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THEORY OF FLUX CREEP IN HARD SUPERCONDUCTORS

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
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A major difficulty in understanding hard superconductors has been the appreciable temperature dependence of critical currents and fields at temperatures as low as 0.1 T_c. None of the properties of the bulk superconducting state vary noticeably at this temperature.

Bean[1] has introduced the notion of the "critical state" of the hard superconductor, and Kim et al.[2,3] have shown that it may be defined, in their experiments, in terms of constants α and B_0:

$$\alpha(T) = J_{\rm cr}(B_{\rm cr}+B_0). \qquad (1)$$

If the current density J or the field B is increased beyond the critical values in (1), the process we shall call "flux creep" sets in, and flux leaks through the material and returns it to the critical state (or, occasionally, complete breakdown occurs). $\alpha(T)$ is a structure-sensitive constant of the material, and a typical temperature dependence is shown in Fig. 1.

We shall show that this behavior of α can be explained by assuming that the mechanism of flux creep is thermally activated motion of bundles of flux lines, aided by the Lorentz force $\vec{J}\times\vec{B}$, over free energy barriers coming from the pinning effect of inhomogeneities, strains, dislocations, or other physical defects. This theory also explains the constant B_0, and predicts time relaxation behavior strikingly similar to those of magnetic aftereffect and some forms of plastic creep, which are explained in a very similar way.[4] This behavior has been verified by Kim et al.[3]

First, we must define, if somewhat imperfectly,

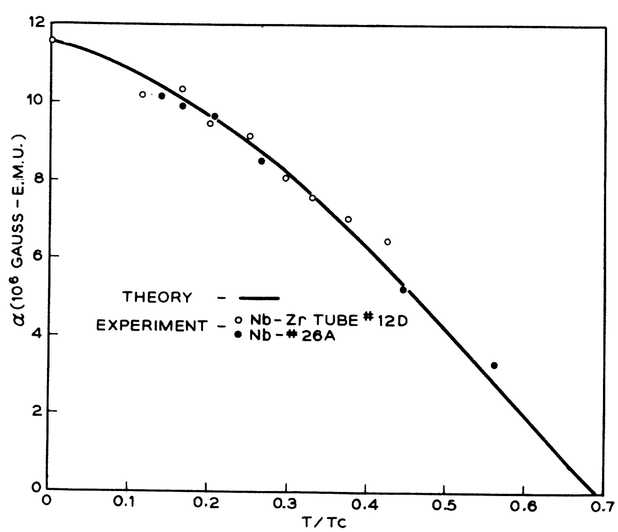

FIG. 1. The critical state constant α as a function of T/T_c. The line is a theoretical fit; the points are experimental ones for two different tubes.

the bundles of flux which are the moving entities. They are not necessarily single quantized flux lines. Theoretically[5] the magnetic field of a flux line extends over a distance of the order of the London penetration depth $\lambda_L \sim 5\times10^{-6}$ cm, and we therefore expect that flux lines closer together than λ_L are to some extent bound together into "bundles" by the interaction of their fields and wave functions. The bundle may be of dimension $d \sim 10^{-5}$-10^{-6} cm. Its internal structure may be visualized as similar to the Abrikosov structure,[5] which we imagine as extending throughout the sample but irregularly so in some degree.[6] We simply assume that the length of the "bundle" which can move independently is of order d also.

The most extreme inhomogeneity which could pin down a flux bundle of volume d^3 would lead to a free energy barrier of

$$\Delta F_{\max} = (F_n - F_s)d^3 = (H_{cB}^2/8\pi)d^3.$$

H_{cB} is the bulk critical field. Actually, in any reasonable case the pinning effect will be very much smaller; for instance, a single dislocation might pin proportionately to a fractional volume $\sim r^2/d^2 \sim 10^{-4}$. An arbitrary structure-sensitive parameter of our theory will be the average fractional amount of pinning p:

$$\Delta F = p\Delta F_{\max} = (pH_{cB}^2/8\pi)d^3. \quad (2)$$

We then suppose that the free energy as a function of bundle position x is a random function $F_i(x)$ with scale length in x of d, with scale height ΔF.

As a result of the penetration of flux lines into the superconductor, there will be a mean current flow J, and a force on the flux bundle:

$$f = \int J\times B d\tau.$$

J is in emu. This leads to a contribution to the free energy as a function of x of

$$-JB\times d^3 = -J\Phi_B dx, \quad (3)$$

where Φ_B is the total flux in the bundle. Note that Φ_B cannot be smaller than $\Phi_0 = hc/2e$. Thus the total free energy is

$$F(x) = F_i(x) - J\Phi_B dx. \quad (4)$$

The free energy hump which the bundle must climb to get from one fairly stable minimum to another will then be of order

$$\Delta F^* \cong pH_{cB}^2 d^3/8\pi - J\Phi_B d^2, \quad (5)$$

and the rate at which the bundle will hop the barrier will be

$$\text{Hop rate} = \boldsymbol{R} = \boldsymbol{R}_0 \times \exp[-(1/kT)(pH_{cB}^2 d^3/8\pi - J\Phi_B d^2)], \quad (6)$$

where R_0 is an appropriate frequency factor, which is hard to estimate but certainly should not be greater than 10^{10}/sec or so.

Let us now, to write down an actual rate equation, specialize to the tubular geometry used by Kim. He uses a tube of radius a and thickness w, with an interior field H' and an outer field H. If $H > H'$, flux creep will lead to the motion of flux bundles into the tube at a rate which may be estimated to be R times the number of bundles in the wall of the tube ($\cong 2\pi awH^*/\Phi_B$) and divided by the number of barriers a bundle may see in traversing w ($= cw/d$, c being a parameter giving the number of barriers which are effective.) It is then quickly shown that the equation determining the rate of creep is

$$(H^*)^{-1}(d/dt)(H - H') = -(2dR_0/ca) \times \exp[-(1/kT)(pH_{cB}^2 d^3/8\pi) - J\Phi_B d^2)]. \quad (7)$$

From this formula we may extract the results mentioned.

(1) <u>The critical state.</u>—We have obviously predicted that there is no precise "critical state," since the creep in (7) continues at any values of J and B. Nonetheless, one may define the critical state as that at which the rate falls below a practically observable limit. For Kim's experiment this may be roughly 10^{-3} times the inverse of the duration of an average experiment—say 1 h. Thus we get

$$(J\Phi_B)_{\text{cr}} = pH_{cB}^2 d/8\pi - (kT/d^2)\ln(3.6\times10^6\times 2dR_0/ac). \quad (8)$$

Let us consider high fields first, where $\Phi_B = d^2H^*$. Then the above agrees with Kim's formula (1) and gives

$$\alpha = pH_{cB}^2/8\pi d - (kT/d^4)(28\pm9). \quad (9)$$

This allows for a total uncertainty in our estimates of R_0 and c of a factor 10^8. Figure 1 shows that (9) fits the observed curves very satisfactorily. Using for illustrative purposes Kim's Nb-Zr tube No. 12D, we estimate

$$d \sim 1.0\times10^{-5}\text{ cm},$$

$$p \sim 7\times10^{-3}\text{ cm},$$

which are reasonable values, although p seems a bit large; perhaps the strongest barriers are the most effective.

As for the constant B_0, we may ascribe this to the fact that the bundle must not contain less than a flux quantum Φ_0. Thus even when H^* is near zero J will have a critical value. It is not possible in view of the possible fluctuations in bundle size to verify the form (1) of the expression, but we can predict

$$B_0 = \Phi_0/d^2 \simeq 2000.$$

This is very close to the observed values.

(2) Time behavior.—This may be obtained very readily by integrating Eq. (7). Let us define

$$K_1 = (2dR_0/ac)\exp(-pH_{cB}{}^2d^3/8\pi kT), \qquad (10)$$

and

$$\alpha(t) = J(t)[B(t) + B_0]. \qquad (11)$$

Then (7) becomes

$$d\alpha/dt = -H^*|d\alpha/dH'|K_1\exp(\alpha d^4/kT). \qquad (12)$$

Assuming that the total variation of α is small, and allowing an initial transient to die out, this gives

$$\delta\alpha \simeq \text{const} - (kT/d^4)\ln t,$$

$$\delta H' = (dH'/d\alpha)(kT/d^4)\ln t,$$

$$= -[k'w/(H' + B_0)](kT/d^4)\ln t. \qquad (13)$$

We have used the critical state equation (1).[3] Where dH'/dH, and thus $dH'/d\alpha$, is steep the rate becomes rapid, and at a point of vertical slope instability would occur.

This characteristic logarithmic behavior is shown in Fig. 3 of the preceding Letter.[3] The dependence of the logarithmic slope on T and H has been verified. The dimensionless constant $kT/\alpha d^4$ is measured by Kim et al. to be about 1/400, while our values (for a slightly different tube) would give about 1/70 theoretically.

In conclusion, it should be emphasized that this is an extremely rough theory of only one of many possible regimes of behavior of the hard superconductor. We have completely neglected such important factors as changes of free energy with current or field, changes of λ_L with these or temperature, shape factors of the flux bundles, details of the interactions between flux lines, etc. The results are surprisingly accurate for Kim's measurements, but in application to any other system or regime one can expect major modifications. Nevertheless, we feel the concept of activated motion of flux structures is a proved one in this case, and one of some importance. A second new feature is the importance of the interaction of quantized flux lines to the problem. We have only speculated as to its size and range; explicit study should be undertaken.

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