CIRCULAR POLARIZATION OF HIGH-ENERGY γ RAYS BY BIREFRINGENCE IN CRYSTALS

N. Cabibbo
Laboratori Nazionali di Frascati del Comitato Nazionale per l'Energia Nucleare, Roma, Italia and
Istituto di Fisica dell'Università di Roma, Roma, Italia

and

G. Da Prato, G. De Franceschi, and U. Mosco
Laboratori Nazionali di Frascati del Comitato Nazionale per l'Energia Nucleare, Roma, Italia
(Received October 18, 1962)

In a previous work[1] the authors have shown that thick single crystals can be used as polarizers and polarimeters for high-energy gamma rays. This possibility is due to the well known coherence effects in pair production, which cause a dependence of the absorption cross section on the linear polarization of the photons.

In this paper we show that as a consequence of the effects discussed in A and B, single crystals are also birefringent. This opens interesting possibilities in the use of single crystals for the handling of circularly polarized gamma rays. In particular it is possible to use a crystal of appropriate thickness as the high-energy analog of a quarter-wavelength plate to convert linear into circular polarization, and vice versa. In this way it will be possible to produce and analyze circularly polarized gamma-ray beams of very high energy. As in the case of linear polarization[1] the efficiency is found to increase with the energy.

As in A we consider a cubic crystal of thickness x and a photon whose momentum $\vec{K}$ is in the (001) plane of the crystal at an angle δ from the (110) axis. The polarization vector $\vec{\epsilon}$ of the incoming photon will in general be a combination of two vectors $\vec{t}$ and $\vec{y}$ (see B), respectively, on the (001) plane and orthogonal to it: $\vec{\epsilon}=\epsilon_1\vec{t}+\epsilon_2\vec{y}$.

$\vec{\epsilon}$ can be considered as a two-component vector: $\vec{\epsilon}\equiv(\epsilon_1, \epsilon_2)$. Beyond the crystal the outgoing amplitude in the forward channel (photon of momentum $\vec{K}$) will be connected to $\vec{\epsilon}$ by a 2×2 diagonal matrix of the form[2]

$$S=\begin{pmatrix}\exp[in^{\parallel}(\omega,x)\omega x] & 0\\ 0 & \exp[in^{\perp}(\omega,x)\omega x]\end{pmatrix},$$

where[3] $\omega=|\vec{K}|$ and the quantities $n^{\parallel}$ and $n^{\perp}$ are the analogs of the refraction index in optics.

The crystal acts as a $\frac{1}{4}\lambda$ plate if the relative phase of the two components is changed by $\frac{1}{2}\pi$, i.e., if $\mathrm{Re}(n^{\perp}-n^{\parallel})\omega x=\frac{1}{2}\pi$.

The imaginary parts of these are connected with the absorption cross section, $\mathrm{Im}n(\omega)=\epsilon(\omega)/2\omega$, and were discussed and evaluated in A and B. The real parts can be derived from them by the use of dispersion relations.[4] We are interested in the difference of the real parts, which enters in the phase relations between the two components:

$$\mathrm{Re}[n^{\perp}(\omega,\delta)-n^{\parallel}(\omega,\delta)]=\frac{1}{\pi}\mathrm{P}\int_0^{\infty}\frac{\Sigma^{\perp}(\omega',\delta)-\Sigma^{\parallel}(\omega',\delta)}{\omega'^2-\omega^2}d\omega'. \tag{1}$$

From B one has[5]

$$\Sigma^{\perp}(\omega,\delta)-\Sigma^{\parallel}(\omega,\delta)=\sum_{\vec{q}}K(\vec{q})\frac{1}{\omega\beta^2}\left[\beta\left(1-\frac{4}{\beta}\right)^{1/2}+2\ln\left(\frac{1+(1-4/\beta)^{1/2}}{1-(1-4/\beta)^{1/2}}\right)\theta(\beta-4)\right], \tag{2}$$

with

$$K(\vec{q})=\frac{8\alpha Z^2r_0^{\,2}}{\pi}\frac{N}{a^3}(2\pi)^3\frac{b^4\exp(-\Lambda q^2)}{(1+b^2q^2)^2}\cos(2\theta)D(\vec{q})\frac{4q^2\omega^2-\beta^2}{\beta^2}, \tag{3}$$

where b is the screening length, N the number of atoms per unit volume, a the lattice constant, and $D(\vec{q})$ and Λ are defined in B; $\beta=2(\vec{k}\cdot\vec{q})$, θ is the angle between the (100) and the $\vec{k}, \vec{q}$ planes. Integrating term by term, we obtain

$$\mathrm{Re}(n^{\perp}-n^{\parallel})=\sum_{\alpha} f(\omega,\delta,\beta) \qquad (4)$$

and

$$f(\omega,\delta,\beta)=\frac{1}{8\pi}\frac{1}{\omega^2}K(\vec{q})\left\{\left[\left(1-\frac{4}{\beta}\right)^{1/2}+\frac{2}{\beta}\ln\frac{1+(1-4/\beta)^{1/2}}{1-(1-4/\beta)^{1/2}}\right]^2 -\frac{4\pi^2}{\beta^2}+\left[\left(1+\frac{4}{\beta}\right)^{1/2}-\frac{2}{\beta}\ln\frac{(1+4/\beta)^{1/2}+1}{(1+4/\beta)^{1/2}-1}\right]^2\right\}$$

for $\beta > 4$,

$$f(\omega,\delta,\beta)=\frac{1}{8\pi}\frac{1}{\omega^2}K(\vec{q})\left\{-\left[\left(\frac{4}{\beta}-1\right)^{1/2}-\frac{4}{\beta}\cot^{-1}\left(\frac{4}{\beta}-1\right)^{1/2}\right]^2 +\left[\left(1+\frac{4}{\beta}\right)^{1/2}-\frac{2}{\beta}\ln\frac{(1+4/\beta)^{1/2}+1}{(1+4/\beta)^{1/2}-1}\right]^2\right\}$$

for $\beta \leqslant 4$. (5)

The summation is extended over the points in the reciprocal lattice for which $(\vec{k}\cdot\vec{q})>0$.

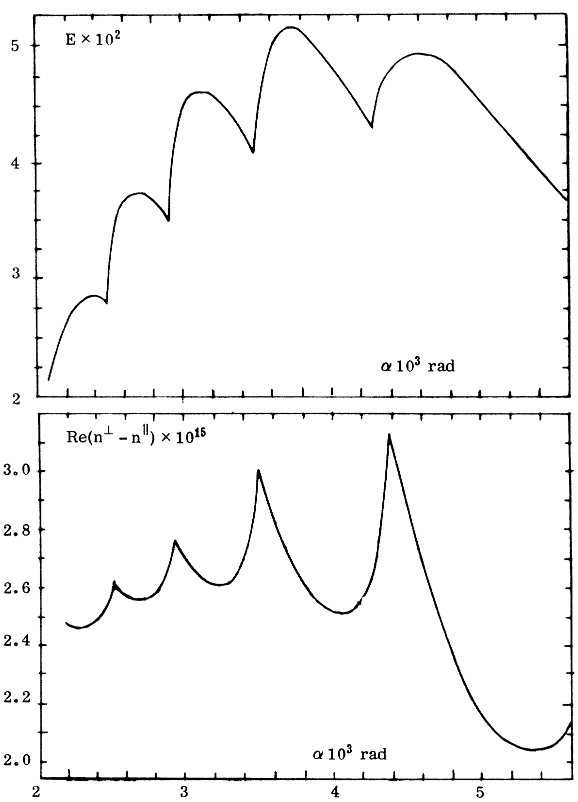

FIG. 1. E (upper part) and $\mathrm{Re}(n^{\perp}-n^{\parallel})$ (lower part) as a function of δ at $\omega = 6$ GeV.

Table I. "Best results" around 1 GeV, 6 GeV, 40 GeV, and the thickness x of a $\frac{1}{4}\lambda$ plate.

ω (GeV)	δ (mrad)	$\mathrm{Re}(n^{\perp}-n^{\parallel})$	x (cm)
1	28	2.62	11.5
6	3.7	2.74	1.84
40	0.46	2.67	0.273

Numerical evaluation of Eq. (4) for the case of a Cu crystal has been done with an IBM-1620 computer. In Table I we give the "best results" around 1 GeV, 6 GeV, 40 GeV, and the thickness of a $\frac{1}{4}\lambda$ plate, $x=(\pi/2\omega)(n^{\perp}-n^{\parallel})$. It is seen that the effect at the best angle is nearly independent of the energy. In Figs. 1 and 2 we give the dependence of $\mathrm{Re}(n^{\perp}-n^{\parallel})$ on the angle and the energy in the 6-GeV region. For comparison, these figures also contain a plot of $E(\delta,\omega) = (\Sigma^{\perp}-\Sigma^{\parallel})/(\Sigma^{\perp}+\Sigma^{\parallel})$.

It is seen that $\mathrm{Re}(n^{\perp}-n^{\parallel})$ has maxima where $dE/d\omega$ is maximum.[6]

It is a suggestive possibility that the effects discussed here and in A and B will allow the introduction of a new family of "optical" instruments for the handling of very-high-energy photon beams.

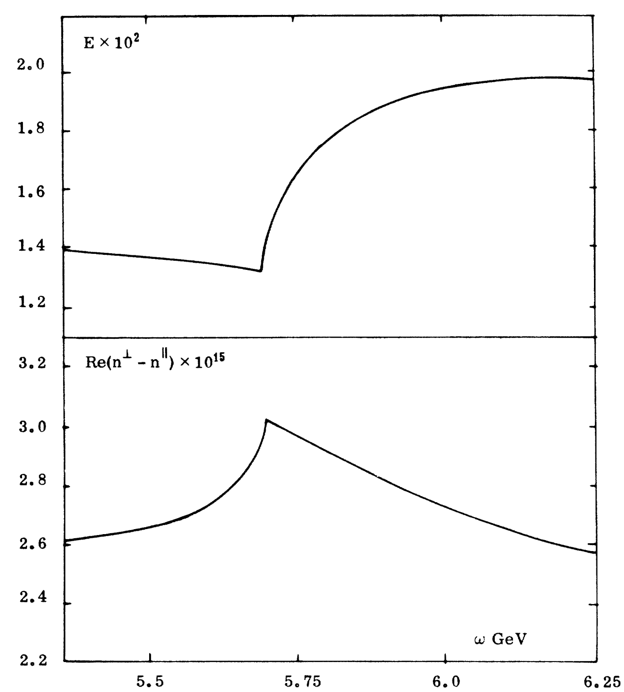

FIG. 2. E (upper part) and $\mathrm{Re}(n^{\perp}-n^{\parallel})$ (lower part) as a function of ω at $\delta = 3.7$ mrad.

We are grateful to the "Gruppo Calcoli Numerici" of the Frascati Laboratories for the assistance given to us in the use of the IBM-1620 computer.

[1]N. Cabibbo, G. Da Prato, G. De Franceschi, and U. Mosco, Phys. Rev. Letters 9, 270 (1962); N. Cabibbo, G. Da Prato, G. De Franceschi, and U. Mosco, Nuovo cimento (to be published), hereafter referred to as A and B.

[2]The diagonality is due to our choice of the basis vectors and to the symmetry of our situation in respect to the (001) plane; the exponential form can be assumed because the coherent action of the crystal is limited to microscopic dimensions because of multiple scattering of the produced electrons.

[3]We use the usual units with $\hbar = c = 1$ and also $m_e = 1$.

[4]The n's introduced here are essentially the amplitudes for forward Delbrück scattering in a crystal.

[5]This is obtained from Eqs. (15′) and (A10) in B neglecting q^2 relative to m_e and β, an approximation which was found very accurate and greatly simplifies the integration.

[6]At these points $\Sigma^{\perp} - \Sigma^{\parallel}$ is also maximum, since $\Sigma^{\perp} + \Sigma^{\parallel}$ is dominated by the incoherent contribution and relatively flat.