Table II. The calculated value of P is compared to the value of P measured by nuclear scattering of alpha particles from the gas target for the two directions of circular polarization of the resonant radiation. $M = \pm I$ corresponds to right- or left-handed polarization, respectively. The alpha-particle energy was 7.33 MeV, and He^3 recoils at 45 degrees (lab) were observed. P (measured) is given by $P = (N_1 - N_2)/(N_1 + N_2)$. $P = P_t P_n$ was calculated from the optically measured value of $P_t = (-M/I)(\Delta I/2I)$, and $P_n = +80\% \times (1 \pm 0.1)$ (see reference 3).

M	P_t (optical)	P (calculated)	P (measured)
$-I$	+8.5% ×(1 ± 0.2)	+6.8% ×(1 ± 0.3)	+6.8% ×(1 ± 0.3)
$+I$	-8.5% ×(1 ± 0.2)	-6.8% ×(1 ± 0.3)	-4.5% ×(1 ± 0.5)

at liquid helium temperatures by "solid" or Overhauser effects. In any case, such low-temperature methods have not as yet been applied successfully to He^3. Although the low atomic density may preclude certain measurements, targets of the type described here should be adequate for a wide variety of nuclear scattering experiments. Furthermore, it is believed that improvements in the optical design may allow a substantial increase in the value of P_t (see reference 1).

These results demonstrate the feasibility of using such polarized He^3 targets for nuclear experiments. It is planned to continue to study the above process and, in addition, to investigate the nuclear reactions produced by the bombardment of such targets by protons, deuterons, He^3, and heavy ions.

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†On leave from North Texas State University, Denton, Texas.

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[2]P. D. Miller and G. C. Phillips, Phys. Rev. 112, 2048 (1958).

[3]G. C. Phillips and P. D. Miller, Phys. Rev. 115, 1268 (1959).

[4]The formula for the polarization P, appearing in reference 1, should read $P = \frac{1}{2}(\Delta I/I)$. This formula applies for polarizations below about 25%, according to calculation, and has been verified experimentally by comparison with nuclear magnetic resonance measurements of the polarization (see reference 1).

K^+-p SCATTERING; THE PHENOMENOLOGY OF A REPULSIVE INTERACTION*

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This Letter is concerned with some recent K^+-proton elastic scattering data[1] in the laboratory energy range 20 MeV to 450 MeV, and their proper interpretation. On the basis of potential scattering, it is demonstrated that these data now yield, besides the range of this dominantly repulsive interaction, a clear indication that the potential is attractive at its outer fringe. It may therefore be possible, by more sophisticated theoretical methods, to learn easily about the elementary K-pion interaction, for example, from this very simple scattering situation.

The experimental data were analyzed[1] to give, in more detail than was previously possible, the energy dependence of the s-wave phase shift δ, the only contributor to scattering at these energies. This energy dependence was then fitted to the effective-range formula $k \cot\delta = a^{-1} + \frac{1}{2}r_0 k^2$, producing the values $a \simeq 0.29$ F, $r_0 \simeq 0.50$ F. We wish to point out that the $k \cot\delta$ expansion is not appropriate to dominantly repulsive interactions, and that when it is used, the parameters a and, especially, r_0 do not have their usual connotation.[2] A more appropriate expansion is obtained, and an inequality established which enables us to draw our conclusion about the sign of the outer fringe of the interaction.

The $k \cot\delta$ expansion has proved very useful in treating attractive interactions, both as a shape-independent representation of potential scattering, and as a phenomenological presumption about interactions of unknown structure. The first term of the expansion, representing scattering by an equivalent zero-range potential, is already a useful approximation. The wave function extends throughout the region of the interaction so that the

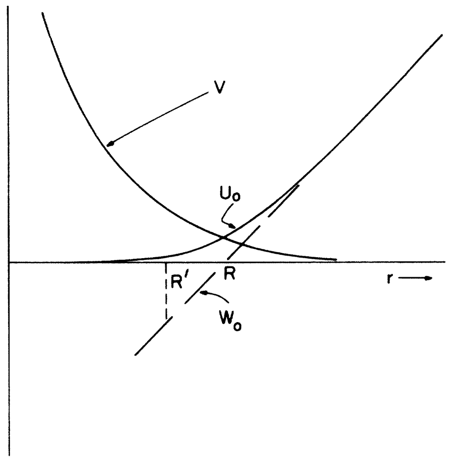

FIG. 1. Sketches of the zero-energy wave function u_0 and its asymptotic form w_0. The value of R' indicated is such as to make $I_0(R')=0$ [see Eq. (5)].

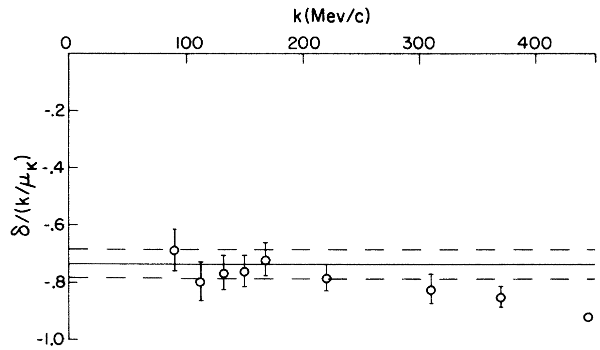

FIG. 2. The quantity $\delta_k/(k/\mu_K)$ is plotted against k, the center-of-mass momentum. μ_K is the K^+ rest energy. The values of δ_k are taken from reference 1. The quoted error on δ at the largest momentum (taken from the second paper) is about as big as that on the neighboring points, but because of possible p-wave modifications it has been omitted here. The solid horizontal line represents the phase shift due to an infinite repulsive core of radius equal to the scattering length $a=0.29$ F of reference 1, and the dashed lines the error of ± 0.02 F on a. Potentials with a soft edge, and everywhere repulsive, would be represented by a parabola, tangent to this at $k=0$, and curving upward.

scattering length represents a spatial average of the potential. The next term corrects for the finite extension of the interaction, r_0 characterizing the range. For a repulsive potential, there is no scattering in the zero-range limit. The wave function drops rapidly to zero near the origin, as shown in Fig. 1. It explores only the outer fringe of the potential, and so the first term in the expansion of δ already represents the range. The natural questions which arise are: What is the appropriate expansion to make for obtaining a more detailed, but shape-independent, picture of the scattering; and what does the next parameter signify? We believe that, with our expansion (2), the sign of the k^3 term gives the sign of the potential in the outer fringe (attractive or repulsive), and its magnitude measures the "softness" of the potential there. The new data on K^+-p scattering seem to predict clearly that the k^3 term is negative, as shown by the downward curvature in Fig. 2, and thus that the tail is attractive.

The formula we are led to, in the case of a dominant repulsion at low energy, is

$$\delta=-kR'-\tan^{-1}k(R-R'), \tag{1}$$

with the very-low-energy approximation[2]

$$\delta\simeq -kR+\tfrac{1}{3}k^3(R-R')^3. \tag{2}$$

R is a constant length which describes the behavior of δ at very low energies. R' is a length which depends on k^2, but which can be assumed constant at low energies. For an infinite square repulsion, R' and R are equal, and it is the difference between these parameters that characterizes the "softness" of the tail of the potential. Simple potentials which can be handled analytically, such as the square and exponential, give phase shifts with a form very similar to Eq. (1). When expressed in these terms, the square repulsive potential, with height V_0, and width a, gives

$$R\simeq a(1-\lambda^{-1}),$$

$$R-R'\simeq 0.793\,\lambda^{-1}-0.219\,\lambda^{-3}(ka)^2,$$

when $\lambda=(2mV_0a^2/\hbar^2)^{1/2}$ is large. As a numerical example, we quote phase shifts for $\lambda=2.6$: The exact phase shift for a kinetic energy of V_0 is -80°, and the value obtained from Eq. (1) is -89°; the relative difference is, of course, less for lower kinetic energies. At this value, the k^2 terms in $R-R'$ are still only 22% of the constant first term. A detailed numerical study for a variety of potentials is in progress.

Since models which assume a repulsive core, or in which the inner region is replaced by a boundary condition at a certain radius, have been used extensively in the analysis of nucleon-nucleon scattering, and even K^+-proton scattering,[2] some overlap of our derivation and discussion with previous work is unavoidable. Those models employ necessarily at least three parameters, a boundary or core radius, and the scattering length and effective range of the exterior region. Clearly, in an expansion such as (2) there are only two significant parameters, indicating a redundancy in the above models from the point of view of a purely phenomenological discussion. Our object here is to give a shape-independent characterization of these two parameters, for the case of an interaction dominated by a repulsion at short distances. We know of no discussions in the literature which illuminate this problem.

The derivation of Eq. (1) parallels the Bethe development[3] of the $k\cot\delta$ expansion, with modifications dictated by the different behavior of the wave functions near the origin, illustrated in Fig. 1. By simple manipulation of the radial Schrödinger equations

$$u_k'' + [k^2 - v(r)]u_k = 0$$

and

$$w_k'' + k^2 w_k = 0,$$

with the wave functions u_k and w_k defined by

$$u_k(0) = 0, \quad u_k(r) \sim w_k(r) = N_k \sin(kr + \delta_k), \tag{3}$$

the following exact expression may be obtained:

$$[1 + k^2(R - R')^2]^{1/2} N_k (k'N_{k'})_0 \times \sin[kR' + \tan^{-1}k(R - R') + \delta_k] = -k^2 I_k(R'), \tag{4}$$

where

$$I_k(R') = -\int_0^{R'} u_k u_0 dr + \int_{R'}^{\infty} [w_k w_0 - u_k u_0] dr. \tag{5}$$

R is defined by the property $\delta_k \to -kR$, $k \to 0$. With the normalization convention $u_k(R) = 1$, which gives $I_k(R)$ a magnitude comparable with the range of the finite part of the potential, the limiting quantity $(k'N_{k'})_0$ is finite (but not dimensionless). The parameter R' is adjustable.

In the spirit of an effective-range expansion we first consider on the right side of Eq. (4) only $I_0(R')$. In the usual effective-range theory, R' is set equal to zero, and $I_0(0)$ then represents a range, being zero for a zero-range potential. We want our expression to be an expansion about the different limiting case of an infinite repulsive core. If, in that case, we set $R' = R$, then $I_k(R) = 0$, and we get the correct result $\delta_k = -kR$. Setting $R' = 0$, however, produces a result which can be brought into this form only after some manipulation by cancellation of the term $\tan^{-1}kR$ with $I_k(0)$. Thus, effects which we wanted to be included in our first approximation are in fact contained in the right-hand term, which is explicitly of higher order in k. We would rather choose R' so as to describe the departure from the $\delta = -kR$ limiting case. The simplest way to do this is to choose R' so as to make the integral $I_0(R')$ zero. The argument of the sine is zero, thus giving Formula (1). The more general requirement that $I_k(R') = 0$ (general k) will give $R' = R'(k^2)$, and the same Formula (1). The expansion (2) is affected only in the k^5 term.

We can now obtain a simple inequality between R' and R. For an entirely repulsive potential, by a simple argument based on Fig. 1, the quantity $I_0(R)$ (no prime on R) must be negative. [This quantity represents the negative of the area between u_0^2 and the two straight lines $w_0^2 = 0$ ($r < R$) and $w_0^2(r)$ ($r > R$). It describes the departure of the potential from an infinite square repulsion with the same low-energy phase shift.] To make $I_0(R') = 0$, we must add a positive contribution to the integral $\int_R^\infty w_0^2 dr$, i.e., we must have $R' < R$. To be specific,

$$R - R' = [-3I_0(R)/(k'N_{k'})_0^{\,2}]^{1/3}. \tag{6}$$

The inequality would not necessarily hold if $u_0(r)$ were convex upwards over a considerable region, i.e., if the potential were attractive in the outer regions. But if $v(r)$ is everywhere repulsive, R' is less than R, and the cubic term in expansion (2) must be positive. It is interesting to examine the $K^+ - p$ phase shift in this respect. In Fig. 2 we have plotted $\delta_k/(k/\mu_k)$ against k, the relativistic center-of-mass momentum (not the P of reference 1). The term we are interested in now appears quadratic in k, and the downward curvature of the experimental points indicates clearly that it is negative. A crude fit to Formula (2) gives $R \simeq 0.29$ F, $R' \simeq 0.65$ F.

The simplest way to explain this result is that outside a repulsive core there is an attractive region to the interaction. A potential of such a shape needs at least three parameters to describe it, so to compare with the phenomenological fit, one of these parameters must be fixed by other arguments. As an example, consider an infinite repulsive core of radius r_∞ surrounded by the tail of an attractive Yukawa potential[4] $V(r) = -V_0 \times \exp(-\mu r)/\mu r$ characteristic of two-pion or ρ-meson exchange (this provides the third parameter). Because the attraction turns out to be very weak, we calculate its contribution to δ by Born approximation.[5] The result is

$$\delta_k \simeq -kr_\infty + (k/\mu)A(\mu r_\infty)[1 - (k/\mu)^2 B(\mu r_\infty) + \cdots], \tag{7}$$

where A and B are simple expressions involving exponential integrals. By comparison with R and R', the parameters are determined to be as follows: exchange of two zero-kinetic energy pions, $\mu^{-1}=0.70$ F, $r_\infty=0.30$ F, $V_0=4.5$ MeV; ρ-meson exchange, $\mu^{-1}=0.38$ F, $r_\infty=0.32$ F, $V_0=13$ MeV. The comparison of r_∞ and R illustrates the fact that because of the outside attraction, the wave function appears to come from inside the repulsive core, an effect which could be more pronounced if the attraction were stronger.

In summary, the phenomenology of a strong repulsion is somewhat different from that of a strong attraction, and the significance of the parameters is quite different. The sign of the cubic term in the expansion of δ gives information about the sign of the tail of the potential. The experimental s-wave phase shifts for K^+-p scattering may possibly indicate that although the center is strongly repulsive, the tail of the interaction is weakly attractive, if the nonrelativistic potential scattering derivation given here has any validity.

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[3]H. A. Bethe, Phys. Rev. 76, 38 (1949).

[4]Potential models of this kind have been used by H. P. Noyes, Phys. Rev. 99, 618(A) (1955), in a discussion of low-energy s-wave π-p phase shifts. Particular examples have been considered by Noyes and Woodruff, in that context (see the above abstract for references) following the original suggestion of R. Marshak, Phys. Rev. 88, 1208 (1952). The object was to describe a phase shift which appeared to be positive at very low energy, and then became negative. The situation would seem to be intermediate between the usual attraction, and our dominant repulsion. The particular results obtained agree qualitatively with our predictions. We thank Dr. Marshak for pointing out this work to us.

[5]A little care must be taken in applying the usual $k \cot \delta$ expression to such an attraction because as the scattering length a tends to zero, r_0 is proportional to a^{-1}.

NOTE ON THE POSSIBLE RESONANT STATE IN THE $K\bar{K}$ SYSTEM

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The recent experimental data[1] for K^--p collisions at 2.24 and 2.5 BeV/c have shown that (1) in the $K^-+p\rightarrow\Xi+\pi+K$ reaction there is evidence for the existence of a Ξ^* state in addition to the well-known K^* state; (2) the mass spectrum of the $K\bar{K}$ system in the final state of the reaction $K^-+p\rightarrow\Lambda+K+\bar{K}$ seems to have a peak.

Bertanza et al.[1] have pointed out on the basis of their results, mentioned above, that the $K\bar{K}$ peak may be due to a $K\bar{K}$ resonance or to an S-wave final-state interaction between the K and $\bar{K}$ mesons. They have said that if the peak is due to the decay of a resonant state (which we will call the Z particle), the mass of this state is equal to 1020 MeV. If this peak is not due to a statistical fluctuation, it would be very desirable to know which isotopic spin state is responsible for the peak, $I=0$ or $I=1$. In this note we shall focus our attention on this problem.

A recent experiment[2] on the $\pi^-+p\rightarrow\pi^++\pi^-+n$ reaction at 3.3 BeV/c has shown the existence of a resonant state in the 2π system whose mass is nearly equal to 1040 MeV. It is said that this resonant state probably corresponds to the Pomeranchuk particle (X particle) with spin $J=2$ which has been predicted by Chew and Frautschi.[3] We assume that this interpretation is correct. Although the mass of the Z particle is nearly equal