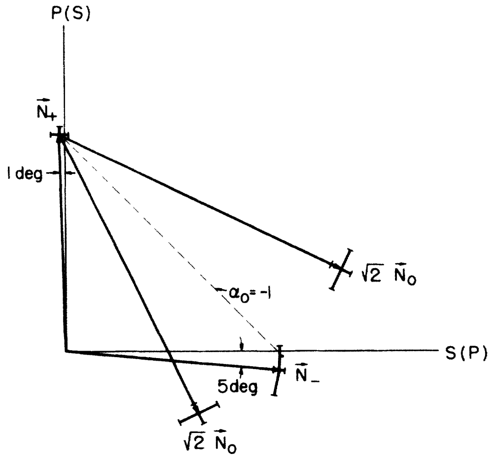

FIG. 2. The Σ-decay amplitudes in the $S-P$ plane. The $|\Delta\vec{T}| = \frac{1}{2}$ rule requires that the three amplitudes form a triangle.

scattering phase shifts. These phase shifts are listed in reference 8 and are seen to be small. Taking the amplitudes to be real (the resulting error is small relative to experimental uncertainties), one then describes the various decay amplitudes as vectors in an S-P plane. The magnitudes of the vectors are determined from the three decay rates.[9] The directions are obtained from Eq. (1) by expressing $\alpha = \sin 2\nu$, where ν is the angle with respect to the coordinate axis. Two ambiguities remain—the labeling of the S and P axes.

Using the combined values for the helicities α listed in Table I, we construct Fig. 2. The two directions for $\vec{N}_0$ arise from $|\alpha_0|$ being less than one, corresponding to S/P greater or less than one. For $\alpha_0 = -1$, the triangle would close well within experimental errors, and the triangular relationship given by the $|\Delta\vec{T}| = \frac{1}{2}$ rule would hold. The inconsistency with the $|\Delta\vec{T}| = \frac{1}{2}$ rule lies between two and three standard deviations.

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*Work supported by the U. S. Atomic Energy Commission.

†National Academy of Sciences Fellow.

[1]M. Ferro-Luzzi, R. D. Tripp, and M. B. Watson, Phys. Rev. Letters 8, 28 (1962).

[2]R. D. Tripp, M. B. Watson, and M. Ferro-Luzzi, Phys. Rev. Letters 8, 175 (1962).

[3]The decay asymmetry is written as $1 + \alpha P_\Sigma \cos\phi$, where ϕ is the angle between the hyperon polarization, P_Σ, and the nucleon direction. This convention has the merit of having the nucleon helicity equal to the decay asymmetry parameter. The more usual, but not universal, convention of following the pion leads to an annoying minus sign relating the helicity to the asymmetry parameter.

[4]M. B. Watson, M. Ferro-Luzzi, and R. D. Tripp (to be published).

[5]E. F. Beall, Bruce Cork, D. Keefe, W. C. Murphy, and W. A. Wenzel, Phys. Rev. Letters 8, 75 (1962).

[6]Bruce Cork, L. T. Kerth, W. A. Wenzel, J. W. Cronin, and R. L. Cool, Phys. Rev. 120, 1000 (1960).

[7]See reference 5. The value we quote uses the $|\Delta\vec{T}| = \frac{1}{2}$ rule, but in only a very weak way involving the πN phase shifts. We have adjusted their uncertainties to correspond to standard deviations.

[8]M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nuclear Sci. 7, 454 (1957).

[9]W. H. Humphrey and R. R. Ross, Phys. Rev. (to be published). The mass difference has been accounted for approximately by dividing the decay rates by the phase-space factor P/E to obtain $|N|^2$. If we assume the $|\Delta\vec{T}| = \frac{1}{2}$ rule, these decay amplitudes alone show that the triangle is nearly a right triangle (94 ± 5 deg).

CORRESPONDENCE BETWEEN τ AND η DECAYS*

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In a recent Letter, Barton and Rosen[1] have proposed a combined test for the decay mechanism of the η meson as envisaged by Gell-Mann et al.[2] and of the adequacy of the pion pole approximation suggested earlier for τ and τ' decays.[3] The argument rests on the observation that the model of Gell-Mann et al. leads in a natural manner to the one-pion intermediate state and that, once this state has interceded, the Dalitz plot for η decay is determined by the $\pi \to 3\pi$ amplitude; in particular, if τ' decay is adequately described by the pion pole term, the π^0 spectrum in $\eta \to \pi^+\pi^-\pi^0$ is identical with the π^+ spectrum in $\tau^{+\prime}$ decay.[4]

The purpose of this note is to point out that this similarity of Dalitz plots [which, incidentally, is in reasonable agreement with the data available

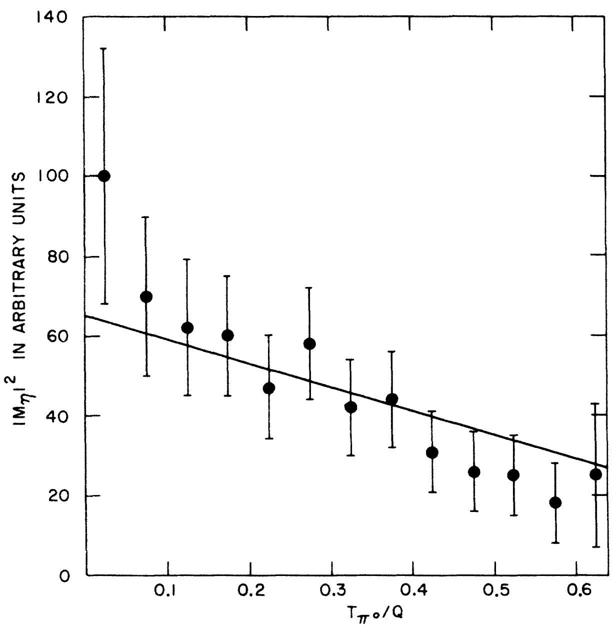

FIG. 1. The experimental points are from a compilation of $\eta\to\pi^+\pi^-\pi^0$ data kindly supplied by A. Pevsner (private communication). The straight line, arbitrarily normalized, is the best linear fit to the π^+ spectrum in $\tau^{+\prime}$ decay.

(Fig. 1)] is a rather general consequence of the quantum numbers attributed (0^{-+}, $T=0$) to the η. Thus, while this result cannot be used as a test of the specific decay models mentioned above, it nevertheless serves a more useful purpose as a check on the assignment of quantum numbers. For this reason the present note should not be taken as detracting in any measure from the contribution in BR but rather as extending its domain of utility.

We assume for the moment that the η meson is 0^{-+} with $T=0$, so that $C|\eta\rangle=|\eta\rangle$. The 3π decay is then induced, to the lowest order in α, through an operator of the form

$$H=\alpha\int d^4x\int d^4y J_\mu(x)D_F^{\mu\nu}(x-y)J_\nu(y), \qquad (1)$$

where J is the electric current and D_F is the photon propagation function.

Clearly, H commutes with C but has no definite transformation properties under G. We can, however, write the currents as the sum of isoscalar and isovector terms so that H may be recast in the form

$$H=H_S^{\,e}+H_V^{\,o}+H_T^{\,e}, \qquad (2)$$

where the subscripts S, V, T imply, respectively, scalar, vector, second rank tensor, in isospace, and the superscripts indicate the G transformation property.[5] Since the part of H which contributes in η decay must be odd under G, a $|\Delta\vec{T}|=1$ rule is immediately seen to be operative.[6]

Once the isotopic spin of the final state has been fixed to be unity, the time-honored "centrifugal barrier" argument[7] can be used to infer that, in the absence of final-state interactions, the dominant configuration is the completely symmetric one. Inclusion of the final-state interactions would then result in mixing with the unsymmetric $T=1$ states. Since this mixing is independent of the primary decay mechanism, the invariant matrix element for K or η decay may be written in the form

$$M_{i;\rho\alpha\beta\gamma}(s_1,s_2,s_3)=\lambda_i[A(s_1s_2s_3)\delta_{\rho\alpha}\delta_{\beta\gamma}+B(s_1s_2s_3)\delta_{\rho\beta}\delta_{\gamma\alpha}+C(s_1s_2s_3)\delta_{\rho\gamma}\delta_{\alpha\beta}], \qquad (3)$$

where $i=K$ or η, $s_1s_2s_3$ are the usual invariants[3] for 3-body decay, and the A, B, C are all normalized to unity at $s_1=s_2=s_3$. The various decay spectra are then given as follows:

$$|M_\tau|^2=\lambda_K^{\,2}|A+B|^2, \qquad (4)$$

$$|M_{\tau'}|^2=\lambda_K^{\,2}|C|^2, \qquad (5)$$

$$|M_{\eta\to\pi^+\pi^-\pi^0}|^2=\lambda_\eta^{\,2}|C|^2=(\lambda_\eta/\lambda_K)^2|M_{\tau'}|^2, \qquad (6)$$

$$|M_{\eta\to3\pi^0}|^2=\lambda_\eta^{\,2}|A+B+C|=(\lambda_\eta/\lambda_K)^2|M_\tau+M_{\tau'}|^2. \qquad (7)$$

Equation (6) provides the requisite identification of Dalitz plots. Furthermore, Eqs. (6) and (7) may be used to infer the branching ratio $R(\eta\to3\pi^0)/R(\eta\to\pi^+\pi^-\pi^0)$. If the τ-decay data are adequately described by the linear (in the kinetic energy of the odd pion) fit of Gell-Mann and Rosenfeld,[8] the above branching ratio is obviously unaffected by the final-state interaction and remains[9] $(9/3!)=1.5$.

Reference to Table I indicates that among the various quantum numbers suggested in the literature[10] the 0^{-+}, $T=0$ assignment is, in fact, unique in predicting a connection between the Dalitz plots for η and τ' decays.

In conclusion we should perhaps emphasize the tacit assumption made above, namely that the

Table I. η decay for various quantum numbers.

Quantum numbers	$\lvert\Delta\vec{T}\rvert$ rule	T_{final}	Relationship to τ decay
$T=0,\ 0^{-+}$	1	1	Charged mode similar to τ' (or τ^0)
$T=0,\ 0^{--}$	0, 2	0, 2	None
$T=1,\ 1^{--}$	0, 2	1, 3	None
$T=1,\ 2^{++}$	1	0, 2	None

structure of the decay amplitudes for K and η is determined by the final-state interactions. If either of the amplitudes contains important intrinsic P-wave structure, the terms in the respective spectra which should be compared are not the linear but rather the quadratic terms calculated, for example, by Barton and Kacser.[11] For these terms the correspondence still holds.

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*Work performed under the auspices of U. S. Atomic Energy Commission.

[1]G. Barton and S. P. Rosen, Phys. Rev. Letters 8, 414 (1962). Hereafter called BR. References to experimental work are contained in this paper.

[2]M. Gell-Mann, Phys. Rev. 125, 1067 (1962); M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters 8, 261 (1962).

[3]M. A. B. Bég and P. C. DeCelles, Phys. Rev. Letters 8, 46 (1962).

[4]This result, as well as the results of the present paper, are correct only to the extent that the 10% mass difference between K and η may be ignored. (See the relevant discussion in BR.) Also the πK and $\pi\eta$ interactions should be negligible over the decay spectrum; this is substantially correct since the relevant thresholds are far removed from the physical region.

[5]G. Feinberg and A. Pais (to be published) have noted independently the utility of exploiting this property.

[6]It should perhaps be noted that C invariance alone gives the final isotopic spin to be 1 or 3; the latter is eliminated if terms of order α^2 are ignored.

[7]R. H. Dalitz, Phys. Rev. 94, 1046 (1954).

[8]M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nuclear Sci. 7, 407 (1957).

[9]Quadratic terms will, of course, affect the branching ratio. The effect of these terms is being investigated by K. C. Wali (private communication from R. F. Sawyer).

[10]G. Shaw and D. Wong, Phys. Rev. Letters 8, 336 (1962); M. Ross, Phys. Rev. Letters 8, 417 (1962).

[11]G. Barton and E. Kacser, Phys. Rev. Letters 8, 228, 353(E) (1962).

EXPERIMENTAL TESTS OF THE $\Delta I = \frac{1}{2}$ RULE, AND THE $\Delta S = +\Delta Q$ RULE IN THREE-BODY DECAYS OF NEUTRAL K MESONS*

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We present the results of an experiment using the three-body decays of netural K mesons to test three rules that have been suggested to hold in strange-particle decay:

1. The rule[1-3] $\Delta I = \frac{1}{2}(NL)$, for nonleptonic ($NL$) decays. This rule is in agreement with many observations.[4]

2. The rule[5] $\Delta I = \frac{1}{2}(L)$, for strangeness-changing leptonic (L) decays. Here ΔI refers to the isotopic spin changes of the strongly-interacting particles.

3. The rule[6] $\Delta S = +\Delta Q$, correlating the changes in strangeness (S) and charge of the strongly-interacting particles in S-changing L decay.

The rules $\Delta I = \frac{1}{2}(L)$ and $\Delta S = +\Delta Q$ are not independent. Three amplitudes $a(\frac{1}{2}, \frac{1}{2})$, $a(\frac{3}{2}, \frac{1}{2})$, and $a(\frac{3}{2}, \frac{3}{2})$ suffice to describe L decays of charged and neutral K mesons. Here, for instance, $(\frac{3}{2}, \frac{1}{2})$ means $\Delta I = \frac{3}{2}$ and $\Delta I_z = \frac{1}{2}$, where ΔI is the isotopic spin difference between the initial and final state, for the strongly interacting particles, and can be thought of as carried by a Wentzel "spurion."[3] The amplitudes are assumed to be independent of the sign of ΔI_z. Assuming CP invariance, using standard spurion technique,[3] and letting L stand for either an electron or a muon, one has, after cancelling a common factor, the decay rates

$$\Gamma(K^0 \to L^+) = \Gamma(\bar{K}^0 \to L^-) = |a|^2 \equiv |-2a(\tfrac{1}{2}, \tfrac{1}{2}) + a(\tfrac{3}{2}, \tfrac{1}{2})|^2,$$

$$\Gamma(\bar{K}^0 \to L^+) = \Gamma(K^0 \to L^-) = |\bar{a}|^2 \equiv |6^{1/2} a(\tfrac{3}{2}, \tfrac{3}{2})|^2,$$

and

$$\Gamma(K^+ \to L^+\pi^0\nu) \equiv \Gamma_+(L^+) = |\sqrt{2}\,a(\tfrac{1}{2}, \tfrac{1}{2}) + \sqrt{2}\,a(\tfrac{3}{2}, \tfrac{1}{2})|^2.$$