our approximate eigenstate is

$$\Gamma/2 = \left| \frac{\mathrm{Im} D(\omega_0 + i\epsilon)}{\mathrm{Re} D'(\omega_0)} \right|. \quad (3)$$

We expect the Debye model to be adequate for very heavy impurities, since then $\omega_0 \ll \omega_D$. Using it we find, with $x = \omega/\omega_D$,

$$D(\omega + i\epsilon) = 1 - \frac{3\Delta m}{m} x^2 \left(1 - \frac{x}{2} \ln \left| \frac{1+x}{1-x} \right| \right) - \frac{3\pi i}{2} \frac{\Delta m}{m} x^3. \quad (4)$$

For $\Delta m/m \gg 1$, we find

$$x_0 = \omega_0/\omega_D = (m/3\Delta m)^{1/2},$$

$$\Gamma = \tfrac{1}{6}\pi (m/\Delta m)\omega_D,$$

$$\Gamma/\omega_0 = \pi x_0/2. \quad (5)$$

It is thus seen that for $\Delta m/m \gg 1$ one may have quite a sharp resonance in the continuum. For example, a mass ratio of 25 in a material of Debye temperature 300°K (such as a heavy metal placed in a light molecular solid by evaporation) would give $\omega_0 \cong 3 \times 10^{-3}$ eV and $\Gamma/2 \cong 3 \times 10^{-4}$ eV. The cross section for absorption of Mössbauer γ rays will have a peak at ω_0 with width Γ and a height which can be shown to be approximately x_0^{-2} times the one-phonon background, a factor of 100 for the example above.

A more direct experiment to pick up the approximate eigenstate is a measurement of neutron transmission. Scattering should be enhanced by x_0^{-2} at ω_0, and a neutron experiment would yield both the location and width of the resonance.

We are grateful to Dr. Paul Craig and Dr. Darragh Nagle for discussions on the feasibility of this experiment.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

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OSCILLATORY INTERBAND FARADAY ROTATION AND VOIGT EFFECT IN SEMICONDUCTORS

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(Received June 21, 1962)

We have performed a preliminary experiment on thin Ge samples in fields up to 90 kG at room temperature in order to investigate the oscillatory behavior of the interband Faraday rotation predicted theoretically.[1] The infrared radiation from a monochromator was polarized with a Polaroid sheet and focussed on a sample 4 microns thick which was placed in the center of a Bitter solenoid. The transmitted radiation was split into two beams, each being focussed on a PbS detector. Analyzers in front of both detectors had their planes of polarization perpendicular to each other and 45 degrees to that of the polarizer. The signal outputs of the detectors were balanced out to zero in the absence of a magnetic field. With the field on, the Faraday rotation in the sample gave rise to an unbalanced component of signal output which was calibrated in terms of actual rotation in the plane of polarization. The results of our experiments are shown in Figs. 1 and 2. The oscillatory character is shown prominently at 23 kG which has eleven minima and maxima within the range of 0.1 eV above the energy gap. With increased field, only a few peaks are observed within this range of energy. However,

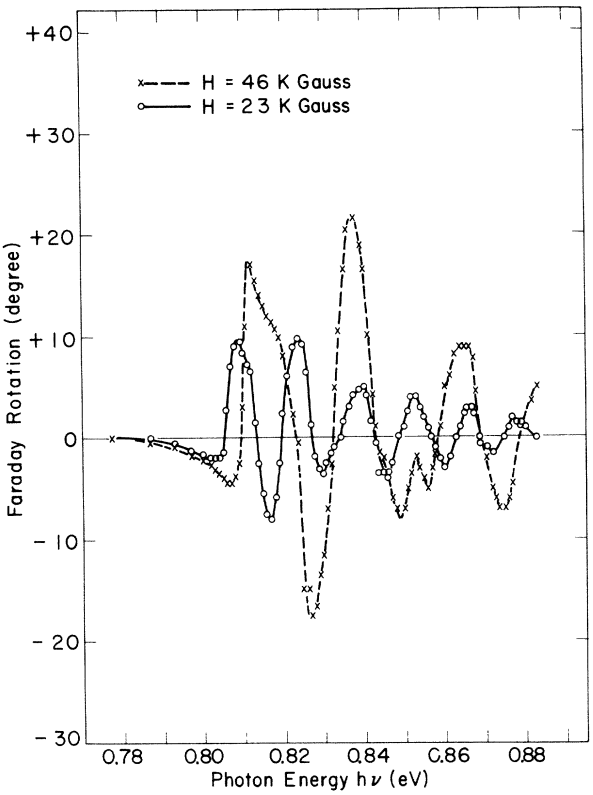

FIG. 1. The Faraday rotation in a Ge single crystal for weak magnetic fields as functions of incident photon energy at room temperature. The field was applied in [110] direction.

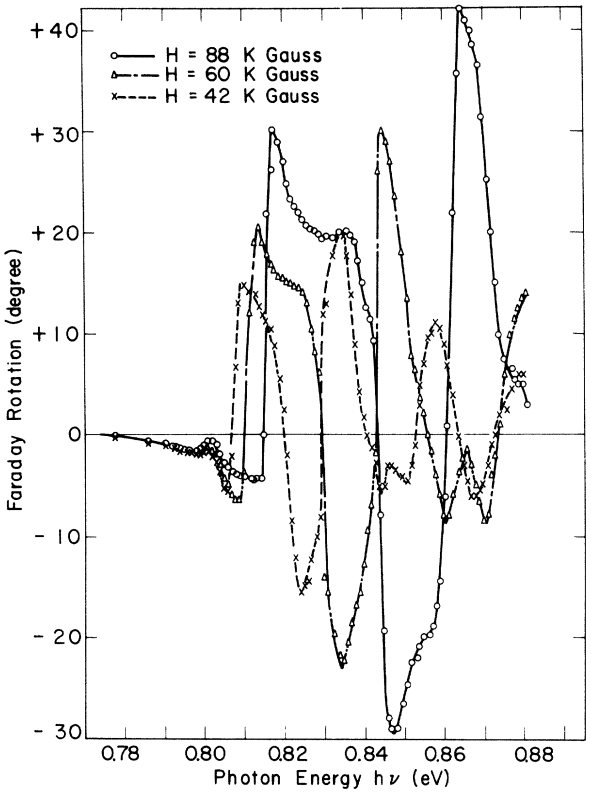

FIG. 2. The Faraday rotation in the second Ge sample for strong magnetic fields at room temperature. The field was applied in [110] direction.

there is evidence of fine structure as seen in Fig. 2. The lines not only broaden, but show subsidiary maxima and minima on the original peaks. The first few peaks correlate quite exactly with the data of Zwerdling, Lax, and Roth[2] obtained in magnetoabsorption experiments and extrapolate to the energy gap of 0.803 eV as before. In the case of Faraday rotation, we are able to deduce to a good approximation a linear relationship of the peak amplitude as a function of the magnetic field. This is most evident for the first few peaks which we have observed up to the maximum field available.

From the theory the important term of the Faraday rotation, θ, is the term with the singularity which is of the form

$$\theta = (A\omega_c/\omega)\sum_n \tau_n^{3/2}\Delta\omega_n (d/dx_n)\mathrm{Re}(X_n + j)^{-1/2}, \quad (1)$$

where A is a constant determined by band parameters, $X_n = (\omega_n - \omega)\tau_n$, $\omega_n = \omega_g + (n + \frac{1}{2})\omega_c$, ω_c = cyclotron frequency for reduced effective mass for electrons and holes, ω_g = frequency for the gap energy, $\Delta\omega_n = \gamma_n H$, γ_n is a phenomenological constant, H is a dc external magnetic field, τ_n is a relaxation time, and ω is optical frequency. The first major line indicated by the positive sharp rise in Fig. 2 is a characteristic dispersion curve which is obtained from the theoretical expression. However, the second major line which is correlated with the ZLR results[2] has the line shape reversed, suggesting a change in the sign of γ_n for this transition. In principle the amplitude of the Faraday rotation for a given transition should also measure the magnitude of this parameter if the value of the coefficient A is known. In practice this is difficult. However, if the Faraday rotation is combined with the oscillatory interband Voigt effect, then it is possible to determine effective g factor or γ_n of each transition from the combined results. From the approximation given by Eq. (1), we can show[3] that the peak-to-peak amplitude is given by

$$\theta_{\max}(n) = 0.6(A\omega_c/\omega)\gamma_n H\tau_n^{3/2} \quad (2)$$

Similarly, for the interband Voigt effect we can show that the phase shift

$$\delta = (A\omega_c/2\omega)\sum_n \tau_n^{5/2}(\Delta\omega_n)^2(d^2/dx_n^2)\,\mathrm{Re}(X_n+j)^{-1/2}, \quad (3)$$

and

$$\delta_{\max}(n) = 0.5(A\omega_c/\omega)(\gamma_n H)^2\tau_n^{5/2}. \quad (4)$$

Therefore,

$$(\delta_{\max}/\theta_{\max})(n) = 0.8\gamma_n H\tau_n. \quad (5)$$

From the line shape study of our experimental data, τ is approximately 10^{-12} sec. If we assume that $|g| \simeq 2$ for an approximate evaluation, then the above ratio given by Eq. (5) becomes the order of unity for our largest field. At lower temperatures, where τ increases, the amplitude of the Voigt effect $\delta_{\max}$ can exceed that of the Faraday rotation $\theta_{\max}$. Furthermore, if the g factor is anomalously large, then the oscillatory Voigt effect becomes more pronounced than the Faraday rotation.

Experiments of ZLR[2] show that the line shape of magnetoabsorption for the indirect transition is a step function with a finite width.[4] It can be represented by $\tan^{-1}X_{ij}$, where $X_{ij} = (\omega_{ij}-\omega)\tau_{ij}$ and $\omega_{ij} = \omega_g + \omega_p + (n_i+\frac{1}{2})\omega_{c1} + (n_j+\frac{1}{2})\omega_{c2}$.[4] From the dispersion $\sim\ln(X_{ij}^2+1)$ it follows that Faraday rotation for indirect transition is of the form

$$\theta \sim \sum_{ij}\tau_{ij}\gamma_{ij}HX_{ij}/(X_{ij}^2+1). \quad (6)$$

The Voigt effect has the phase shift, δ, of the form

$$\delta \sim \sum_{ij}(\tau_{ij}\gamma_{ij}H)^2(1-X_{ij}^2)/(1+X_{ij}^2)^2. \quad (7)$$

It is evident from the analytic form of Eqs. (6) and (7) that an oscillatory pattern for the indirect interband Faraday rotation and Voigt effect would be expected above the energy gap. The ratio of $\delta_{\max}/\theta_{\max}$ (indirect) is of the order $\gamma_{ij}H\tau_{ij}$. Since the indirect exciton exhibits the same line shape experimentally as that of the corresponding magnetoabsorption, Eqs. (6) and (7) should approximate the Faraday and Voigt effect of the exciton. Hence the g factor for the exciton as well as for the magnetic transitions will be determined from these experiments.

We are grateful to Dr. S. Zwerdling for loaning us Ge samples, and to Mr. R. Newcomb for his assistance in the experiments.

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†Operated with support by the Air Force through the Air Force Office of Scientific Research.

‡Operated with support from the U. S. Army, Navy, and Air Force.

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RESONANT HARMONIC GENERATION IN RUBY

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(Received March 19, 1962)

Harmonic generation in a nonlinear medium is a standard procedure at low and microwave frequencies, and has recently been accomplished at optical frequencies.[1] In this harmonic generation the nonlinear medium is nonresonant with respect to either the fundamental or the harmonic. In the following, an experiment is described in which harmonic generation occurs in a medium which is resonant with respect to both the fundamental and the harmonic. It is related to phenomena which are often referred to as multiquantum processes.

Senitzky[2] has predicted that in a quantum-mechanical three-level system, where the transition probabilities do not vanish, there should occur a mixing effect if the rf fields of frequencies corresponding to two of the energy level separations are present; i.e., if two rf fields are absorbed, there is emission at the sum frequency. The two absorbed frequencies used, in this case, were