NONLINEAR MAGNETORESISTANCE IN BISMUTH

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Recently Esaki[1] reported an experiment on Bi in which he observed an abrupt increase of the current in magnetic fields at low temperatures above a certain critical electric field. When the densities of negative and positive carriers are assumed to be equal, the Hall field becomes almost zero at high magnetic fields and low temperatures where the condition $\omega_c\tau \gg 1$ is fulfilled (where ω_c = cyclotron frequency, τ = carrier relaxation time). Therefore, the electric field E lies in the direction, x, of the total electric current. In this case, contrary to a weak magnetic field case, the carriers of both sign drift in the direction perpendicular to both the electric and magnetic field which are perpendicular to each other. Then, only the scattering gives rise to a net electric flow by shifting gliding centers. According to Esaki, the scattering processes in which the carriers emit phonons increase rapidly when the drift velocity, $v_d = cE/H$, exceeds the velocity of sound, c_p. This increase is a manifestation of the energy conservation and the Pauli principle for degenerate carriers.

This theory seemed very attractive, so we were tempted to make it quantitative. Our result shows, however, that the effect to be predicted by this mechanism seems too small if the phonons are assumed to be in thermal equilibrium. The calculation, which is described briefly in the following, assumes noncoherent emission of phonons. It may be that a coherency actually exists, as Esaki suggested, as a possible explanation of spontaneous appearance of electrical oscillation. Such a coherency may increase the probability of scattering considerably. A phenomenological approach was reported recently by Hopfield.[2]

For the sake of simplicity, carriers are assumed to be electrons only. The unperturbed Hamiltonian for electrons is obtained from an electronic band energy $E(p)$ by replacing the crystal momentum p by the quasi-momentum π. The perturbation due to the electron-phonon interaction is given by

$$\mathcal{H}_1 = \sum_{\vec{q}} \left(\frac{\hbar}{2M\omega_{\vec{q}}}\right)^{1/2} \left[C(\vec{q}) b_{\vec{q}} e^{i\vec{q}\cdot\vec{r}} + C(\vec{q})^* b_{\vec{q}}^{\,} e^{-i\vec{q}\cdot\vec{r}} \right]. \tag{1}$$

The calculation starts from an expression of current

$$j_x = e^2 E \int_0^\infty dt \int_0^\beta d\lambda \, \mathrm{Tr}\left[\rho_0(\mathcal{H} + eE\xi) \left(e^{\lambda(\mathcal{H}+eE\xi)} \dot{X} e^{-\lambda(\mathcal{H}+eE\xi)} \right) \times \left(e^{i(\mathcal{H}+eEx)t/\hbar} \dot{X} e^{-i(\mathcal{H}+eEx)t/\hbar} \right) \right], \tag{2}$$

which is a slight generalization of the basic expression of current obtained by one of the authors.[3] This generalization enables us to take account of the nonlinear effect to be considered here. In (2), $\mathcal{H}$, ξ, x, and X represent second-quantized operators. The center coordinates (X, Y) and the relative coordinates (ξ, η) have been defined previously.

Evaluation of Eq. (2) to the lowest order in $\mathcal{H}$ yields in the limit $T = 0°\mathrm{K}$

$$j_x \cong \frac{e^2}{\omega_c^2} \frac{1}{6(2\pi)^3 \hbar \rho c_p} (E - E_k) \left(1 - \frac{E_k(E+E_k)}{2E^2} \right) \times \int_0^{(8m^*\zeta)^{1/2}/\hbar} dq q^2 |C(q)|^2 \quad (\text{for } E > E_k)$$

$$\cong 0 \quad (\text{for } E \leqslant E_k), \tag{3}$$

where $E_k = Hc_p/c$ is the critical field above which electrons at 0°K are allowed to emit phonons. The differential conductivity above E_k is given by

$$\frac{dj_x}{dE} \simeq \frac{e^2}{\omega_c^{\,2}} \frac{C^2(8m^*\zeta)^{5/2}}{30(2\pi)^3\hbar^7\rho c_p}\left(1-\frac{E_k^{\,3}}{E^3}\right), \tag{4}$$

where the electron-phonon coupling $C(q)$ is assumed to be Cq. In deriving Eqs. (3) and (4) we assume an isotropic electronic band with an effective mass m^*. Furthermore, the matrix element for transition is replaced by that taken between plane waves, and the oscillation of de Haas-van Alphen type is omitted.

The calculation is very tedious at finite temperatures. As a reasonable approximation, however, we assume that the nonlinear current above the critical field, E_k, is temperature-insensitive. Therefore, the current-voltage curve at finite temperatures can be constructed from the usual Ohmic part and the additional non-Ohmic current calculated above.

The curves thus obtained show kinks at critical fields, E_k, which depend on the magnetic field. These curves resemble those obtained experimentally but two important discrepancies are immediately observed. In the first place, the theoretical nonlinear effect is too small. At 2°K the ratio of the differential conductivity at $E = 1.2E_k$, for instance, to that at smaller fields ($E < E_k$) is calculated to be 1.23 if m^* is assumed to be about $0.06m$, ζ/k about 205°K, and c_p about 10^5 cm/sec. The reason for this small nonlinearity is the fact that the phonons with small q's are present in fairly high concentration even at 2°K and thus the electrons can be scattered by absorbing phonons. On the other hand, the experimental value for this ratio is 16 for $H = 10$ kilogauss and 45 for $H = 20$ kilogauss. This is too great a discrepancy. Secondly, the variation of differential conductivity above E_k as given by Eq. (4) does not agree with the experiment. The theory predicts H^{-2} dependence whereas the experiment shows negligible dependence on the magnetic field.

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SUPERCONDUCTIVITY NEAR IMPURITIES

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Recently a connection has been established between resonant scattering of conduction electrons by transition element impurities in dilute solid solution, and localization of a magnetic moment around the impurity.[1-3] One of the authors has speculated[4,5] that resonance effects could also lead to a similarly localized region of coherent pairing of electrons, such as is characteristic of the Bardeen-Cooper-Schrieffer theory of superconductivity.[6]

In this note we shall make this idea more precise and derive some conditions for such local superconductivity. We consider a model in which the impurity potential is strongly localized (has a constant matrix element) and the one-electron conduction band is characterized by a single peak in its density-of-states curve. In contrast with the localized moment result, the resonance must not be too close to the Fermi level. When this condition is met we arrive at the following conclusions:

If the electron-electron interaction is repulsive, local superconductivity does not occur in this model.

When the electron-electron interaction is attractive, a local superconducting solution can arise. Since the solvent as a whole can then be superconducting, the present solution is of possible interest when the bulk of the solvent is made normal by a magnetic field in excess of critical. Energetic considerations then could favor the locally superconducting regime ("hard" superconductors). Another relevant case (presently under study) could be the increase in transition temperature, occurring upon the quenching of certain alloy systems.[7]

Our analysis employs the Green's function method[8] (here leading to the same result as the Bogoliubov method[9] employed in reference 4). We assume a charged, nonmagnetic impurity, exerting a very short range self-consistent potential V.