PRODUCTION OF UNNATURAL-PARITY STATES BY INELASTIC ALPHA-PARTICLE SCATTERING*

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An unusual opportunity for studying nuclear reaction mechanisms is afforded by the existence in even-even nuclei of unnatural-parity states $[\Pi \neq (-1)^J$, e.g., $2^-, 3^+]$ which can be resolved in inelastic alpha-particle scattering experiments. In such experiments both initial particles and one final particle are spinless and of positive parity, and the production of unnatural-parity levels cannot occur under the assumptions ordinarily made in direct interaction calculations, i.e., a single scattering interaction with a single transfer of angular momentum.

This can be shown as follows: If the wave functions of the initial and final states of an inelastic scattering reaction can be separated into the product of a function of the internal nuclear coordinates β_j and a function of the coordinates $\vec{r}$ of the external system, then the inelastic scattering amplitude for a single scattering occurring at point $\vec{r}_1$ can be written

$$S_{\text{inel}} = (\Psi_f(\beta_j)\varphi_f(\vec{r}_1)\,|\,V\,|\,\Psi_i(\beta_j)\varphi_i(\vec{r}_1)), \tag{1}$$

where Ψ_i and Ψ_f are the initial and final internal nuclear wave functions, and φ_i and φ_f are the initial and final wave functions for the system outside the nucleus, both evaluated at $\vec{r}_1$. We will also assume a potential which is independent of all derivatives of $\vec{r}$. Then the potential can be written

$$V = V(\vec{r}, \beta_j), \tag{2}$$

and V can be expanded in terms of the two sets of coordinates as

$$V = \sum_{lm} W_l^m(\beta_j) v_l(r) Y_l^{m*}(\vec{r}), \tag{3}$$

where v_l contains the radial dependence of $V(\vec{r})$, Y_l^m is a spherical harmonic of rank l, and W_l^m is a tensor operator, so that the scattering amplitude separates to give the form

$$S_{\text{inel}} = \sum_{lm} (\Psi_f(\beta_j)\,|\,W_l^m\,|\,\Psi_i(\beta_j))(\varphi_f(\vec{r}_1)\,|\,v_l Y_l^{m*}\,|\,\varphi_i(\vec{r}_1)). \tag{4}$$

Here the first factor determines the effect of the interaction on the final nucleus and is dependent on the rank and parity of W_l^m.

The potentials V and v_l are scalars with rank zero and even parity, while Y_l^{m*}, a spherical harmonic, has rank l and parity $(-1)^l$. Therefore, the tensor operator W_l^m must also have rank l and parity $(-1)^l$. The total angular momentum J of the final nucleus is the vector sum of the transferred angular momentum l, the spin of the bombarding particle, and the spin of the initial nucleus, the latter two being zero. Therefore, $J=l$, and the parity of the final nucleus must be $(-1)^J$, i.e., the final nucleus must be in a natural-parity state. Thus, any interaction which can be described by the formalism above cannot contribute to the production of unnatural-parity states in any (α, α') reaction on a spinless target nucleus.

This argument is quite general and applies to a wide variety of reaction mechanisms, but there are several well-known processes to which the above argument does not apply. A compound nucleus interaction cannot be separated in the manner indicated above, since an intermediate state is produced and $\varphi_f \neq \varphi_f(\vec{r}_1)$, i.e., the wave function φ_f of the final state is not evaluated at $\vec{r}_1$. Another way of saying this is that the angular momentum of the intermediate state can couple with that of the final system to produce a state of unnatural parity in the final nucleus.

A spin-orbit interaction[1] between the orbital angular momentum of the initial system and the spin of one or more of the target nucleons would involve a potential which depends on the derivatives of $\vec{r}$ as well as on $\vec{r}$, and such a potential could not be expanded in the form given by (3). Therefore, a spin-orbit interaction cannot be excluded as a possible reaction mechanism on the basis of the above argument. However, Satchler[2] pointed out that the intensity of such an interaction would be expected to increase with bombarding energy, while existing experimental evidence (Table I) indicated a decrease. Further, the approximately equal cross sections of O^{16} and Ne^{20} cast doubt on the possibility of a sizable spin-orbit contribution, since one would expect the pairing forces in O^{16}, a doubly closed-shell nucleus, to inhibit strongly a spin-orbit interaction, while in Ne^{20} the inhibition would be considerably weaker.

Table I. Approximate differential cross sections of known unnatural-parity levels for inelastic alpha-particle scattering.

Target nucleus	Level excited (MeV)	J^{Π}	E (c.m.) (MeV)	$(d\sigma/d\Omega)_{\text{average}}$ (mbn/sr)	Reference number
O^{16}	8.88	2^-	14.7	2.5	5
Ne^{20}	4.97	2^-	15.0	2.0	6
Mg^{24}	5.22	3^+	19.3	0.7	Present work
Si^{28}	6.27	3^+	19.7	0.4	Present work
Mg^{24}	5.22	3^+	36.9	0.2	10

Satchler[2] has also suggested an exchange process as a possible reaction mechanism for production of these states. Here again the wave functions cannot be separated as in (1) because the wave function Ψ_f of the final nucleus is not independent of $\vec{r}$. In the case of knockout this could be viewed as a coupling of the angular momentum of the knocked-out alpha particle with the angular momentum of the initial system to give a final unnatural-parity state. This also holds for target stripping, but in that case it is the angular momentum of the core which couples with that of the initial system to produce an unnatural-parity final state. Satchler estimates that a knockout interaction would be expected to decrease with increasing bombarding energy for $E_\alpha > 20$ MeV.

When unnatural-parity vibrational states are excited by inelastic alpha particles scattered by a potential containing terms related to the deformation of the nucleus,[3,4] an interesting situation arises. The processes which might ordinarily be regarded as dominant,[4] i.e., single phonon production and simultaneous production of multiple phonons, are forbidden for the production of unnatural-parity states because both processes satisfy criteria (1) and (2). However, if two or more phonons are produced in at least two successive steps, condition (1) breaks down since an intermediate state is involved and, as before, $\varphi_f \neq \varphi_f(\vec{r}_1)$. In other words, successive phonon production involves an intermediate state with a definite angular momentum, and, as in the case of compound nucleus formation, a coupling of the angular momentum of the intermediate state with that of the final system can produce a final state of unnatural parity. Moreover, Buck[3] has shown that the contribution from such a "second-order" process may not be negligible. The intermediate-state argument given above holds not only for successive phonon production, but for any multiple-scattering process.

Production of unnatural-parity levels has been reported in several alpha-particle scattering experiments on even-even nuclei. The first evidence that a state known to be of unnatural parity could be excited by a direct (α, α') process was reported by Corelli, Bleuler, and Tendam.[5] They measured the differential cross section for inelastic alpha-particle scattering to the 8.88-MeV (2^-) level of O^{16} at $E_\alpha = 18$ MeV (lab). Earlier, the differential cross section was measured for 18-MeV (lab) inelastic alpha-particle scattering[6] to the 4.97-MeV level of Ne^{20}. This state has since been shown[7] also to have $J^{\Pi} = 2^-$. Both distributions mentioned above exhibit strong forward peaking and average differential cross sections of about 2.5 mb/sr in O^{16} and 2.0 mb/sr in Ne^{20} over scattering angles between 30° and 100° (c.m.). The measurements in these experiments were not extended to larger scattering angles.

A recent investigation[1] of the $Ne^{20}(\alpha, \alpha')$ reaction at $E_\alpha = 22.5$ MeV (lab) provides evidence for production of several unnatural-parity levels and an angular distribution for the 2^- (4.97-MeV) level mentioned above. However, since absolute differential cross sections were not measured, a quantitative comparison of these data with those mentioned above cannot be made.

In a study of the $Mg^{24}(\alpha, \alpha')$ reaction at $E_\alpha = 43$ MeV, Shook[8] reported evidence for excitation of the 5.22-MeV state of Mg^{24}, now known[9] to have $J^{\Pi} = 3^+$. Subsequent work by Naqib on the same level (also at $E_\alpha = 43$ MeV) gives a preliminary estimate for the differential cross section of 0.1 to 0.2 mb/sr.[10]

The availability of high-resolution, solid-state, charged-particle detectors now permits efficient (α, α') differential cross-section measurements for unnatural-parity states. The present experiment was undertaken to measure the differential cross sections of the fourth excited states ($J^{\Pi} = 3^+$) of Mg^{24} ($Q = -5.22$ MeV) and Si^{28} ($Q = -6.27$ MeV) using 22.5-MeV (lab) alpha particles from the Indiana University cyclotron. Targets of natural magnesium (0.27 mg/cm²) and natural silicon (0.43 mg/cm²) were used in conjunction with standard experimental arrangement and techniques which are described elsewhere.[11] Differential cross sections of all resolvable states were measured at scattering angles from 12.5° to 165° (lab). The differential cross sections of the ground states and the fourth excited states of the two nuclei are shown in Fig. 1.

The diffraction-pattern maxima and minima, the lack of symmetry about 90° (c.m.), and the peaking at the forward angles of the angular distributions together with the high excitation energy (about 28 MeV) seem to exclude any sizable contribution from a compound-nucleus process to the excitation of these unnatural-parity levels.

Average differential cross sections are about 0.4 mb/sr for the 6.27-MeV level of Si^{28} and 0.7 mb/sr for the 5.22-MeV level of Mg^{24}. Table I summarizes the average differential cross sections from the available measurements on unnatural-parity states and indicates a general trend that the cross sections decrease as the center-of-mass bombarding energy is increased.

There is no clear phase relationship between the elastic and 3^+ distributions (see Fig. 1) of either Mg^{24} or Si^{28}. However, one can say that in Mg^{24} the distribution seems to change from being "in-phase" at the forward angles to "out-of-phase" at the backward angles, while Si^{28} shows the opposite tendency. The anomalous phase of the 3^+ levels suggests a destructive interference effect between two contributing mechanisms, similar to that discussed by Buck[3] for $J^{\Pi} = 4^+$ levels. The Ne^{20} distributions[1,6] mentioned above also exhibit an anomalous phase relationship and the O^{16} 8.88-MeV (2^-) group distribution[5] has minima in phase with the elastic but at double the frequency, i.e., with twice the number of maxima and minima, further indications of interfering mechanisms in the production of unnatural-parity states.

Another feature of the Mg^{24} and Si^{28} distributions measured in the present work is a pronounced peaking at backward angles, which is suggestive of an exchange process, e.g., target stripping. As mentioned earlier, such a process does not seem to be forbidden by spin and parity considerations.

The excitation of unnatural-parity states in even-even nuclei by the inelastic scattering of spinless particles imposes restrictions on the excitation process. Simple, one-step direct-interaction mechanisms are forbidden by spin and parity considerations, but the excitation can

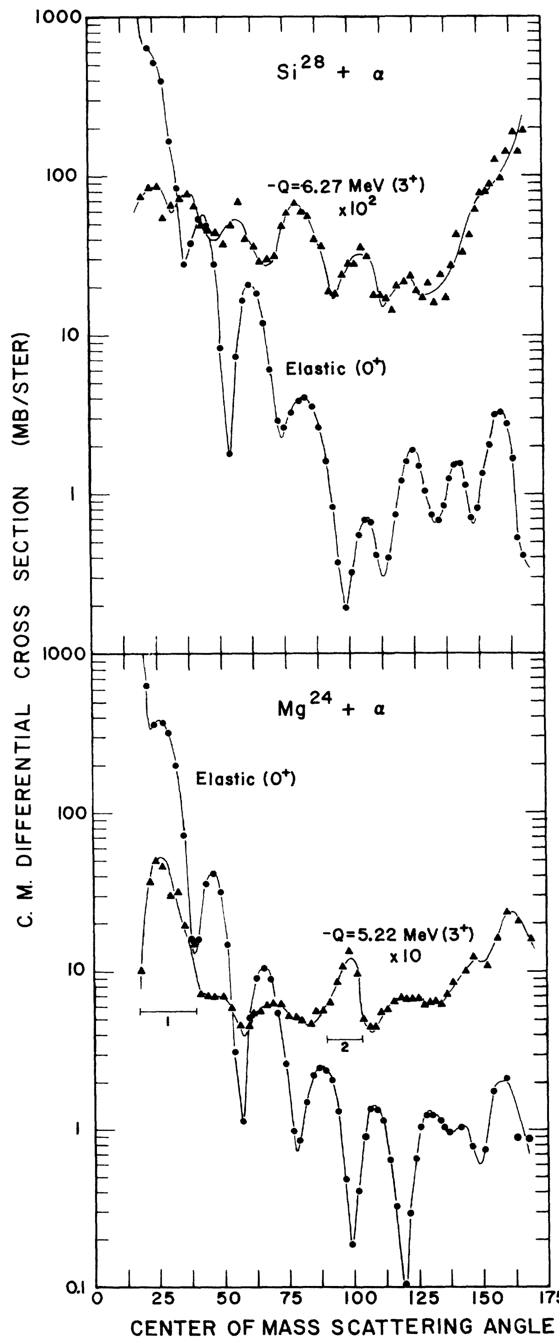

FIG. 1. Experimental differential cross sections for 22.5-MeV alpha particles incident on Mg^{24} and Si^{28}. The differential cross sections of the ground states (0^+) of both nuclei are shown for comparison with the differential cross sections of the unnatural-parity states (3^+). The differential cross sections of the latter have been multiplied by scale factors as indicated. In the distribution for the Mg^{24} 3^+ state, regions 1 and 2 indicate angles where interference from the reactions $C^{12}(\alpha, \alpha')C^{12*}_{4.43}$ and $C^{12}(\alpha,\alpha)C^{12}$, respectively, due to carbon contamination of the target, produced larger uncertainties in the relative differential cross section. Generally, relative errors in the cross sections are of the order of the point size except in the bracketed regions, where errors of ±20% must be quoted; absolute cross-section errors are approximately ±30%.

proceed through (a) compound nucleus formation, (b) spin-orbit interaction, (c) nonsimultaneous, multiple-phonon excitation, and (d) a direct exchange process. Other mechanisms could contribute which have not been considered here.

On the basis of existing evidence concerning shapes of angular distributions and energy dependence of cross sections, processes (a) and (b) do not seem to contribute significantly. Indication of interference in the angular distributions suggests contributions from both (c) and (d). Clearly, to make a more quantitative analysis of these possible reaction mechanisms, more experimental information is needed. On the basis of existing data, however, production of unnatural-parity states in (α, α') scattering is surprisingly intense, since their production by most "first-order" processes is forbidden is forbidden by parity conservation. Thus the study of these levels in even-even nuclei is a potentially powerful tool for investigation of "second-order" reaction processes.

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EFFECT OF EXCHANGE ON L TO K CAPTURE RATIOS*

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Two years ago, Robinson and Fink[1] called attention to a systematic discrepancy between observed and predicted L to K electron capture ratios. Many experiments have since been performed to investigate this discrepancy. The observed L to K ratio has been found[2-5] to exceed by 5 to 25 percent the predicted L to K ratio for all nine of the precisely measured allowed electron captures with Z between 18 and 36. The calculations reported in this note remove the systematic disagreement between theory and experiment by including atomic states in the description of the radioactive system.

Following the suggestions of Benoist-Gueutal[6] and Odiot and Daudel,[7] we have generalized the usual allowed theory of electron capture to include atomic variables in the initial and final states. Using closure to sum over all possible final states of the outer electrons, we find[8-10]

$$\frac{\lambda_{L_{\rm I}}}{\lambda_K} \cong \left(\frac{\lambda_{L_{\rm I}}}{\lambda_K}\right)^0 \left\{\frac{1-[2R_{1s}(0)/R_{2s}(0)]\langle 1s'|2s\rangle}{1-[2R_{2s}(0)/R_{1s}(0)]\langle 2s'|1s\rangle}\right\}, \quad \text{(1a)}$$

where

$$(\lambda_{L_{\rm I}}/\lambda_K)^0 = [q(2s')R_{2s}(0)/q(1s')R_{1s}(0)]^2 \quad \text{(1b)}$$

is the usual[11] $L_{\rm I}$ to K capture ratio, $\langle 1s'|2s\rangle$ is the overlap of the final $1s'$ electron state with the initial $2s$ electron state, and $R_{1s}(0)/R_{2s}(0)$ is the ratio of the electron radial wave functions evaluated at the nuclear surface.

An $L_{\rm I}$ capture can occur in two important ways: (a) direct annihilation of a $2s$ electron and (b) annihilation of a $1s$ electron with the initially present $2s$ electron jumping into the final $1s'$ shell. The probability of the direct process, (a), is proportional to $R_{2s}^2(0)$; the probability of the exchange process, (b), is proportional to $R_{1s}^2(0)\langle 1s'|2s\rangle^2$. The interference between amplitudes for direct and exchange decay is proportional to $-2R_{1s}(0) \times R_{2s}(0)\langle 1s'|2s\rangle$, the minus sign arising from the exclusion principle. The interference between direct and exchange amplitudes for K capture produces a term proportional to $-2R_{1s}(0)R_{2s}(0) \times\langle 2s'|1s\rangle$. Hence, the bracketed expression in