dinal and have their highest frequency at about ω_p. The plasma electrons, driven by the beam through the longitudinal oscillation mode, continue to make ionizing and exciting collisions, and the limiting plasma density is presumably governed by the rate of leakage out of the system.

It is anticipated that the use of longer pulses, higher power beams, and better and stronger mirror fields will result in the production of denser and hotter plasmas. Such experiments are now being prepared.

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MICROWAVE EMISSION AND ABSORPTION AT CYCLOTRON HARMONICS OF A WARM PLASMA

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We present results of measurements on the microwave emission from and absorption by a plasma column in a magnetic field. In the experiments the plasma frequency ω_p was of the order of, or greater than, the microwave radian frequency ω, and the electron collision frequencies were much less than ω ($10^{-4} \leq \nu/\omega \leq 10^{-1}$). Under such conditions, the measured magnitudes and spectra of the emitted and absorbed radiation are found to differ greatly from those predicted theoretically on the basis of models of an infinite, uniform plasma near thermal equilibrium. The magnitudes of the emission and absorption lines of the electron cyclotron harmonics are several orders of magnitude greater than those calculated for a warm plasma with a Maxwellian distribution of electron velocities. Furthermore, the observed line shapes are usually not Lorentzian.

Calculations[1] show that the power absorption coefficient $\alpha = 2\,\mathrm{Im}k$ of a monochromatic plane wave, with a propagation vector $\vec{k}$ in a uniform warm plasma with a Maxwellian velocity distribution at a temperature T, exhibits a series of maxima at the harmonics of the electron cyclotron frequency $\omega_b = eB/m$, where B is the applied magnetic field. A condition for the appearance of these harmonics is that $\vec{k}$ be not exactly parallel to $\vec{B}$. The relative magnitude of the successive harmonics decreases approximately as $(\mathrm{Re}n)KT/mc^2$, where n is the complex refractive index of the plasma $\vec{n} = c\vec{k}/\omega$. In our experiments the refractive index $\mathrm{Re}n$ of the plasma was determined to be close to unity, and the mean electron energy lay between 1 and 10 eV; thus, contrary to observation, no significant variation in α should occur at harmonics higher than the second.

Since, in a plasma whose electrons have a Maxwellian distribution, the absorption and the incoherent emission are related by Kirchhoff's law, the emission spectrum should be similar to the predicted absorption spectrum. Therefore, successive harmonics in emission should decrease as $(\mathrm{Re}n)KT/mc^2$, and, as in the case of absorption, no emission peaks beyond the second should be observed.

In Fig. 1 are shown plots of the absorption of a microwave signal at a fixed frequency of 5000 Mc/sec as a function of the magnetic field B, for three different electron densities. In this experiment the positive column (diameter = 1 cm) of a weakly

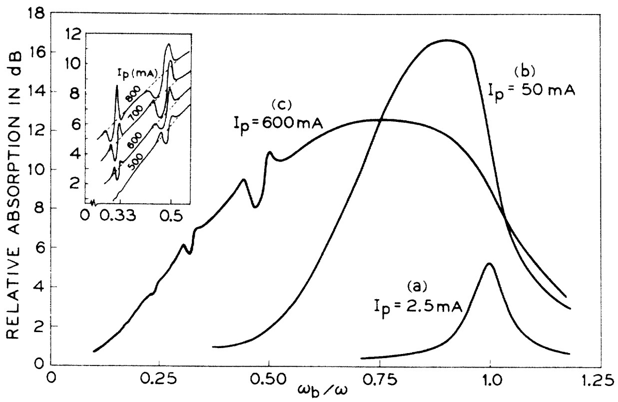

FIG. 1. Microwave power absorption in a cavity-plasma system as a function of magnetic field. The plasma is the positive column of a helium discharge at a pressure of 0.5 mm Hg.

ionized hot-cathode arc discharge in helium was placed coaxially in a cylindrical cavity that was excited in the TE_{011} mode. The static magnetic field was oriented parallel to the plasma column. Such a plasma-cavity system simulates[2] the interaction between a plasma and a plane extraordinary wave propagating at right angles to B. The quantity plotted in Fig. 1 is proportional to the $1/Q$ value of the plasma-cavity system and thus is a measure of the absorption coefficient α, averaged over the column of the plasma.

Curve (a) of Fig. 1 is for a low-density plasma for which $\omega_p/\omega \ll 1$. Here the absorption occurs predominantly at $\omega = \omega_b$ and the line can be interpreted on the basis of single-particle absorption leading to a collision-broadened line whose shape approaches a Lorentzian (a strict Lorentzian occurs only in cases where the collision frequency for momentum transfer is independent of the electron velocity).

Curve (b) is obtained when ω_p/ω is close to, but less than, unity. Theory for a uniform plasma[2] predicts a resonant absorption at $\omega^2 = \omega_p^2 + \omega_b^2$, so that, at a fixed frequency of observation, the resonance shifts to a lower magnetic field relative to the cyclotron field $B_c = m\omega/e$. In the experiment, the electron density and thus ω_p are functions of the radial position in the plasma column. This leads to an asymmetrical broadening of the absorption line, in addition to the shift, as is exhibited by curve (b). As ω_p/ω increases, the broadened line smears out until eventually it occupies the whole range from $B \approx 0$ to $B \approx B_c$.

Curve (c), $\omega_p \approx \omega$, exhibits the unexpected anomalous behavior near the second, third, fourth, and fifth cyclotron harmonics. As many as eight harmonics have been observed. The amplitudes of the successive harmonics, although they are decreasing, are of the same order of magnitude. The study of the amplitude of a given harmonic with discharge current reveals a rapid onset at currents of the order of 400 mA, and a rapid increase in amplitude with current from 500 to about 800 mA. The line shape is peculiar (insert in Fig. 1) because the center portion of the line is inverted and dips below the background absorption. The line broadens asymmetrically with increasing current (see insert Fig. 1). The broadening of the lines is not wholly collisional; the third harmonic is narrower than the second. Also, varying the neutral gas pressure from 0.8 to 0.3 mm Hg had no appreciable effect on the linewidths of the higher harmonics. Studies were also made of the size and shape of the harmonics as a function of the microwave power fed into the microwave cavity. No changes were observed for power inputs ranging from 4 mW to 0.1 mW.

In addition to the absorption, we measured the emission of radiation from plasmas similar to those described above. Curve (a) of Fig. 2 shows a plot of the power radiated from the positive column of a hot-cathode arc discharge in argon, as a function of magnetic field. In this experiment the positive column was mounted within, and nearly coaxially with, a section of S-band waveguide, and the applied magnetic field was oriented along the plasma column. The emitted microwave power flowing down the waveguide was measured at a fixed frequency of 3000 Mc/sec and within a bandwidth of 2 Mc/sec. We observe emission lines superimposed on an emission background. The explanation for this background is similar to that given for the absorption background of Fig. 1. This experimental arrangement, in contrast with that for absorption, simulates plane waves being propagated at an angle $\theta = \sin^{-1}(\omega_c/\omega)$ to the applied magnetic field, where ω_c is the cutoff frequency for the waveguide containing the plasma.

Again, the amplitudes of the successive harmonics are essentially of the same order of magnitude. By varying the pressure from 0.3 to 0.9 mm Hg, no significant change in the linewidths or heights was observed; however, as the pressure was increased to 2 mm Hg, a sharp falloff in the amplitudes was found to occur. Measuring the shape of a spectral line at different frequencies of observation ω gives a rough estimate of the dependence of the effective angle of propagation θ. We found that for frequencies between 3800

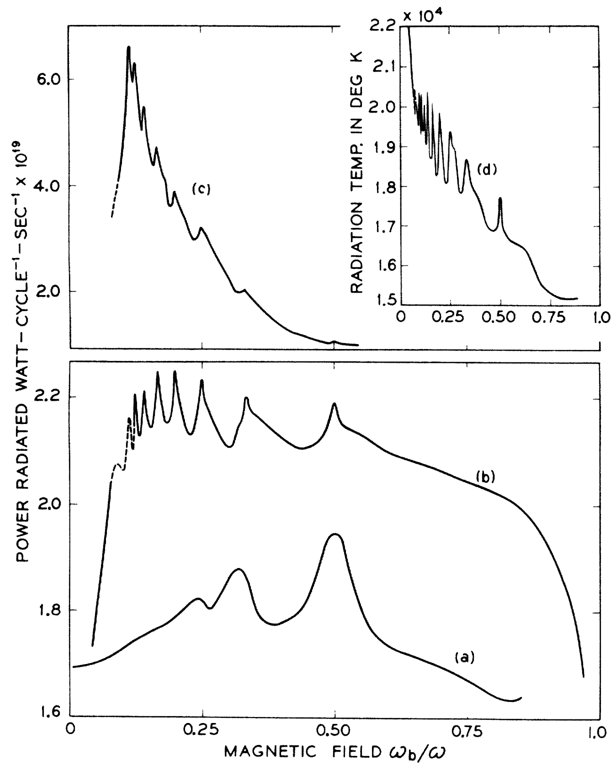

FIG. 2. Microwave power radiated as a function of the applied magnetic field. Curve (a) is the emission from a weakly ionized arc discharge in argon; gas pressure = 0.6 mm Hg, discharge current = 1 A. Curve (b) is the emission from a weakly ionized discharge in mercury vapor; pressure = 0.0054 mm Hg, current = 1 A. Curve (c) is the emission from a highly ionized arc discharge in argon; pressure = 0.002 mm Hg, current = 20 A. Curve (d) is the radiation temperature of the mercury vapor discharge; pressure = 0.003 mm Hg, discharge current = 1 A.

and 2400 Mc/sec $(33^\circ < \theta < 59^\circ)$ the amplitude of a given line relative to the background did not change, but its width increased with decreasing frequency.

In addition to the above emission measurements on weakly ionized plasmas at intermediate gas pressures, we studied the emission spectra from a weakly ionized, low-pressure discharge in mercury vapor [curve (b)], and from a highly ionized, low-pressure hollow-cathode-arc discharge[3] in argon [curve (c)]. In the last case, the radiation was measured at a frequency of 9000 Mc/sec with a microwave horn oriented so as to receive radiation emitted at right angles to the magnetic field, that was colinear with the arc column. In plotting curve (c) the receiving characteristics of the horn are eliminated by a calibration of the system and the results are given in terms of the effective power that the arc would radiate into a waveguide, rather than into free space.

The emission spectra of the three plasmas shown in Fig. 2 have characteristics that are similar to those of the absorption spectrum of Fig. 1, that is, the amplitudes of the successive harmonics are of the same order of magnitude and their widths decrease slightly with increasing harmonic number. The emission lines, notably curves (b) and (c), exhibit an asymmetry which appears to be correlated with the shape of the absorption lines shown in Fig. 1. This is particularly noticeable in the third, fourth, and fifth harmonics of Fig. 2(c) where the lines appear to exhibit an inversion similar to that shown in Fig. 1.

The physical mechanism for the anomalous absorption and emission described here is not understood. We have observed the phenomena in weakly and highly ionized plasmas of low and intermediate gas pressures. Anomalous emission was found also in a P.I.G. discharge.[4] The effects cannot be interpreted on the basis of relativistic motions of the electrons, and calculations show that it is unlikely that the effects are the result of a distortion of the Larmor orbits by ambipolar space charge fields. The mechanisms responsible for the harmonics could be different in the low- and the high-pressure regimes. At low pressures, and especially in a P.I.G. discharge or in the highly ionized arc, there generally exist groups of electrons with high drift velocity, which could lead to a beam-plasma type of interaction. It is difficult to conceive of such a mechanism at the intermediate pressures ($p \approx 0.5$ mm Hg), where the electron drift velocity is very much smaller than its random velocity so that the electron distribution function should be very nearly spherically symmetric.

The anomalous absorption and emission appear at magnetic field strengths that are often smaller than the critical fields for instability measured by Hoh and Lehnert[5]; that is, the plasmas were generally operated in the quiescent regime where the Kadomtsev and Nedospasov[6] instability has not yet set in.

The anomalous emission and absorption may have its origin in the fact that the electron distribution function in an active plasma generally departs from a Maxwellian. We have ascertained that such departures from a Maxwellian exist in the mercury plasma of Fig. 2(b) by measuring, in a separate experiment, the plasma radiation temperature [Fig. 2(d)] defined[7] to be proportional to the ratio of emission to the absorption coefficients. For a plasma with a Maxwell distribution the radiation temperature should be a monotonically decreasing function with increasing magnetic field,[7] and should be representative of blackbody emission from the plasma. The fact that the plot in Fig. 2(d) exhibits pronounced maxima at cyclotron harmonics is evidence that the electron distribution is not Maxwellian.

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OBSERVATION OF QUANTUM PERIODICITY IN THE TRANSITION TEMPERATURE OF A SUPERCONDUCTING CYLINDER*

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Deaver and Fairbank[1] and Doll and Nabäuer[2] have shown experimentally that the flux which is trapped in a superconducting cylinder is an integral multiple of the unit $hc/2e$. It has been pointed out[3,4] that this result follows because the free energy of the superconducting state is periodic in this unit of the flux if the electrons are paired in the manner described by the Bardeen-Cooper-Schrieffer (BCS) theory.[5] The free energy of the normal state, on the other hand, is virtually independent of the flux. Consequently, the transition temperature T_c, which is the temperature at which the free energy of the normal and superconducting states are equal, must also be a periodic function of the enclosed flux ϕ. The magnitude of the change in T_c was calculated for a thin cylindrical sample using the BCS model in which the possible pairing of particles with net momentum was included. This calculation showed that the binding energy of each pair was reduced by the amount of energy required to provide the center-of-mass motion necessary to maintain the fluxoid,

$$\frac{1}{h}\oint\left(m\vec{v}_s+\frac{2e}{c}\vec{A}\right)\cdot d\vec{s},$$

an integer. Each integer n corresponds to a different superconducting state characterized by a particular pairing arrangement and a different transition temperature. The transition temperature is found to vary as

$$\Delta T_c=\frac{\hbar^2}{16m^*R_0^{\,2}}\left(\frac{2e}{hc}\phi+n\right)^2.$$

The choice of n which gives the tightest binding and the highest transition temperature switches from 0 to -1, -1 to -2, etc., when ϕ is given by $\frac{1}{2}(hc/2e)$; $\frac{3}{2}(hc/2e)$, etc. We note also that the binding energy of the pair is a minimum at these points and varies periodically with the flux. At the transition temperature the penetration depth becomes infinite and consequently the flux ϕ, enclosed by the cylinder, is determined entirely by the external field. T_c is given by a periodic array of parabolas, each of which is centered on a flux unit (see Fig. 1). One can estimate the expected magnitude of ΔT_c by taking $m^*=m_e$ and a reasonable diameter of say 1 micron for the cylinder. ΔT_c is then approximately 5×10^{-5} K° which is of measurable magnitude in the liquid helium temperature range.

We have observed such an effect with a thin