TOTAL MUON CAPTURE RATES AND THE UNIVERSAL FERMI INTERACTION*

R. Klein and L. Wolfenstein
Carnegie Institute of Technology, Pittsburgh, Pennsylvania
(Received September 21, 1962)

The most detailed experimental information on muon capture consists of measurements of the total capture rate for many complex nuclei.[1,2] A formula has been given by Primakoff[3] expressing this rate as a function of A and Z in terms of the coupling constants for capture by a proton and a single parameter describing nuclear matter. Sens[1] and, more recently, Telegdi[4] have shown that this formula provides a good fit to the data and have deduced from this fit a magnitude of the coupling in excellent agreement with the predictions of the universal Fermi interaction (UFI).[5]

In this note we wish to point out that the derivation of the Primakoff formula involves unjustified mathematical approximations.[6] The basic assumptions of Primakoff lead rather to Eq. (3) below, or, to a fair approximation, to Eq. (7). The experimental data can also be reasonably fit to this equation, but the value of the square of the coupling constant needed is twice the UFI value.

We do not believe that this result is valid evidence against UFI for reasons discussed below. However, it is clear that no quantitative verification of UFI has been correctly deduced from the total capture rates on nonlight nuclei. This conclusion re-emphasizes the importance of accurate experiments on muon capture in hydrogen, deuterium, and He^3.

The Primakoff analysis is based on the following apparently reasonable assumptions and approximations:

(1) The closure approximation for summing over final nuclear states.

(2) The neglect of relativistic matrix elements proportional to the target proton's momentum.

(3) The neglect of certain terms proportional to G_p^2, where G_p is the effective pseudoscalar coupling.

(4) A particular model for the correlations present in the initial state a of the target nucleus. Primakoff defines two functions [Eq. (P-7b)] $F_a^{(+)}(\vec{r},\vec{r}\,')$ and $F_a^{(-)}(\vec{r},\vec{r}\,')$, associated with the probability of finding two nucleons at positions $\vec{r}$ and $\vec{r}\,'$ when they are, respectively, in a space-symmetric and space-antisymmetric state of relative motion. For these functions he assumes the forms

$$F_a^{(\pm)}(\vec{r},\vec{r}\,')=C_a^{(\pm)}\mathfrak{D}_a(r)\mathfrak{D}_a(r')[1\pm f(|\vec{r}-\vec{r}\,'|)], \qquad (1)$$

where $C_a^{(\pm)}$ is a normalization factor and $\mathfrak{D}_a(r)$ is the proton density distribution (normalized to unity). Primakoff further assumes that $f(r)$ can be considered as different from zero only for small values of r and that for all nuclei

$$\int\mathfrak{D}_a(r)\mathfrak{D}_a(r')f(|\vec{r}-\vec{r}\,'|)d\vec{r}d\vec{r}\,'=\delta'/A, \qquad (2)$$

with δ' a constant.

The first three approximations lead directly to Eq. (P-8). This equation and the fourth assumption then show that to a very good approximation the capture rate Λ_a by nonlight nuclei ($A>30$) is given by[7]

$$\Lambda_a=Z_{\text{eff}}^{\;4}\gamma\Lambda(1,1)\left(1-\frac{A-Z}{2A}\delta_a-J_a\Delta_a+\frac{A-Z}{2A}\Delta_a\delta'\right), \qquad (3)$$

where

$$\frac{\delta_a}{A}=\frac{\int j_0(\bar{\nu}|\vec{r}-\vec{r}\,'|)\varphi_a(r)\varphi_a(r')\mathfrak{D}_a(r)\mathfrak{D}_a(r')f_a(|\vec{r}-\vec{r}\,'|)d\vec{r}d\vec{r}\,'}{\int|\varphi_a(r)|^2\mathfrak{D}_a(r)d\vec{r}}, \qquad (4)$$

$$\Delta_a=\frac{\int j_0(\bar{\nu}|\vec{r}-\vec{r}\,'|)\varphi_a(r)\varphi_a(r')\mathfrak{D}_a(r)\mathfrak{D}_a(r')d\vec{r}d\vec{r}}{\int|\varphi_a(r)|^2\mathfrak{D}_a(r)d\vec{r}},$$

$$=\frac{[\int j_0(\bar{\nu}r)\varphi_a(r)\mathfrak{D}_a(r)d\vec{r}]^2}{\int|\varphi_a(r)|^2\mathfrak{D}_a(r)d\vec{r}}, \qquad (5)$$

$$J_a=\left[\frac{A}{2Z}-\frac{\langle a|G_V^{\,2}(T^2-T_z^{\,2})+\Gamma_A^{\,2}(\vec{Y}^{(+)}\cdot\vec{Y}^{(-)}+\vec{Y}^{(-)}\cdot\vec{Y}^{(+)})a\rangle}{Z(G_V^{\,2}+3\Gamma_A^{\,2})}\right], \qquad (6)$$

$$\vec{Y}^{(\pm)} = \sum_i (\tau_1{}^{(i)} \pm i\tau_2{}^{(i)})\vec{\sigma}^{(i)}/2\sqrt{2}\,.$$

φ_a is the muon wave function, γ is proportional to $\bar{\nu}^2$, and $\bar{\nu}$ is the mean neutrino momentum.[8] $\Lambda(1, 1)$ is proportional to the capture rate by a free unpolarized proton and contains as a factor the coupling constants in the combination $G^2 \equiv G_V{}^2 + 3\Gamma_A{}^2$. Equation (3) differs from the final formula of Primakoff [Eq. (P-11a)] by the presence of the last two terms proportional to Δ_a. It is the dropping of these terms which is unjustified. Values of Δ_a for several nuclei have been calculated using $\bar{\nu} = \frac{3}{4}m_\mu$ and the functions φ_a and $\mathfrak{D}_a$ given by Ford and Wills.[9] Our results are $\Delta_a = 0.47$ for Ti, 0.36 for Mo, 0.25 for Gd, and 0.21 for Pb. Δ_a has the form of the square of a nuclear form factor and represents, in a sense, a long-range correlation contribution to the exclusion-principle inhibition.

We may simplify Eq. (3) by setting[10] $\delta' = \delta_a$ and letting J_a equal its maximum value of unity, which is exact for nuclei for which (1) isotopic spin is a good quantum number and (2) for every open proton shell, both neutron shells with the same value of l are closed. The resulting equation[11] is

$$\Lambda_a = Z_{\rm eff}{}^4\gamma\Lambda(1, 1)(1 - \Delta_a)[1 - \delta'(A - Z)/2A]. \tag{7}$$

In Fig. 1 we show the experimental values of Λ_a divided by $Z_{\rm eff}{}^4(1 - \Delta_a)$, using our calculated (or interpolated) values for Δ_a. Nuclei with $Z < 20$, for which assumption 4 is particularly unlikely

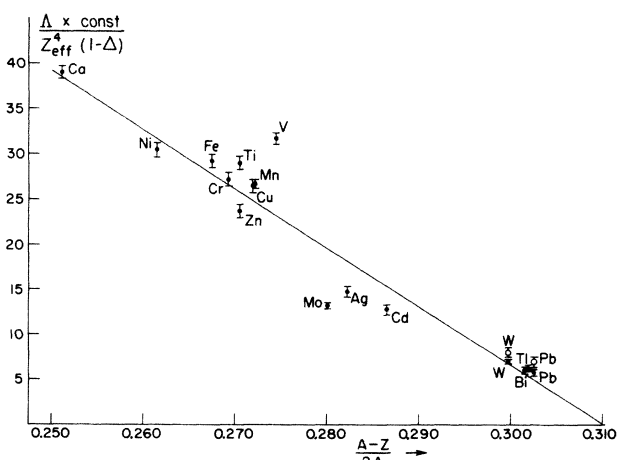

FIG. 1. Experimental values of $\Lambda/Z_{\rm eff}{}^4(1 - \Delta)$ vs $(A - Z)/2A$. The ordinate is given in the same units used by Sens and Telegdi. Errors shown are experimental only. Solid circles are from reference 1; open circles are those points from reference 2 that disagree significantly with reference 1.

to be valid, have been omitted. The plot is reasonably consistent with a constant value of δ', the deviations being, in general, similar to those found by Telegdi,[4] though somewhat larger for certain elements. Fitting with the solid line shown yields the values $\delta' = 3.23$ and

$$\gamma\Lambda(1, 1) = 340 \text{ sec}^{-1}. \tag{8}$$

The value of δ', fixed by the x intercept and thus determined primarily by the points near Pb, is close to that found by Telegdi; however, the value of $\gamma\Lambda(1, 1)$ and thus of G^2 is twice as large as the predicted UFI value of 160 sec^{-1} (for $\bar{\nu} = \frac{3}{4}m_\mu$). We may note that the result of our approximation setting J_a equal to unity is most likely to minimize the value deduced for $\gamma\Lambda(1, 1)$.

Considerable quantitative evidence in favor of UFI now exists, although none of it definitely establishes the presence of the vector current.[12] We therefore do not believe the result in Eq. (8), but rather feel that not all the assumptions of reference 3 are justified. Indeed, there are many reasons for doubting that δ_a and δ' defined by Eqs. (4) and (2) are sufficiently constant over the range of nuclei used. A detailed analysis indicates that the computed capture rate depends quite sensitively on the tail of the correlation function, $f(r)$, which should vary with the neutron-to-proton ratio. Furthermore, even if one assumes for $f(r)$ the truncated form of Eq. (P-10), δ_a and δ' would vary significantly with the nuclear radius. A recent calculation[7] of muon capture in Ca^{40} using assumptions 1 through 3 plus a statistical nuclear model yields (when compared to experiment) a value of G^2 equal to about 0.6 times the UFI value, a factor of over three times lower than the result from Eq. (8).

We therefore believe that the value of G^2 has not been determined within better than a factor of two by the analysis of the data on total muon capture rates by nonlight nuclei. However, if UFI is assumed, the original Primakoff formula [Eq. (7) with $\Delta_a = 0$ and δ' constant] does represent a reasonably good fit to the data. It remains an interesting open problem in nuclear physics to see if this equation can be derived in a convincing manner.

We are grateful to Professor Primakoff for helpful correspondence and discussion.

---

*Work supported in part by the U. S. Atomic Energy Commission.

[1]J. C. Sens, Phys. Rev. 113, 679 (1959).

[2]I. M. Blair, H. Muirhead, and T. Woodhead (to be

published).

[3]H. Primakoff, Revs. Modern Phys. 31, 802 (1959). Our notation follows closely that of Primakoff. Equations from this reference are listed as Eq. (P-); in this form, the formula referred to is Eq. (P-11a).

[4]V. L. Telegdi, Phys. Rev. Letters 8, 327 (1962).

[5]We will use UFI to refer to the following set of assumptions[3]: (1) g_A and g_V are the same for μ capture as for β decay except for small form-factor corrections. (2) The induced pseudoscalar coupling g_P equals $8g_A$ for μ capture. (3) The weak-magnetism coupling g_M is given by the conserved vector current theory. All experiments on total or partial capture rates are primarily sensitive only to combinations of G_V and Γ_A, where G_V is the Fermi coupling constant (closely equal to g_V), and Γ_A, the Gamow-Teller coupling constant, is a combination of g_A, g_P, and g_M.

[6]This point has been stated previously without a convincing demonstration [L. Wolfenstein, Proceedings of the Tenth Annual International Rochester Conference on High-Energy Physics, 1960 (Interscience Publishers, Inc., New York, 1960), p. 529; H. A. Tolhoek, Nuclear Phys. 10, 606 (1959)].

[7]J. R. Luyten, H. P. C. Rood, and H. A. Tolhoek (to be published) give this equation for the special case of closed-shell nuclei, for which $J_a = 1$.

[8]Since no approximation has been made in Eq. (3) as to the value of $\bar{\nu}$, this result with Δ_a and δ_a as functions of $\bar{\nu}$ might be useful in evaluating the cross section for the inverse process in which neutrinos are incident on complex nuclei. See S. Berman, International Conference on Theoretical Aspects of High-Energy Phenomena, CERN Report 61-22, 1961 (unpublished).

[9]K. Ford and J. Wills, Nuclear Phys. 35, 295 (1962); and private communication.

[10]If we assume $f(r)$ is different from zero only for $r < \bar{\nu}^{-1}$, then we obtain $\delta'/\delta_a \simeq 1 + \frac{1}{10}\bar{\nu}^2 d^2$, where $\frac{3}{5}d^2$ is the mean-square radius of the distribution defined by $f(r)$. With the value of d in reference 3, $\delta'/\delta_a = 1.05$.

[11]This equation was first derived by H. Primakoff in a private communication to one of us (L. W.).

[12]The rate of the reaction $\mu^- + C^{12} \rightarrow B^{12} + \nu$ [E. J. Maier, B. L. Bloch, R. M. Edelstein, and R. T. Siegel, Phys. Rev. Letters 6, 417 (1961); E. J. Maier, Ph.D. thesis, Carnegie Institute of Technology, 1962 (unpublished)] indicates that Γ_A^2 agrees with the UFI value within about 25 %, while the rate of $\mu^- + He^3 \rightarrow H^3 + \nu$ [I. V. Falomkin et al., Physics Letters 1, 318 (1962); International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN, Geneva, Switzerland, to be published)] gives a value of G^2 in agreement with UFI to better than 25 %. Recent measurements on muon capture in hydrogen [R. Hildebrand, Phys. Rev. Letters 8, 34 (1962); E. Bleser et al., Phys. Rev. Letters 8, 288 (1962); E. Bertolini et al., International Conference on High-Energy Nuclear Physics, Geneva, 1962 (CERN, Geneva, Switzerland, to be published)] are most likely consistent with UFI, although the rate appears somewhat low. If it is assumed that the axial-vector contributions (g_P and g_A) to μ capture are given by UFI, then both the He^3 and hydrogen results require a vector contribution. If we relax the condition on the axial-vector contributions {either because of some deviation from universality form-factor effects, or second-class current contributions [J. B. Adams, Phys. Rev. 126, 1567 (1962)]}, the analysis of present experiments does not exclude $g_V = 0$.

CONSTRAINTS IMPOSED ON MANDELSTAM REPRESENTATION BY UNITARITY

André Martin*
University of Washington, Seattle, Washington and CERN, Geneva, Switzerland
(Received August 27, 1962; revised manuscript received October 8, 1962)

It is the hope of several physicists[1] that by combining Mandelstam representation with unitarity one could determine completely a two-body scattering amplitude. Therefore, it seems important to reduce as much as possible the amount of information one has to put in a Mandelstam representation for a given process, to determine uniquely the corresponding scattering amplitude, by making use of unitarity. Much progress was made in this direction by Froissart[2] who showed that in the Mandelstam representation with a finite number of subtractions,

$$F(s,t,u) = \frac{1}{\pi^2} s^N t^N \iint \frac{\rho_u(s't')\,ds'dt'}{(s'-s)(t'-t)s'^N t'^N} + P_{stu} + \sum_{p=0}^{M<N} t^p s^M \frac{1}{\pi} \int \frac{\rho_{p,s}(s')\,ds'}{(s'-s)s'^M} + P_{stu}$$

$$+ \sum_{pq=0}^{L} t^p s^q \rho_{pq} + \text{pole terms} \qquad (1)$$

(where P_{stu} denotes a circular permutation), the subtraction terms, determined by $\rho_{p,s}$, etc., ρ_{pq},