PRECISE NEUTRON AND PROTON FORM FACTORS AT LOW MOMENTUM TRANSFERS*

D. J. Drickey and L. N. Hand†

High-Energy Physics Laboratory, Stanford University, Stanford, California

(Received November 13, 1962)

We have measured precise values (1-2%) of the ratio of elastic electron-deuteron scattering to elastic electron-proton scattering in an attempt to determine the neutron charge form factor G_{En}.[1] The precision of the well-known experiments on the neutron-electron interaction[2-4] is so great that previous electron scattering data are essentially qualitative by comparison. Thus, the only firm conclusions regarding the sign and magnitude of G_{En} involve, for typical electron scattering momentum transfers, an extrapolation of at least eight orders of magnitude in the momentum transfer. We felt it desirable to have an independent measurement of G_{En}.

Some byproducts of this experiment, which spans the range of momentum transfer $0.3 \leq q^2 \leq 2.2$ F^{-2}, are absolute values for the proton charge form factor G_{Ep} and, at $q^2 = 1.6$ F^{-2}, a straight-line plot verifying the Rosenbluth formula and yielding the proton magnetic form factor G_{Mp} relative to G_{Ep}.

The neutron-electron interaction experiments measure $dG_{En}/d(q^2) = +0.021 \pm 0.001$ F^2 at $q^2 = 0$. The best previous data on electron-deuteron scattering are those of Lehmann[5] and of Friedman, Kendall, and Gram.[6] The latter deduce an average value for $G_{En} \approx -0.04 \pm 0.04$ $(1 \leq q^2 \leq 8)$. At $q^2 = 1.0$ F^{-2}, Lehmann has reported $G_{En} = +0.014 \pm 0.009$, a value consistent with the neutron-electron interaction. In principle, other experiments, e.g., pion electroproduction and electron-deuteron quasi-elastic scattering, can yield information on G_{En}, but the sensitivity is more than an order of magnitude less. For quasi-elastic electron scattering, the neutron charge scattering is incoherent with the proton charge scattering and a 4% combined theoretical and experimental uncertainty in the cross section can reveal only the limit $G_{En} \leq 0.2\, G_{Ep}$.

In order to interpret the elastic scattering of electrons by deuterons in terms of nucleon structure, it is necessary to have very good knowledge of the long-range structure of the deuteron itself. This seems to be possible, at least with the neglect of purely relativistic corrections.[7]

Justification of the errors quoted here is reserved for a longer article, but it is useful to describe the experiment briefly in order to list the factors we believe have permitted improved accuracy.

A well-defined beam of electrons from the Stanford linear accelerator passed through one of two identical aluminum cups containing either liquid hydrogen or liquid deuterium and then into a Faraday cup previously shown to be 100.0 ± 0.1% efficient for electrons in this energy range.[8] The scattered electrons were momentum analyzed by a double-focusing, zero-dispersion spectrometer[9] calibrated with alpha particles to an absolute accuracy of 0.2%. Linearity and reproducibility of the momentum setting were better than 0.1% over the range 100-280 MeV/c. The variable resolution of the spectrometer allowed the maximum accepted momentum to be set at 1.5% above the central ray and the lowest accepted momentum to be fixed at 1.8 MeV below the central ray. Independent calibrations of the machine energy for each data point were taken by changing the spectrometer resolution to $0.1\% = \Delta p/p$ and measuring both peak shape and peak centering. A single Lucite Cherenkov counter selected for its excellent pulse-height spectrum (14% full width at half-maximum) detected the scattered electrons.

The spectrometer and the two magnets in the energy-analyzing system of the incident beam could be set to better than 0.01% using nuclear magnetic resonance. Any field drift was instantly observed as a resonant frequency shift. This was an important feature of the experiment, because at the highest q^2 value the spectrometer momentum cutoff was only 1.8 MeV or 0.63% below the center of the elastic peak. This high resolution required an understanding of the various contributions to the observed finite peak width and an experimental determination of the actual location of the peak with respect to the spectrometer momentum slits. The radiative correction and predicted actual peak shape under the conditions of observation were calculated by a computer, and the experimental shape was always in good agreement with the predicted shape, being in the range 0.45-0.55%, full width at half-maximum, depending on the point. The radiative correction formula allowed for multiple photon emission[10] and contained small terms involving radiation by the proton or the deuteron. The radiative correction itself was about 25%.

For scattering angles less than 90°, cycling on both sides of the beam was adopted to cancel

Table I. Measured absolute cross sections.

q^2 (F^{-2})	(θ)	$d\sigma/d\Omega$ (10^{-32} cm^2/sr)	% absolute error[a]	Calculated % magnetic scattering[b]
		Proton		
0.300	45°	927	1.1	3.4
0.600	45°	429	1.3	6.5
0.997	45°	229.9	1.2	10.4
1.612	60°	66.7	1.5	18.8
1.607	90°	22.81	1.5	29.6
1.596	120°	10.24	2.3	49.6
1.592	135°	6.94	2.2	64.0
2.200	60°	42.8	1.5	24.2
		Deuteron		
0.300	45°	674	1.3	0.3
0.600	45°	225.4	1.1	0.6
1.002	45°	86.8	1.1	1.1
1.607	60°	15.95	1.7	2.1
1.590	120°	1.71	4.0	5.7
2.200	60°	6.74	2.2	2.9

[a]Errors assigned to relative cross sections are slightly smaller. A 2% error was allowed in the radiative correction.

[b]Assumes scaling of magnetic scattering. See text.

any beam angular misalignment.

Experimental conditions were rigidly controlled to minimize contamination by electrons scattered inelastically from deuterium, but some counts were recorded from this reaction. A correction, never larger than 2%, was deduced by simulating the inelastic continuum with the elastic peak, using a variable incident beam energy. It was also necessary to make a rough determination of the average inelastic cross section at threshold.

Several corrections were necessary to obtain G_{Ep} and G_{En} from the cross sections listed in Table I, as follows:

Proton and deuteron magnetic scattering. The magnetic scattering from the deuteron is expected to be much smaller than from the proton. We have assumed the deuteron magnetic form factor to have the same shape as the deuteron electric form factor, an assumption consistent with our results for $q^2 = 1.6$ F^{-2} (see Table II and Fig. 1), with the best experimental values for G_{Mn} and G_{Mp}, and with reference 9. There is some evidence from the Freidman, Kendall, and Gram 145° data that at larger q^2, $G_{Md} > \mu_d G_{Ed}$, where μ_d is the static deuteron magnetic moment. If this were the case for $q^2 \leq 2.2$ F^{-2}, our values for G_{Ed} and hence G_{En} should be decreased slightly. The largest magnetic scattering contribution is 2.9% at $q^2 = 2.2$ F^{-2}.

Proton magnetic scattering is considerably larger, reaching 24.2% at $q^2 = 2.2$ F^{-2}. We have here made use of our data at $q^2 = 1.6$ F^{-2} and the data of Lehmann et al.[11] at $q^2 = 1.0$, 2.0, and 2.98 F^{-2} in setting $G_{Mp}(q^2) = \mu_p G_{Ep}(q^2)$. The present experiment at $q^2 = 1.6$ F^{-2} yielded

$$G_{Mp} = (1.008 \pm 0.014)\mu_p G_{Ep}$$

(χ^2 for four points equals 3.2).

The error from the uncertainty in this assumption is negligible in our final answer. All values of G_{Ep} quoted in Table II except at $q^2 = 1.6$ F^{-2} are deduced using the above relation between G_{Mp} and G_{Ep}.

Deuteron quadrupole scattering. In the Rosenbluth formula, G_E^2 should be replaced by the incoherent sum of convection current and quadrupole moment scattering. The latter never exceeds 2.2% of the total.

Table III contains a list of the values of F_d and the experimental values for $G_{Ed}/G_{Ep} = F_d(1 + G_{En}/G_{Ep})$ (F_d is the deuteron form factor). G_{Qd} and F_d have been calculated from deuteron wave functions supplied by Lomon.[12]

Deuteron form factor. It may be premature to draw conclusions about the charge structure of the neutron until a better, relativistic, theory of the deuteron is available. For the nonrelativistic theory, however, it may be easily seen from an examination of the results of reference 9 that F_d for $q^2 \leq 2.25$ F^{-2}, at any given q^2, appears to depend

Table IIa. Proton form factors.

q^2 (F^{-2})	(θ)	G_{Ep} [a]	Remarks
0.300	45°	0.9731 ± 0.0054	
0.600	45°	0.9399 ± 0.0061	
0.997	45°	0.8866 ± 0.0055	
1.612	60°	0.8523 ± 0.0066	Straight-line best fit,
1.607	90°	0.8458 ± 0.0064	corrected to 1.600 F^{-2}:
1.596	120°	0.8598 ± 0.0099	$G_{Mp} = (1.008 \pm 0.014)\mu_p G_{Ep}$,
1.592	135°	0.8534 ± 0.0094	$G_{Ep} = 0.850 \pm 0.010$, $G_{Mp} = (0.857 \pm 0.009)\mu_p$.
2.200	60°	0.7900 ± 0.0059	

[a] Assumes $G_{Mp} = \mu_p G_{Ep}$ except where noted.

Table IIb. Deuteron form factors.

q^2 (F^{-2})	(θ)	$G_{Ed}^2 + G_{Qd}^2$ [a]	% scattering by quadrupole moment
0.300	45°	0.6784	...
0.600	45°	0.4611	0.16
1.002	45°	0.3009	0.45
1.607	60°	0.1803	1.2
1.590	120°	0.1814	1.2
2.200	60°	0.1059	2.2

[a] Error here is same as an absolute cross section in Table I, but a smaller error is assigned for the G_{Ed}/G_{Ep} ratio calculation.

only on the value for the triplet scattering length a_t and not specifically on the inner form of the potential. It is evident that models of this type, with a repulsive core and a potential asymptotic to the one-pion-exchange potential lead to near-minimum values of F_d. Increasing the charge near the deuteron center will increase F_d and thus decrease the measured G_{En}. The particular model used in Table III does not differ in F_d significantly from those of reference 9 for these points.

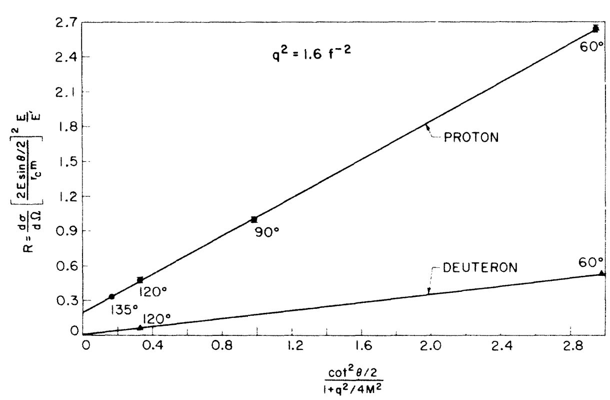

FIG. 1. Straight line plot at $q^2 = 1.60\ F^{-2}$.

Figure 2 and the last column of Table III contain our final answers for G_{En}/G_{Ep} under the above assumptions. The dashed line in the figure represents an extrapolation of the neutron-electron interaction assuming a constant slope for G_{En}. The complete disagreement between this line and the experimental result is clear. A breakdown in quantum electrodynamics, either in the electron vertex or photon propagator, might appear as a single multiplicative factor in electron scattering experiments, thus accounting for our results, but this does not explain an inconsistency with the neutron-electron interaction.

To summarize, we have found the following:

(1) Precise absolute values for G_{Ep}, which agree with those of Lehmann *et al.*

(2) That the proton rms magnetic and electric radii are equal, to 4%.

(3) That $G_{En} = 0.00 \pm 0.01$, $0.3 \leq q^2 \leq 2.2\ F^{-2}$ (F_{1n} is therefore negative).

Alternatively, one may take our results to be an indication of large relativistic corrections

Table III. G_{En}/G_{Ep}.

q^2 (F^{-2})	F_d (calculated)	$G_{Es}/G_{Ep} = F_d(1 + G_{En}/G_{Ep})$	G_{En}/G_{Ep}
0.3	0.842	0.8465 ± 0.0051	+0.005 ± 0.006
0.6	0.728	0.7230 ± 0.0051	-0.007 ± 0.007
1.0	0.615	0.6169 ± 0.0036	+0.002 ± 0.006
1.6	0.494	0.4951 ± 0.0045	+0.002 ± 0.009
2.2	0.410	0.4075 ± 0.0045	-0.006 ± 0.011

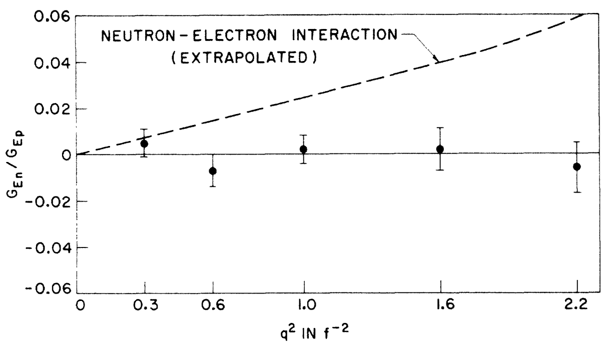

FIG. 2. Experimental values of G_{En}/G_{Ep}.

to the current theoretical picture of the deuteron, yielding an 11 % correction to the cross-section ratio at $q^2 = 2.2$ F^{-2}.

We wish to thank many people for their indispensable support and encouragement, including Professor W. C. Barber, Professor W. K. H. Panofsky, and Professor R. Wilson. A. Browman contributed both the electronics and several excellent ideas. L. Buss was responsible for our successful use of a nuclear magnetic resonance probe in an inhomogeneous magnetic field. We take this opportunity to express our gratitude for the cooperation we have received from the staff of the Stanford High Energy Physics Laboratory and from the Stanford Computation Center.

*This work was supported in part by the Office of Naval Research, the U. S. Atomic Energy Commission, and the Air Force Office of Scientific Research.

†Now at Harvard University, Cambridge, Massachusetts.

[1] $G_E^* = F_1 - (q^2/4M^2)\mu F_2$; $G_M = F_1 + \mu F_2$ are discussed in the reference: L. N. Hand, D. G. Miller, and R. Wilson, Phys. Rev. Letters 8, 110 (1962).

[2] For a review of the neutron-electron interaction, see L. L. Foldy, Revs. Modern Phys. 30, 471 (1958).

[3] D. J. Hughes, J. A. Harvey, M. D. Goldberg, and M. J. Stafne, Phys. Rev. 90, 497 (1953).

[4] E. Melkonian, B. M. Rustad, W. W. Havens, Bull. Am. Phys. Soc. 1, 62 (1956).

[5] P. Lehmann, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN, Geneva, Switzerland, to be published).

[6] J. I. Freidman, H. W. Kendall, and P. A. M. Gram, Phys. Rev. 120, 992 (1960).

[7] N. K. Glendenning and G. Kramer, Phys. Rev. 126, 2159 (1962).

[8] D. Yount, thesis, Stanford University, 1962 (unpublished).

[9] R. A. Alvarez, K. L. Brown, W. K. H. Panofsky, and C. T. Rockhold, Rev. Sci. Instr. 31, 556 (1960).

[10] Yung-Su Tsai, Phys. Rev. 22, 1898 (1961). We have used an approximation to the Tsai formula supplied by D. R. Yennie (private communication to R. Wilson).

[11] P. Lehmann (private communication to R. Wilson); P. Lehmann, R. Taylor, and Richard Wilson, Phys. Rev. 126, 1183 (1962).

[12] The particular model used corresponds to a Hamada potential with an OPEP tail, an effective range $r_t = 1.76$ fermi, and 7 % D state. The form factor was computed by E. Erickson.