

FIG. 3. Average total fragment kinetic energy as a function of mass ratio. Results for three groups of alpha-particle energies in ternary fission are compared with those for binary fission. Note that E_K is the sum of the fragment energies only, and does not include E_α .

sponds to a light-fragment mass of 90 amu, which contains the closed $N = 50$ shell. The total fragment kinetic energy averaged over all mass ra-

tios and over all alpha-particle energies was found to be 155 ± 2 MeV, in slight disagreement with the value of 150 MeV reported for ternary fission by Schröder *et al.*¹¹

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³The authors very gratefully acknowledge the collaboration of C. D. Moak in the bromine-ion experiment.

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CROSS-SECTION PROBABILITY DISTRIBUTIONS*

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The purpose of this note is to introduce the notion of cross-section probability distributions and to give an example.

Energy-averaged cross sections are well known and form the basis of the optical model concept. Cross-section dispersions were first considered under another guise¹ and were subsequently emphasized by Thomas² when he derived a simple connection between the dispersion of the total cross section and the fluctuation cross section. Cross-section correlation coefficients were introduced at a later date.³ It is, of course, in principle, possible to compute higher and higher

moments and in this way to define a cross-section probability distribution.

In order to clarify what we mean, we present a simple example. The example we choose is that of the completely soluble "picket fence" model in which the spacings between and widths of fine structure levels are assumed to be constant. We confine our discussion to S waves; in this case the relevant logarithmic derivative can be exhibited exactly by

$$f(E) = \frac{1}{\pi s} \cot \left[\frac{\pi}{D} (E + i \frac{1}{2} \Gamma_r) \right], \quad (1)$$

where s is the usual strength function, D is the mean distance between levels, and Γ_γ is the mean reaction width. For our example, we consider the reaction cross section σ_γ which is given by

$$\sigma_\gamma = \pi\lambda^2(1 - |\eta|^2), \quad (2)$$

where

$$\eta = \exp(-2ix)[1 + 2ix/(f - ix)], \quad (3)$$

and $x = kR$ with k the incident wave number and R the radius of the interaction region. It is convenient to define the variable

$$\begin{aligned} v &= \delta\sigma_\gamma / \bar{\sigma}_\gamma, \\ &= -\delta|\eta|^2 / (1 - |\eta|^2), \end{aligned} \quad (4)$$

in terms of which we will display the cross section probability distribution. The result for dP/dv is

$$\frac{dP}{dv}(v) = \frac{1}{\pi} \frac{1+v}{[2\xi/(1+\xi)+v]^{1/2}[2\xi/(1-\xi)-v]^{1/2}}, \quad (5)$$

where

$$\begin{aligned} \xi &= [p(1-T)]^{1/2}, \\ p &= \exp(-2\pi\Gamma_\gamma/D), \\ T &= 1 - |\bar{\eta}|^2, \\ &= 4x\pi s / (1 + x\pi s)^2. \end{aligned} \quad (6)$$

Thus we see that p is the probability that no reaction has taken place during one cycle of the compound nucleus motion while T is the usual transmission coefficient. Equation (5) is plotted in Fig. 1 for various values of ξ . From the figure and from (5) it is seen that the range of fluctuation in the reaction cross section is from v

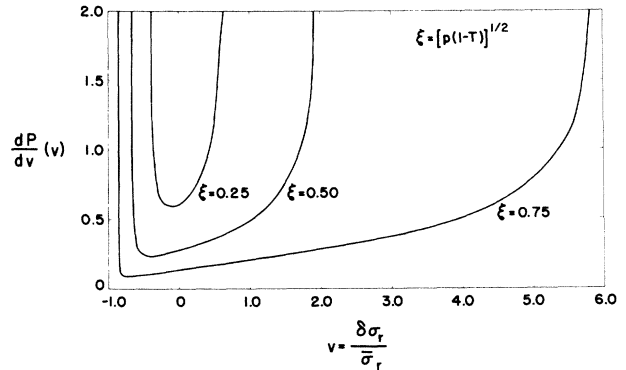


FIG. 1. Plot of the differential cross section probability distribution dP/dv as a function of v for various values of the parameter ξ as indicated.

$= -2\xi/(1 + \xi)$ to $v = 2\xi/(1 - \xi)$ with integrable singularities in the distribution function at both ends of the range. It is therefore highly probable that the cross section take on either a minimum or a maximum value and not so probable that it be found in between the minimum or maximum values.

There is clearly every reason to investigate these distributions for more realistic models since they constitute the natural extension of the optical model concept. Further work along these lines is presently in progress.

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STATISTICAL DISTRIBUTION OF REACTION WIDTHS*

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Porter and Thomas¹ have proposed an analysis of (reduced) reaction widths which yields a quantity ν interpretable as the number of open decay channels. They give for the scatter in reaction widths the so-called chi-squared distribution function for ν degrees of freedom,

$$P_\nu(\Gamma) \propto \Gamma^{\frac{1}{2}\nu - 1} \exp(-\nu\Gamma/2\langle\Gamma\rangle), \quad (1)$$

where ν is an integer.

In the present note, the analysis is generalized to include partially open channels or channels of varying width; at the same time a method of analyzing data emerges which is simpler than the usual curve plotting and comparison technique.

First, we recall the physical concepts which lead to Eq. (1). Consider a nucleus which has