points, so the energy interval has been made $\Delta E = 0.3$ eV, with the result for $h\nu \simeq 3.5$ eV and 5.3 eV shown in Fig. 4. The two peaks agree well with those observed[12] and confirm the basic assignments $\Gamma_{25'} \rightarrow \Gamma_{15}$ centered at 2.5 eV (more accurately, $\Delta_5 \rightarrow \Delta_1$ and $\Lambda_3 \rightarrow \Lambda_1$) and $L_{3'} \rightarrow L_3$. The width of the low energy peak is determined by the spread in Δ_1 and Λ_1 energy levels. The spread in the $L_{3'} \rightarrow L_3$ peak is also given qualitatively by the variation of Λ_3.

We are grateful to W. E. Spicer and R. E. Simon as well as G. W. Gobeli and F. G. Allen for discussions of their data prior to publication. We are particularly grateful to Dr. E. O. Kane for stimulating discussions concerning escape probabilities.

---

†Work supported in part by the Office of Naval Research and the U. S. Atomic Energy Commission.

*Resident Student Associate at Argonne National Laboratory from University of Chicago, Chicago, Illinois.

‡Shell Graduate Fellow.

‖Permanent address: University of Chicago, Chicago, Illinois.

[1]J. C. Phillips, Phys. Rev. 112, 685 (1958).

[2]F. Bassani and V. Celli, J. Phys. Chem. Solids 20, 64 (1961).

[3]D. Brust, J. C. Phillips, and F. Bassani, Phys. Rev. Letters 9, 94 (1962).

[4]J. C. Phillips, Phys. Rev. 125, 1931 (1962).

[5]H. R. Philipp (private communication). See also H. R. Philipp and E. A. Taft, Phys. Rev. 120, 37 (1960).

[6]M. H. Cohen and V. Heine, Phys. Rev. 122, 1821 (1961).

[7]M. Cardona and G. Harbeke, Phys. Rev. Letters 8, 87 (1962).

[8]D. T. F. Marple and H. Ehrenreich, Phys. Rev. Letters 8, 92 (1962).

[9]Transformation of the theoretical curve by a Lorentzian function of width $\Gamma = 0.05$ eV for $E > 3.4$ eV and $\Gamma = 0.02$ eV for $E < 3.4$ eV reproduces the experimental curve. However, the conduction-band density of states is nearly constant near this energy. The lifetime is also affected by unknown deformation potentials.

[10]F. G. Allen and G. W. Gobeli, Phys. Rev. 127, 141 (1962).

[11]E. O. Kane, Phys. Rev. 127, 131 (1962).

[12]W. E. Spicer and R. E. Simon, preceding Letter [Phys. Rev. Letters 9, 385 (1962)].

[13]G. W. Gobeli and F. G. Allen (to be published).

SCATTERING APPROXIMATION FOR LONG RANGE FORCES*

K. R. Greider

Yale University, New Haven, Connecticut

(Received June 11, 1962; revised manuscript received September 24, 1962)

Many important contributions in scattering theory have been made through the use of approximations, like the impulse approximation,[1] which treat many-body scattering processes in terms of the individual physical two-body scattering amplitudes. Although these methods have provided powerful tools in the analysis of complicated processes, they have been useful in general for short-range potentials and for large energies and momentum transfers. In this note we describe an approximation which, although considerably different from the impulse approximation, is also formulated in terms of the physical two-body scattering amplitudes. The method is apparently valid for all particle energies and momentum transfers, and requires at least one of the interactions to be long range. Consequently it appears useful in treating a wide variety of elastic and inelastic scattering problems in atomic and nuclear physics. To illustrate and substantiate the theoretical arguments, we show that our methods give rather good agreement with experiments for two different inelastic nuclear processes: the neutron transfer reaction in low-energy ion-ion scattering, and the high-energy (p, d) pickup process.

Before proceeding to the actual approximation we derive an expression that formally obtains the many-body scattering matrix in terms of the two-body amplitudes. For elastic and inelastic (non-rearrangement) scattering a suitable expression can be obtained from a slight modification of existing formulations, such as that of Gell-Mann and Goldberger.[2] Consider the scattering of one particle by another via two potentials U and V; the transition matrix element for this process is given in Eq. (4.4) of reference 2. The total wave function $\psi_a^{(+)}$ defined there can be written in terms of a factorable wave operator acting on an asymptotic

plane wave state:

$$\psi_a^{(+)} = \Omega_{U,V}\varphi_a = \left[1 + \frac{1}{E - K - U - V + i\epsilon}(U+V)\right]\varphi_a$$
$$= \left[1 + \frac{1}{E - K - U - V + i\epsilon}U\right]\left[1 + \frac{1}{E - K - U - V + i\epsilon}V\right]\varphi_a = \Omega_U''\Omega_V\varphi_a, \quad (1)$$

where

$$\Omega_V = 1 + \frac{1}{E - K - V + i\epsilon}V. \quad (2)$$

Using this result in Eq. (4.4) of reference 2, we find for the matrix element

$$T_{ab} = \langle\chi_b^{(-)}|t_V|\chi_a'^{(+)}\rangle + \langle\varphi_b|t_U|\varphi_a\rangle, \quad (3)$$

where

$$\chi_a'^{(+)} = (\Omega_V^{-1}\Omega_U''\Omega_V)\varphi_a = \Omega_V'\varphi_a,$$

and we have used the relations that define the two-body amplitudes, $t_V = V\Omega_V$, $t_U = U\Omega_U$, where Ω_U is defined by Eq. (2) with U replacing V.

In general, Ω_U' is quite complicated since it describes the distortion of the asymptotic state φ_a by the potential V, the subsequent distortion by U (in a V field), and finally the removal of the original V distortion. In practical applications this operator may be simplified in several ways. The greatest simplification is afforded by the impulse approximation in which the effects of U in the first term of Eq. (3) are neglected and both Ω_U' and Ω_U are set equal to unity. However, this procedure would give a poor approximation in many problems of interest, and one would do better to use distorted-wave methods to approximate the effects of the various operations in Ω_U'. A third method, that appears most promising in applications of the type considered in this note, is the expansion of Ω_U' in powers of V. If $U \gg V$ in one region of space and $U \to 0$ or $U \ll V$ everywhere else (as is approximately realized with combined Coulomb and nuclear potentials), such an expansion of Ω_U' is feasible, and yields Ω_U as its lowest order term in the region where $U \gg V$. One may then approximate $\Omega_U' \approx \Omega_U$ and replace $\chi_a'^{(+)}$ by $\chi_a^{(+)}$ in Eq. (3).

Rearrangement collisions may also be treated by methods similar to those just described. In the case where scattering occurs in both initial and final states via the two potentials U and V, the matrix element for the rearrangement process is given by only the first term of Eq. (3),[3] if the wave functions are defined as follows:

$$(H_a - E_a)\varphi_a = 0,$$
$$(H_b - E_b)\varphi_b = 0,$$
$$(H_a + U - E_a)\chi_a = 0.$$

H_a and H_b are the Hamiltonians for the initial and final asymptotic states, respectively. The total wave function ψ is the solution of the Schrödinger equation using the total Hamiltonian, $H = H_a + U + V + W' = H_b + U + V + W$, where W and W' are potentials that give binding only in the initial and final asymptotic states, respectively. The exact matrix element for the rearrangement scattering is obtained by again factoring the wave matrix as in Eq. (1):

$$T_{ab} = \langle\Omega_U'^{(-)}\varphi_b|t_V|\Omega_U^{(+)}\varphi_a\rangle, \quad (4)$$

where Ω_V and Ω_U are the same as in the nonrearrangement case if K and E are replaced by H_a and E_a, and

$$\Omega_U'^{(-)} = \Omega_V^{-1}\left[1 + \frac{1}{E_b - H_b - U^\dagger - V^\dagger - W^\dagger - i\epsilon}(U^\dagger + W^\dagger)\right]\Omega_V.$$

In this formalism W can be treated in the usual manner[4] as a scattering potential whose effects are incorporated into U.

Although the matrix element of Eq. (4) is exact, it appears quite different from the usual expressions for rearrangement collisions in direct interaction theory (as in stripping reactions) where either W or W' appears explicitly and serves as the "mechanism" for the reaction. We argue here that the use of the scattering interaction t_V instead of W is justified in those cases where the V interaction is so strong that it dominates all two-body processes. This is the case in many applications of the usual Chew-Wick impulse approximation where strong short-range (e.g., pion-producing) interactions dominate any rearrangement potentials; it is also the case here for long-range pro-

cesses such as ion-ion scattering where the two-body Coulomb interaction clearly dominates the reaction. Our approach may be justified too by the success of semiclassical theories in nuclear and atomic physics[5] in which rearrangement collision cross sections are calculated as the particles move in classical orbits of the dominant scattering potential, V. Equation (4) may therefore be considered as the completely quantum-mechanical analog of these semiclassical theories.

Now that the transition matrix element is written formally in terms of the two-body amplitudes in Eq. (3) for nonrearrangement, and Eq. (4) for rearrangement processes, it can be evaluated either by the usual impulse approximation (for short-range t_V) or by the long-range approximation, the main subject of this note. But it should be emphasized that these particular formal expressions [Eqs. (3) and (4)] do not exhaust the uses of the long-range approximation which, like the impulse approximation, should have quite general application in reducing the complexity of many-body scattering problems.

Consider the momentum-space representation of the matrix element of Eq. (4) by taking the Fourier transform of each of the three functions $\Omega_U^{(-)}\varphi_b$, t_V, and $\Omega_U^{(+)}\varphi_a$. (We assume here that such transforms exist and for simplicity we have set $\Omega_U' = \Omega_U$.) The form of the expression obtained in this manner will depend on the specific process under consideration. As an illustrative example we consider a rearrangement collision like that of reference 5, and neglect recoil effects to keep the treatment clear. After a little algebra we find

$$T_{ab} \sim \int d\vec{k} X_b^*(\vec{k}) \tau_V(\vec{k} - \vec{\Delta}) X_a(\vec{k}), \tag{5a}$$

$$= \int d\vec{k}' X_b^*(\vec{k}' + \vec{\Delta}) \tau_V(\vec{k}') X_a(\vec{k}' + \vec{\Delta}), \tag{5b}$$

where $X_b(\vec{k})$ is the Fourier transform of the distorted state $\Omega_U\varphi_b$ and $X_a(\vec{k})$ is the transform of $\Omega_U\varphi_a$, $\vec{k}_i$ and $\vec{k}_f$ are the center-of-mass momenta in the initial and final states, respectively, and $\vec{\Delta}$ is the physical momentum transfer, $\vec{\Delta} = \vec{k}_i - \vec{k}_f$.

We now state our new approximation as follows: If the known physical τ_V is highly peaked in a region near or on the path of integration in $\vec{k}$ space, Eq. (5) should yield reasonably results using only this physical (asymptotic) τ_V, provided that the momentum functions $X_{a,b}(\vec{k})$ are smoothly varying in $\vec{k}$.

For instance, in Eq. (5b) if τ_V is singular at the origin, then the main contributions to the integral above arise at values of $\vec{k} \approx 0$, provided that the $X_{a,b}$ possess no singularities or resonances in other regions along the path of integration. Then the lack of knowledge of τ_V in unphysical regions where energy and momentum are not conserved should not significantly affect our result, since these regions contribute only a negligible amount of the integral of Eq. (5). It is evident that we must ignore the possibility of further singularities in τ_V in unphysical regions—a subject that will be dealt with in a later publication.

From the arguments above, it is expected that these methods are particularly applicable to problems in which the Coulomb interaction plays a dominant role in the scattering. Fortuitously, the analytic form of the Coulomb amplitude is known, and satisfies our criteria since it is singular at the origin of the path of integration in k space:

$$\tau(k) \sim \exp[-i\eta \ln(k^2)]/k^2 = k^{-2i\eta}/k^2,$$

where $\eta = ZZ'e^2/\hbar v$ is the usual Coulomb parameter.

To illustrate the utilization of the Coulomb t matrix, we consider in this note only one of several promising applications: the low-energy ion-ion inelastic process, $N^{14} + N^{14} \rightarrow N^{15} + N^{13}$. The principal interaction in the Ω operator here is the nuclear potential between the two ions, and we have included this effect by considering an extremely simplified distorted-wave model. We assume that χ_a is equal to the asymptotic wave function φ_a, modulated by a real function, $f(\vec{r})$, of the relative coordinate between the two ions. $f(\vec{r})$ describes the large absorptive effects of the nuclear potential: It is zero at $r = 0$ and increases smoothly to unity around $r = R_0$, the sum of the radii of the two N^{14} nuclei. The one adjustable parameter in $f(r)$ describes the rate at which the function increases from zero to unity.

For the function φ_a, we again use a simple reasonable model. We let φ_a represent an ion "core" of N^{13} to which is bound a neutron forming the ground state of N^{14}, and a second ion "core" of N^{14}. Then the final state, φ_b, represents the N^{15} ion, composed of a neutron bound to the N^{14} core, and the remaining ion core, N^{13}. The results of the calculation are shown in Fig. 1 along with the recent experimental data of Jobes and McIntyre.[6] We also include the results of our method when all effects of U (the absorptive nuclear interaction) are ignored, i.e., with $\Omega_U^{(+)} = 1$. These latter results are similar to those obtained from the earlier work of Breit and Ebel[5] on the semiclassical theory of neutron transfer in ion-

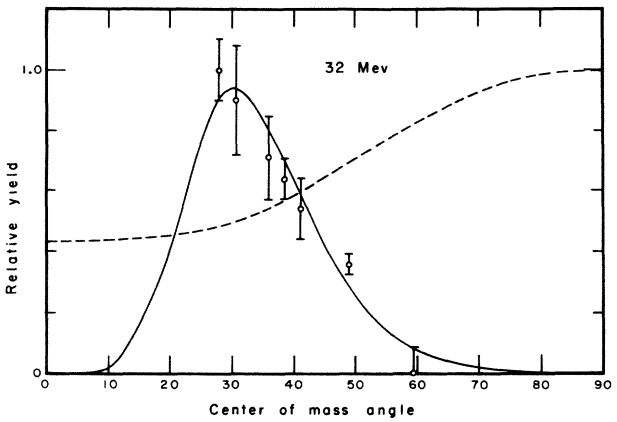

FIG. 1. Differential cross section for N^{13} ions from the reaction $N^{14}+N^{14}\rightarrow N^{13}+N^{15}$ at 32 MeV. The experimental points of Jobes and McIntyre are shown, along with the curves obtained from the present theory. The solid curve includes the effects of the nuclear potential between the two ions, while the dashed curve neglects these same effects.

ion reactions.

For the second application of the theory, we consider stripping and pickup processes. These provide rather good tests of the approximation because in addition to many experiments, there exist fairly accurate distorted-wave calculations with which we can compare our new methods. Specifically, we calculate the cross section for the process $p+C^{12}\rightarrow d+C^{11}$ at high energies (~100 MeV). In order to keep the treatment simple, and in view of the large kinetic energies of the particles, we now ignore the Coulomb potential and consider instead that the dominant interaction τ_V is due to the absorptive effects (imaginary nuclear potential) felt by the proton. (One can also include the deuteron's absorptive potential in a symmetric way.) We assume τ_V to have the usual form due to diffraction of particles by a completely absorbing sphere of radius R:

$$\tau(k)=J_1(kR)/kR\approx j_1(kR')/kR'.$$

This function should also be suited to our methods since in momentum space it is peaked at the origin, due to the fact that the absorptive potential is characterized by large impact parameters in configuration space. The spherical Bessel function is substituted only for calculational purposes and gives virtually the same results as J_1, if R and R' are properly related.

In this calculation we make the additional approximation of neglecting all other interactions

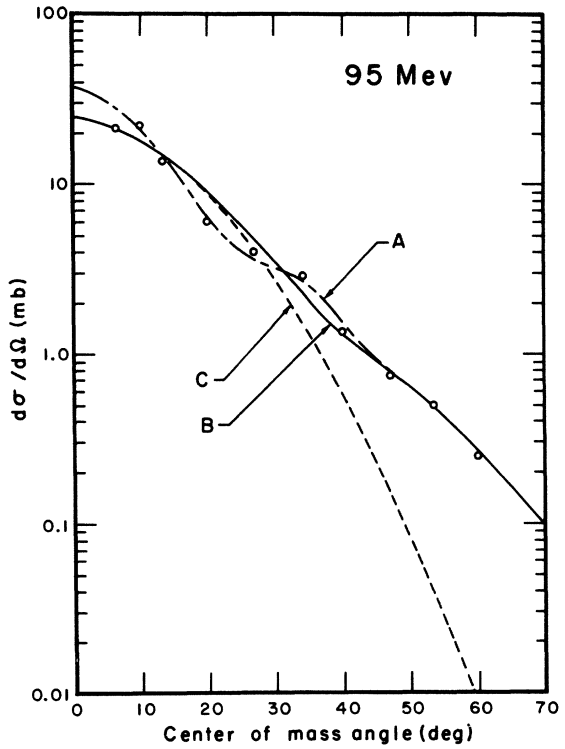

FIG. 2. Differential cross section for deuterons from the pickup reaction $p+C^{12}\rightarrow d+C^{11}$ at 95 MeV. The experimental points of Selove are shown by circles. The result of the Born approximation (Chew-Goldberger theory) is shown by curve C, and the result of a distorted-wave Born approximation by curve A. The present theory is given by the solid curve B.

except τ_V, that is, we set $\Omega=\Omega'=1$. Thus the momentum functions used in Eq. (5) are merely the Fourier transforms of the asymptotic plane-wave functions $\varphi_{a,b}$, for which we use a model based on a C^{11} core, similar to the model used in the neutron transfer problem above.

Figure 2 shows our results along with Selove's experiments for the (p,d) process on C^{12} at 95 MeV.[7] For comparison, the results of both the Born approximation (Chew-Goldberger theory)[8] and a distorted-wave optical-model calculation by the author[9] are included. It should be emphasized that even with the extremely simplifying conditions $\Omega=\Omega'=1$, one achieves significant improvement over the Born approximation, and rather good agreement both with the distorted-wave calculation and with experiment. By including distortion effects (of the Coulomb potential, for instance), one could obtain significant improvement over results of current distorted-wave theories.

The other approximate method of solving Eq. (5) is that of Chew-Wick impulse approximation. The strong short-range interactions in configuration space will yield extremely slowly varying functions, $\tau_V(\vec{k})$, in momentum space. From Eq. (5a) we see then that if the momentum functions $X(\vec{k})$ (which in the impulse approximation are the transforms of the asymptotic functions φ, i.e., $\Omega = 1$) are peaked at $k = 0$, the major contributions to the integral come only from this region. Thus for large momentum transfers we can approximate $\tau_V(\vec{k} - \vec{\Delta})$ by $\tau_V(\Delta)$, and take it outside the integral as a multiplicative factor. The resulting integral over only the momentum functions $X(\vec{k})$ give the effects on the scattering amplitude of the "spectator" particle as in the usual formulation of the impulse approximation. Another advantage of our formalism is that it offers a relatively simple way of calculating corrections to the impulse approximation. Such corrections appear in a multiplicative rather than additive way if one uses $\chi = \Omega\varphi$ ($\Omega \neq 1$) in a distorted-wave approximation.

A forthcoming paper will give results for other applications of the theory (in particular to the scattering of electrons by hydrogen atoms) as well as expand on both the details of these calculations and the general formalism.

The author would like to thank Professor G. Breit for bringing up the specific problem from which this approximation evolved, and Professor L. Durand, III, and Professor G. Rawitscher for many helpful discussions and comments.

*This research was supported by the U. S. Atomic Energy Commission and by the Air Force Office of Scientific Research.

[1]G. F. Chew, Phys. Rev. 80, 196 (1952); G. F. Chew and G. C. Wick, Phys. Rev. 85, 636 (1952).

[2]M. Gell-Mann and M. L. Goldberger, Phys. Rev. 91, 398 (1953), Eq. (4.4). It should be noted that $\chi_a^{(+)}$ of their first term of this equation should read $\psi_a^{(+)}$. This example of a nonrearrangement process is used to illustrate conveniently the formulation of the two-potential problem in terms of the single-potential scattering amplitudes, and this formulation is easily extended to describe also the scattering of a particle by a bound two-body system.

[3]B. A. Lippmann, Phys. Rev. 102, 254 (1956).

[4]N. C. Francis and K. M. Watson, Phys. Rev. 93, 313 (1954).

[5]G. Breit and M. E. Ebel, Phys. Rev. 103, 679 (1956).

[6]F. C. Jobes, thesis, Yale University, 1962 (unpublished), and F. C. Jobes and J. A. McIntyre (to be published). The author would like to thank Professor McIntyre for advance communication of the data.

[7]W. Selove, Phys. Rev. 101, 231 (1956).

[8]G. F. Chew and M. L. Goldberger, Phys. Rev. 77, 470 (1950).

[9]K. R. Greider, Phys. Rev. 114, 786 (1959).

MEASUREMENT OF THE POLARIZATION OF THE FRASCATI 1-GeV ELECTRON SYNCHROTRON γ-RAY BEAM FROM A DIAMOND CRYSTAL RADIATOR

G. Barbiellini, G. Bologna, G. Diambrini, and G. P. Murtas
Laboratori Nazionali di Frascati del Comitato Nazionale per l'Energia Nucleare, Roma, Italy
(Received October 8, 1962)

In a previous Letter[1] we showed experimental evidence for a line spectrum in the 1-GeV bremsstrahlung γ-ray beam obtained using a diamond radiator. In this Letter we give some results of the calculations and measurements concerning the polarization of a 150-MeV photon line.

In high-energy electron bremsstrahlung, the radiated photons are emitted preferentially with a state of linear polarization parallel to the plane determined by the direction of the primary electrons and the recoil momentum of the nucleus. In a crystal only those recoil momenta are allowed which, in suitable units, are coincident with the reciprocal lattice vectors. Therefore the process is not symmetrical around the direction of the primary electron. It follows that the entire bremsstrahlung beam has a net polarization, with respect to a crystal plane.

The first calculation of this polarization was performed by Überall[2]; we have repeated it in order to take into account the discrete structure of the lattice planes. The importance of this structure has already been stated in other works.[1,3] A few numerical results of this calculation were given by us in reference 1. At the same time Überall also had recalculated the polarization, but his numerical data[4] are not comparable with our experiment.

We define the polarization of the entire γ-ray beam as

$$P = (I_H - I_V)/(I_H + I_V), \qquad (1)$$