ferent, peaking at 40 μ Hg and falling by a factor of 5 at 10 μ Hg. Similar results for reversed trapped field experiments have been reported in detail by others.[3]

The effect on neutron yield of bias field direction is shown in Fig. 3(b). It is clear that neutrons are produced not only with zero bias field, but with positive bias fields as high as 1 kG. The yield falls off smoothly as the positive bias field is increased. The conditions with negative bias field are more complicated. With negative bias above 1.5 kG, the neutron output rapidly increases with field. The increase may be related to additional plasma energy produced by axial contraction[4] or by trapped field diffusion. Streak pictures show a large increase in the plasma diameter typical of the collision of axial shocks produced by contraction. At negative bias fields larger than 3 kG the yield becomes constant and streak pictures indicate the axial contraction becoming so energetic that plasma hits the wall when the shocks collide, and the neutron production is quenched earlier in the current cycle.

For negative bias fields below 1.5 kG no evidence of axial contraction is observed and the time distribution of the neutrons for this condition is essentially the same as for the zero bias field condition.

We conclude that, at these low densities, large yields of neutrons may be produced without reversed trapped magnetic fields and under initial conditions of negligible parallel and B_θ trapped fields. It should be noted that at the low pressures and in the short time preceding the neutron production it is unlikely that the plasma is dominated by Coulomb collisions.

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ENHANCEMENT OF HYDRODYNAMIC STABILITY BY MODULATION

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We have investigated the stability of flow between rotating cylinders (Couette flow) in the case where the inner cylinder rotates and the outer cylinder is at rest. All experiments and theories to date[1] have dealt with steady rotation of the cylinders; this Letter describes the effect of modulating the rate of rotation. In many systems (e.g., classical inverted pendulum,[2] charges in an electrostatic field,[3] ferromagnetic resonance,[4,5] etc.), modulation of a suitable parameter can have marked effects on the motion and can result in increased stability of the system. The present experiments seek to explore the extent to which similar modulation techniques lead to enhancement of stability in hydrodynamics.

The technique for studying the instability is new[6] and has been developed at the University of Chicago during the past year. The apparatus shown in Fig. 1 consists of a pair of coaxial metal cylinders, electrically insulated from each other. The space between the cylinders is filled with carbon tetrachloride, the liquid under study. The current reaching the ring electrode, formed by a 0.005-in. long insulated section of the outer cyl-

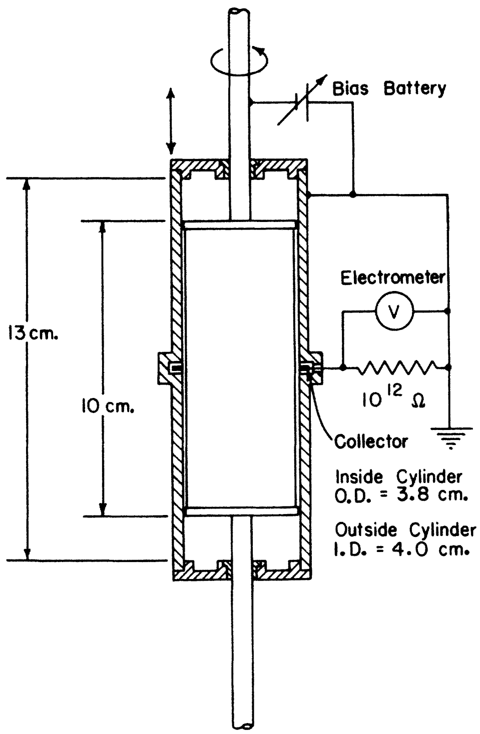

FIG. 1. Sketch of the ion apparatus for studying hydrodynamic stability. The inner cylinder rotates and the outer cylinder is at rest. The outer cylinder may be moved slowly up and down with respect to the inner cylinder in order to study the distribution of velocity in the fluid at the position of the insulated collector ring.

inder, is measured by a vibrating reed electrometer. With zero applied bias the collected current is due to impurity ions in the carbon tetrachloride and is of the order of 10^{-12} amperes. The current depends upon the mobility of the ion species present and their diffusion constants. When the inner cylinder rotates it produces motions in the fluid which change the current. The outer cylinder is so arranged that it can be moved slowly in the axial direction in order to study the distribution of velocity in the fluid. This instrument brings a new and versatile tool into use, without which the present experiments might have proved to be quite difficult.

At 25°C the kinematic viscosity of carbon tetrachloride is about 5.80×10^{-3} cm^2/sec. In this case, with cylinders of radii $R_1 = 1.9$ cm and $R_2 = 2.0$ cm, the critical angular velocity for the onset of instability is $\Omega_c = 5.63$ radians/sec. Below Ω_c the flow is laminar, while above Ω_c the flow breaks up into a series of toroidal vortices of nearly square cross section spaced uniformly in the axial direction. A recording of collected current as a function of axial position of the outer cylinder is shown in the left portion of Fig. 2 for a speed $\Omega = 5.80$ rad/sec. The variation of the signal amplitude A with speed of rotation is shown on the left curve of Fig. 3. We have yet to determine the relationship between fluid velocity and signal amplitude; so the results are expressed in terms of the signal at $\Omega = 5.80$ rad/sec, which is adopted as the unit amplitude of velocity A_0. It should be emphasized that we have no assurance that the relative signal A/A_0 is a linear function of velocity so that our curves of A/A_0 vs $\bar{\Omega}$ must be regarded as qualitative in shape. Nonetheless, it will be seen that very useful information can still be obtained from such curves.

The method of modulating the speed is to arrange two Graham variable speed transmissions in series. The first transmission fixes the mean speed $\bar{\Omega}$. The control of the second transmission is varied continuously over a definite range to set the period of modulation P and the depth of modulation $\Delta\Omega$. Thus, the angular velocity is given by $\Omega = \bar{\Omega} + \Delta\Omega\cos(2\pi t/P)$ and in any series of runs $(\Delta\Omega/\bar{\Omega})$, the relative depth of modulation, is near-

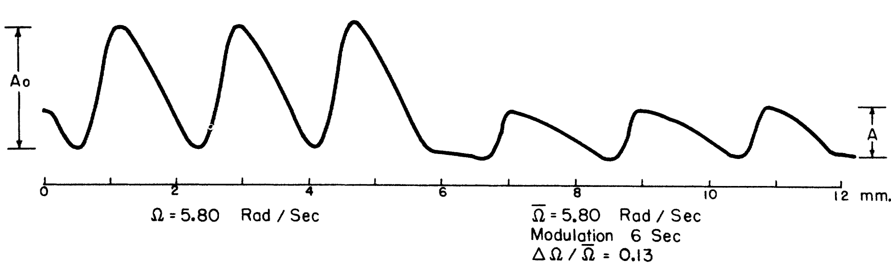

FIG. 2. Recorder traces of ion current as a function of axial position of the collecting ring. The three vortex cells at the left are taken without modulation; those on the right are taken with the same mean speed, but with modulation. The reduction of velocity in the vortices is readily observed.

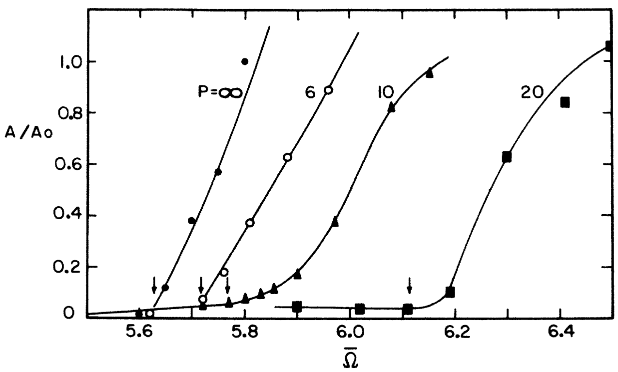

FIG. 3. Relationship between the velocity of the vortices and the mean angular velocity of the inner cylinder. With no modulation ($P=\infty$) the critical angular velocity is $\Omega_c = 5.63$ rad/sec as indicated by the small arrow. With modulation the vortex velocity begins to increase at $\bar{\Omega}_c$, as indicated by the small arrows for $P=6$, 10, and 20 sec.

ly constant.

Preliminary results of the experiments are shown in Fig. 2. It can be seen that at any given mean angular velocity of rotation, $\bar{\Omega}$, applying modulation decreases the circulation of the vortices. A particular example is seen in Fig. 2. A number of results are collected in Fig. 3 for a constant modulation depth $\Delta\Omega/\bar{\Omega} = 0.13$. The curves show that although traces of vortex motion remain at lower speeds (due to transient instability from the modulation), a definite speed $\bar{\Omega}_c$ may be located at which the signal A/A_0 begins to grow, and this speed lies higher than Ω_c. In other words, the stability of the flow may be said to be enhanced by modulation, the degree of enhancement being greater, the longer the period of modulation. The reason for the increased efficiency of slow modulation is that the modulation produces a viscous wave in the fluid which penetrates a distance proportional to the square root of the period P. This trend has been observed to reverse when P is sufficiently large (~40 sec) that the flow is approaching a quasi-stationary state at all times during each cycle. The curve for $P=20$ sec in Fig. 3 illustrates the degree of enhancement of stability attainable through such modulation. Here the effective critical speed is as high as $\bar{\Omega}_c = 6.12$ rad/sec, although during its excursion between $\Omega = 5.33$ and $\Omega = 6.91$ rad/sec, the speed Ω remains only for a small fraction of each cycle in the normal laminar range below $\Omega_c = 5.60$ rad/sec.

These results are in qualitative agreement with a rather complicated theory, which will be given in a later paper. On an inviscid analysis, one obtains two simultaneous first-order equations with periodic coefficients describing the time variation of the radial and azimuthal flow disturbances. An especially simple case (which does not, however, reflect the conditions of the present experiment) is that of a viscous liquid modulated sufficiently slowly so that there is no phase shift (the condition for this is modulation frequency times mean radius squared, much less than the kinematic viscosity). In that case, if the inner and outer cylinder are modulated in phase, but with depths of modulation inversely proportional to the square of their radii, the equations reduce to a single Mathieu equation, with the usual stability and instability regions. On this theory, if the outer cylinder is held fixed, no stabilization results. The present experiments, however, suggest that even when one is far from the condition of zero phase shift through the fluid, and with the outer cylinder fixed, stabilization still results.

These results suggest a number of important investigations. In particular, if modulation can stabilize parallel flows such as the boundary layer over a flat plate, the consequences should be of considerable interest to both physicists and engineers as a method of boundary layer control.

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