Gamma_1/\Gamma_2=9$, a compromise between our result (7) and that of Ely et al., (8). The result is $\Gamma_2(L^\pm)=(8.5\pm2.8)\times10^6$ sec^{-1}, in excellent agreement with our present result (6). [Our result (6) does not, of course, depend on any assumption for Γ_1/Γ_2.]

In terms of the three amplitudes for $(\Delta I, \Delta I_z)$ our results for L decays can be summarized as follows:

(i) $a(\frac{3}{2},\frac{3}{2})$ and $a(\frac{3}{2},\frac{1}{2})$ are not both zero [from the absolute rate $\Gamma_2(L^\pm)$].

(ii) $a(\frac{3}{2},\frac{3}{2})$ is not zero [from the ratio $\Gamma_2(L^\pm)/\Gamma_1(L^\pm)$].

(iii) $a(\frac{3}{2},\frac{1}{2})$ can easily be zero. [The best fit is $a(\frac{3}{2},\frac{1}{2})/a(\frac{1}{2},\frac{1}{2})=+0.2$.]

(iv) $a(\frac{1}{2},\frac{1}{2})$ cannot be zero. [With $a(\frac{3}{2},\frac{3}{2})$ alone we cannot fit both $\Gamma_2(L^\pm)$ and $\Gamma_1(L^\pm)/\Gamma_2(L^\pm)$.]

We now turn to the rule $\Delta I=\frac{1}{2}(NL)$ and its consequences for the decays $K^+\to\pi^+\pi^+\pi^-$ (++-) and (+00), and $K_2^0\to$ (+-0) and (000). If there are no restrictions on ΔI in the decay, then the final 3π states available are those with $I=1$, 2, or 3 for K^+ decay, and $I=1$ or 3 for K_2^0 decay. (Invariance under CP excludes $I=0$ and 2 in K_2^0 decay; charge conservation excludes $I=0$ in K^+ decay.) Table II contains the branching ratios corresponding to each of these states.

If $\Delta I=\frac{1}{2}(NL)$ holds, then only the final states with $I=1$ are allowed. From Table II we see that in that case $\Gamma_2(+-0)/\Gamma_+(+00)=2/1$, and that this holds for any one and also, it turns out, for any admixture of the three symmetry states with $I=1$. (We are indebted to Professor S. B. Treiman for this observation.)

After taking into account the phase space[20] we have the prediction

$$\Gamma_2(+-0)=2(1.032)\Gamma_+(+00), \qquad (11)$$

that is

$$\Gamma_2(+-0)=(2.87\pm0.23)\times10^6\ \text{sec}^{-1}. \qquad (12)$$

(See footnote f, Table I.)

We now compare this prediction with experiment. Based on our four events $K_2^0\to$ (+-0), we find

$$\Gamma_2(+-0)=(2.66\pm1.34)\times10^6\ \text{sec}^{-1}. \qquad (13)$$

Table II. Charge branching ratios in the decays $K^+\to3\pi$ and $K_2^0\to3\pi$. Intensities should be compared only within a single state. There are two $I=1$ states of mixed symmetry, both having the same branching ratios. Similarly there are two $I=2$ states, both having mixed symmetry and having the same branching ratios. Invariance under CP excludes the $I=0$ and $I=2$ states for K_2^0. Charge conservation excludes $I=0$ for K^+. Phase-space corrections are contained in reference 20.

I	3π state Symmetry of state	K^+ (++-)	K^+ (+00)	K_2^0 (+-0)	K_2^0 (000)
1	Totally symmetric	4	1	2	3
1	Mixed	1	1	2	0
2	Mixed	1	1	0	0
3	Totally symmetric	1	4	3	2

Our result (13) agrees with the prediction (12), but is based on only four counts, so we try a separate approach. We first compare our ratio $\Gamma_2(+-0)/\Gamma_2(L^\pm)$ with that of Luers et al.,[21] who find

$$\Gamma_2(+-0)/\Gamma_2(L^\pm) = 0.155 \pm 0.022. \qquad (14)$$

Based on our 14 L decays Eq. (14) predicts that we should see 2.2 decays into (+-0). This is not in disagreement with the 4 ± 2 seen.

It is reasonable to combine our rate $\Gamma_2(L^\pm)$, given by (6), with the branching ratio (14) of Luers et al., omitting our four $\pi^+\pi^-\pi^0$ as (relatively) statistically insignificant, to obtain a combined experimental rate

$$\Gamma_2(+-0) = (1.44 \pm 0.43) \times 10^6 \text{ sec}^{-1}. \qquad (15)$$

This result differs from the prediction (12) by 2.95 std. dev. We calculate odds of about 100/1 against a statistical fluctuation this large or larger, by the method of reference 16.

We therefore believe that our data, when combined with K^+ and K_2^0 branching ratios from other experiments, are inconsistent with the rule $\Delta I = \frac{1}{2}(NL)$. This conclusion is independent of the symmetry of the $I=1$ final 3π states.

As our final result we determine the total K_2^0 decay rate. We assume that all other states besides the symmetric $I=1$ state can be neglected. In that case, from Table II we find[5]

$$\Gamma_2(000) = \tfrac{3}{2}(1.218)\Gamma_2(+-0), \qquad (18)$$

where again phase space is included.[20] Combining our rate for $\Gamma_2(L^\pm)$, the branching ratio $\Gamma_2(+-0)/\Gamma_2(L^\pm)$ of Luers et al.[21] and Eq. (18), we obtain the total K_2 decay rate

$$\Gamma_2 = \Gamma_2(L^\pm)\{1 + [\Gamma_2(+-0)/\Gamma_2(L^\pm)][1 + \tfrac{3}{2}(1.218)]\}, \text{ i.e.,}$$

$$\Gamma_2 = (13.38 \pm 3.62) \times 10^6 \text{ sec}^{-1}, \qquad (19)$$

that is

$$\tau_2 = (\Gamma_2)^{-1} = (7.47^{+2.77}_{-1.59}) \times 10^{-8} \text{ sec}. \qquad (20)$$

This result may be compared with the lifetime measured directly by attenuation with distance of Bardon et al.,[12] who obtain

$$\tau_2 = (8.1^{+3.3}_{-2.4}) \times 10^{-8} \text{ sec}.$$

Most of the predictions and experimental results are summarized in Table I.

It is a pleasure to thank Professor Luis Alvarez for his encouragement and advice throughout the course of this work. We are indebted to Professor S. B. Treiman for illuminating remarks. We are indebted to J. A. Anderson and J. L. Lloyd for assistance in obtaining and processing the data, and to Dr. P. G. Burke for help in the computer programs.

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[15]We have included a correction factor of 0.987 for K_2^0 attenuation by decay.

[16]To calculate the probability we use Eq. (3) to predict the number $\bar{n}$ of counts expected, and then calculate the Poisson probability to see the observed 14 counts, or

less. We take into account the error in Eq. (3) through a (Gaussian) weighted distribution in $\bar{n}$.

[17]The quoted error in (7), and thus also in (9), was obtained by going to values where the likelihood function decreases by a factor $e^{1/2}$. The probability 6.5% for $\bar{a} = 0$ was obtained separately by using an especially simple likelihood function, namely a binomial distribution with respect to counts n_1 and n_2 earlier and later than $2\tau_1$, normalized to a total of 22 counts. For $\bar{a} = 0$, $n_1 = 2.96$ is "expected." Six counts are observed. The chance for at least six is 6.5%. The binomial distribution with the observed $n_1 = 6$ also gives the result (7) and the same calculated errors, as does the complete (multinomial) likelihood function. The quoted errors are thus essentially given by $n_1 = 6 \pm 6^{1/2}$.

[18]If we remove from the sample the six possible decays into $\pi^+\pi^-\gamma$ we find $\Gamma_1/\Gamma_2 = 4.7^{+5.9}_{-3.6}$.

[19]The (fractionally) smaller error in $\bar{a}/a$ as compared to Γ_1/Γ_2 is somewhat misleading, since it arises only by virtue of taking (essentially) square roots of counts, through the relation: $\bar{a}/a = [(\Gamma_1/\Gamma_2)^{1/2} - 1]/[(\Gamma_1/\Gamma_2)^{1/2} + 1]$.

[20]We use the phase-space formula of Dalitz, reference 2, with $m(K^0) = 497.8$, $m(K^+) = 493.9$, $m(\pi^+) = 139.59$, and $m(\pi^0) = 135.0$ to find phase-space factors ϕ for K^+ or K^0 decaying into three pions (1,2,3), and find $\phi_+(++-) = 6.282 \times 10^3$, $\phi_+(+00) = 7.81 \times 10^3$, $\phi_0(+-0) = 8.067 \times 10^3$, and $\phi_0(000) = 9.825 \times 10^3$. The appropriate ratios of these factors are then used. (The factors ϕ were not included in the predictions of reference 5.)

[21]D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. Letters 7, 255 (1961). Preliminary results by A. Astier, L. Blaskovic, M. M. de Courreges, B. Equer, A. Lloret, P. Rivet, and J. Siad [Proceedings of the Aix-en-Provence Conference on Elementary Particles, 1961 (C. E. N. Saclay, France, 1961)] are consistent with those of Luers et al.

ONE-PION-EXCHANGE MODEL AND EVIDENCE THAT THE SPIN OF THE ζ IS EVEN*

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A number of experiments now indicate that in the reaction

$$\pi^{\pm} + p \rightarrow \pi^{\pm} + \pi^0 + p \qquad (1)$$

there is a peak in the di-pion mass spectrum in the vicinity of $E_\zeta = 575$ MeV in cases where the laboratory energy of the incident pion is less than about 1 BeV; this peak is now generally denoted by ζ.[1-4] Indications of the existence of the ζ have also been observed by Sechi Zorn in a different type of experiment.[5] The ζ appears to be singly charged[3] and thus to have isotopic spin $T = 1$. Its spin has been the subject of some discussion[6,7] and it is the purpose of this note to point out that the existing experimental evidence strongly suggests that this spin is even.

The argument is based on the fact that if the spin of the ζ is odd, then it can be coupled by strong interactions to a two-pion $T = 1$ state. One would, therefore, expect that the ζ would appear as a resonance in the π-π cross section at E_ζ. This cross section has been measured as a function of energy by Carmony and Van de Walle who study reaction (1) for an incident momentum of 1.25 BeV/c.[8] They make use of the Chew-Low extrapolation procedure[9] and also analyze the π^+ data by applying the Chew-Low formulas in the low momentum transfer part of the physical region, a procedure equivalent to assuming that reaction (1) is dominated by the one-pion-exchange (OPE) diagram of Fig. 1 for events with small momentum transfer to the proton. This assumption is well justified experimentally for the π^+ data by the excellent experimental agreement with the predicted momentum distribution for the recoil protons, and also by the consistency of the total cross section and angular distribution[10] obtained with the presumed $T = 1$, $J = 1$ character of the resonance at about 750 MeV. The cross sections obtained by extrapolation and those obtained directly from the data in the physical region are in agreement, and neither shows any enhancement in the vicinity of E_ζ; the measured cross sections fall short by about a factor of four from what would be expected from a $J = 1$ resonance at the energy of the ζ. Earlier measurements by the Berkeley group[11] using the extrapolation procedure indicated a possible peak in the π-π cross section in the vicinity of E_ζ. These measurements involved a lower incident energy and required an extrapolation over a longer distance, so that it would appear that the later measurements are

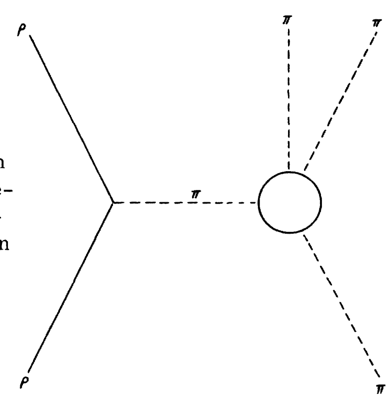

FIG. 1. Diagram associated with one-pion-exchange contribution to reaction (1).