The error of the relative Q's for a single gas is about 1-2%. The relative error in Q from gas to gas, which is caused by nonreproducibility of the McLeod, is about ±4%, except for He, where it is ±10%. The absolute values of Q, which are also subject to errors in the absolute McLeod calibration and the scattering path length, are probably correct within ±10%.

An examination of Fig. 1 shows the major features. Li^7-Xe, in the velocity range studied, exhibits two maxima and two minima. This system is expected to have the largest attractive forces. The Kr curve shows two maxima, a minimum, and a relatively steep decline at high v_r. It appears that the extrema which are analogous to those of Xe have shifted toward lower velocities. Argon displays only one maximum and one minimum. These are at even lower velocities. At low v_r, Ne appears to be at the end of the long decline which was seen in Ar and Kr, and then decreases slowly at higher velocities. Helium shows only a gradual decrease of Q with v_r, which resembles the behavior at the higher Ne velocities.

This behavior is as predicted[1]: it demonstrates a low-velocity undulatory region, an intermediate region of sharp decline in Q, and a high-energy region characteristic mainly of the repulsive part of the potential.

A detailed theoretical analysis of these data is deferred to a paper in preparation.

We are indebted to R. B. Bernstein, R. H. Neynaber, and S. M. Trujillo for valuable discussions.

---

*Work supported by Advanced Research Projects Agency (Project Defender) through the Office of Naval Research.

†On leave from Laboratory for Mass-Separation, Amsterdam, The Netherlands.

[1]R. B. Bernstein (to be published).

[2]H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc. (London) A144, 188 (1934).

[3]R. B. Bernstein, J. Chem. Phys. 34, 361 (1961).

[4]E. W. Rothe, P. K. Rol, S. M. Trujillo, and R. H. Neynaber, Phys. Rev. 128, 659 (1962).

SPIN EXCHANGE IN ELECTRON-HYDROGEN COLLISIONS

M. H. Mittleman*
Physics Department, University College, London, England
(Received November 5, 1962)

The problem of rearrangement collisions is plagued by an overabundance of formal expressions for the transition matrix.[1-4] Each exact form of the T matrix contains a wave function or propagator which represents an exact solution of the problem. Thus approximations are necessary. With approximations for the exact wave functions the various forms of T give different results. Until very recently there has been no physically meaningful scattering problem in which rearrangement is possible which has been solved exactly. (This is not surprising in that a rearrangement collision is intrinsically a three-body problem.) There has therefore been no means of testing the approximations to the various forms of T.

Recently the low-energy scattering of electrons by hydrogen atoms has been solved by elaborate numerical methods in both singlet and triplet states.[5-7] The three widely different methods all yield similar results which are presumably correct. From these results we may extract the spin-exchange amplitude and test the approximate forms of T. We shall confine our attention to incident S-wave electrons. The partial cross section is given by

$$\sigma_S = (1/4p^2)\sin^2(\delta_+ - \delta_-) \tag{1}$$

below threshold, or

$$\sigma_S = (1/16p^2)|A_+ - A_-|^2 \tag{2}$$

above threshold, where

$$A_\pm = \exp(2i\delta_\pm) - 1, \tag{3}$$

where $\delta_\pm$ are complex.

We shall test two forms of T. The first is

$$T_1 = \langle \phi_f V_f \Psi_i^{(+)} \rangle, \tag{4}$$

and the second is

$$T_2 = \langle \Psi_f^{(-)} [\pi_f, V_f] \Psi_i^{(+)} \rangle. \tag{5}$$

Here $\Psi_i^{(+)}$ is the exact wave function with initial state ϕ_i, and $\Psi_f^{(-)}$ is the exact time-reversed wave function with ϕ_f as initial state, where[8]

$$\phi_f = (\sin pr_2/pr_2)\exp(-r_1)/\sqrt{\pi}, \tag{6a}$$

$$\phi_i = (\sin pr_1/pr_1)\exp(-r_2)/\sqrt{\pi}, \quad (6b)$$

V_f is the interaction in the final state,

$$V_f = 2/r_{12} - 2/r_2, \quad (7)$$

and π_f is the projection operator onto the final bound electron,

$$\pi_f = [\exp(-r_1)/\sqrt{\pi}]\delta(\vec{r}_2 - \vec{r}_2')\exp(-r_1')/\sqrt{\pi}. \quad (8)$$

These are the so-called "post" forms. The "prior" forms, in which the interaction in the initial state appears, will yield the same results because of the symmetry in this problem.

The Born-Oppenheimer approximation is obtained by replacing $\Psi_i^{(+)}$ by ϕ_i in (4). Similarly we shall replace $\Psi_i^{(+)}$ and $\Psi_f^{(-)}$ by ϕ_i and ϕ_f, respectively, in (5). These will be the approximations for T_1 and T_2 which we shall test. The cross section is then given by

$$\sigma_i = (T_i/4\pi)^2. \quad (9)$$

The results are

$$\sigma_1 = 4(p^2-3)^2/(p^2+1)^6 \quad (10)$$

and

$$\sigma_2 = 4[(p^2+5)(p^2+9)^2 - 16(33+p^2)]^2/(p^2+1)^6(p^2+9)^4. \quad (11)$$

A sharp resonance has been found numerically[6,7] around $p^2 = 0.7$ in the singlet state. The phase jumps by about π. The triplet phase is smooth in this region so that (1) must vanish someplace between $p^2 = 0.70$ and 0.73. Equation (11) vanishes at $p^2 \approx 0.72$. In Fig. 1 we have plotted the cross sections σ_S, σ_1, and σ_2. The results of Schwartz[5] were used below $p^2 = 0.64$ and those of Burke and Schey[7] above. It seems clear that T_2 is a much better approximation than T_1.

In all fairness it should be pointed out that a formulation of T also exists[4] in which π_f is replaced by a sum over all bound states of (r_1). The cross section from this form has been evaluated numerically. It gives essentially the same results as (11) above $p^2 = 1.5$ but extremely bad results below this.

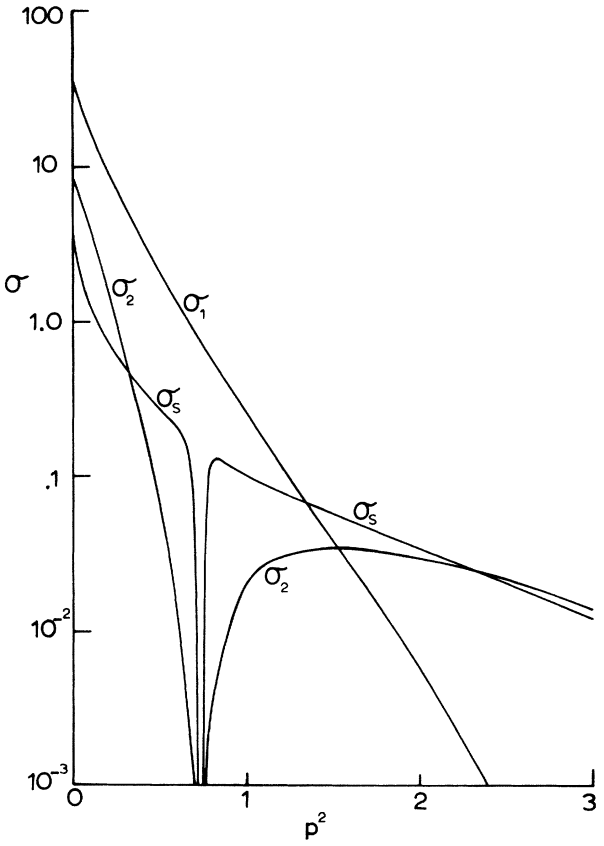

FIG. 1. Cross section in units of Bohr radii squared vs energy in units of 13.6 eV.

I would like to express my gratitude for the support of the Department of Scientific and Industrial Research during my visit to University College.

*Permanent address: Lawrence Radiation Laboratory, University of California, Livermore, California.

[1]B. A. Lippman, Phys. Rev. 102, 264 (1956).
[2]K. Greider, Phys. Rev. 114, 786 (1959).
[3]M. H. Mittleman, Phys. Rev. 122, 1930 (1961).
[4]M. H. Mittleman, Phys. Rev. 126, 373 (1962).
[5]C. Schwartz, Phys. Rev. 124, 1468 (1961).
[6]A. Temkin, Phys. Rev. Letters 4, 566 (1960).
[7]P. G. Burke and H. M. Schey, Phys. Rev. 126, 147 (1962). I am indebted to Dr. Burke for furnishing me with the elements of the T matrix above threshold prior to publication.
[8]Energies are measured in units of 13.6 eV, lengths in Bohr radii.