CRITICAL PERSISTENT CURRENTS IN HARD SUPERCONDUCTORS

Y. B. Kim,* C. F. Hempstead, and A. R. Strnad
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received September 12, 1962)

We have been investigating persistent currents induced in tubular samples by measuring the resulting magnetic effects. This technique of "tube magnetization" is particularly useful for studying critical currents in hard superconductors. While the details of this technique will be communicated elsewhere,[1] in this Letter we present evidence that (a) in hard superconductors the Lorentz force plays a crucial role in determining the critical current density, and (b) the critical persistent currents in hard superconductors decay with a measurable rate.

Hard superconductors (HSC) have critical magnetic field values much higher than those of ideal soft superconductors, and do not exhibit appreciable Meissner effect. These behaviors are usually attributed to meshes of superconducting filaments imbedded throughout the material, a picture first suggested by Mendelssohn.[2] There is considerable indirect evidence for this filamentary model.[3-5] One consequence of the filamentary structure is that HSC exhibit magnetic hysteresis. The magnetic properties of HSC have been treated theoretically on the basis of thermodynamic energy density, including a surface energy term resulting from the filamentary structure.[3,6,7] A more direct, but less fundamental, approach was made by Bean,[8] who predicted the magnetic behavior of HSC from the critical current which can be carried by the filaments. In the present work, Bean's approach is extended, and the critical currents are measured by the magnetic effects they produce.

A suitable sample geometry is that of a tube of uniform wall thickness w, and length large compared to its diameter (see Fig. 1). An external field H is applied parallel to the tube axis, and the axial field H' at the tube center is measured with a sensitive magnetoresistance probe wound noninductively of high-purity copper wire. As the external field H is changed, currents are induced in the tube which counteract the change in H. Starting from the origin, for example, H' will remain zero as H is increased. When H reaches H_s the tube walls are carrying the maximum supercurrents permitted by the sample at the given temperature. If H is further increased, the wall currents cannot increase and continue to shield the tube interior, so H' rises with H. For increasing H, the "shielding" portion of the $H'(H)$ curve in the first quadrant lies below the 45° line until H reaches the filamentary critical field of the sample. If H is now decreased, H' remains at a constant value until the "trapping" portion (above the 45° line) of the $H'(H)$ curve is reached. When H is reduced to zero, the tube is left with a trapped field H_s'.

If H is changed sufficiently slowly, so that H' and H remain close to their equilibrium values, the magnetization curve $H'(H)$ is remarkably reproducible and predictable. When the values of H' and H lie on this curve, the sample is said to be in a critical state.[8] In this state, every macroscopic region in the sample carries a critical or maximum current density $J(B)$ determined by the local magnetic field B at that region. This hypothesis of a critical state has been fully verified by the present experiments.

The analytic expression for the critical state is readily derived as follows. The internal field H' is the algebraic sum of H and the field produced by current induced in the sample, i.e.,

$$H' = H + k\int_0^w J[B(r)]dr, \tag{1}$$

where $k = 0.4\pi$ gauss-cm/amp and r is a radial variable measured from the outer surface of the tube. Eliminating r, we obtain an integral rela-

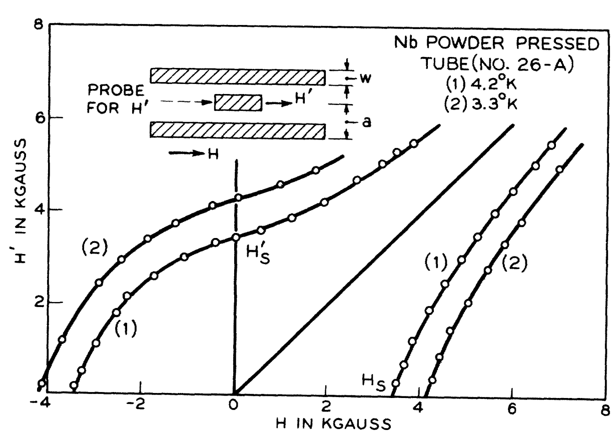

FIG. 1. Experimental magnetization curves for a tube sample. Magnetoresistance probe for H' is 0.86 cm long in the axial direction. Dimensions of tube samples in the low-field runs are i.d. = 0.36 cm, wall thickness from 0.1 to 0.25 cm, and tube length from 2.5 to 5 cm.

tion

$$kw = \int_H^{H'} \frac{dB}{J(B)} = \int_{H^* - \frac{1}{2}M}^{H^* + \frac{1}{2}M} \frac{dB}{J(B)}, \tag{2}$$

where $H^* = \frac{1}{2}(H' + H)$ is the mean field in the sample wall, and $M = H' - H$ is the field produced by the induced supercurrents. The average current density in the wall is then $J^* = M/kw$.

The general behavior of $J(B)$ suggests a power series expansion

$$\alpha/J = B_0 + B + a_2 B^2 + a_3 B^3 + \cdots, \tag{3}$$

and using the second expression of (2) we obtain

$$\alpha kw/M = \alpha/J^* = B_0 + H^* + a_2(H^{*2} + M^2/12) + \cdots. \tag{4}$$

k and w are constants for a given experiment, and $1/M$ and H^* come directly from measurements of H' and H. If the coefficients a_2, a_3, $\cdots$ are sufficiently small, then (3) reduces to

$$\alpha/J = B_0 + B, \tag{5}$$

and (4) implies that a plot of $1/M$ vs H^* should give a straight line. Since this was obtained, we derived the critical state curve from (5). The result is two hyperbolas,

$$(H' + B_0)^2 - (H + B_0)^2 = \pm 2\alpha kw, \tag{6}$$

in the first quadrant, and a circle,

$$(H' + B_0)^2 + (H - B_0)^2 = 2(\alpha kw + B_0^2), \tag{7}$$

in the second quadrant.

These predictions are verified by the experimental curves shown in Fig. 1. At a lower temperature, α increases and the curves shift outward from the 45° line, but the shapes remain unchanged. An even more sensitive test of the validity of (5) is linearity of the $1/M$ vs H^* plot.

Early measurements on Nb_3Sn and 3Nb-Zr samples[1] were made using a Bitter-type solenoid for external fields up to 100 kilogauss. The results of these measurements strongly implied (5), and led to the development of the theory of tube magnetization and the prediction of critical state curves (6) and (7). Later measurements and verification were carried out using a well-stabilized and controllable Varian 12-inch magnet. As will be shown later, critical states are inherently unstable, and $|dH/dt|$ must remain small while moving from one critical state to another. Otherwise the entire sample will go normal momentarily, causing most of the supercurrent to decay quickly. When this occurs, the value of H' suddenly approaches H, and we say that a "flux jump" has taken place. The degree of instability, or ease with which a flux jump is caused, depends upon the location on the critical state curve—where the slope is large, the instability is large. In fact, it is possible to trace the complete curve in the second quadrant only with certain samples. The degree of instability also depends on temperature and on the total induced current. In regions where the instability is not severe, however, an appreciable time lag between changes in H and changes in H' was observed. Measurements of this effect will be described at the end of this Letter.

Values of α obtained from low-field data are shown in Fig. 2 as a function of reduced temperature, T/T_C. B_0 varies from 1 to 2 kilogauss for annealed 3Nb-Zr tubes and from 0.5 to 2 kilogauss for Nb powder sintered tubes. The sintering and annealing temperatures have considerable control

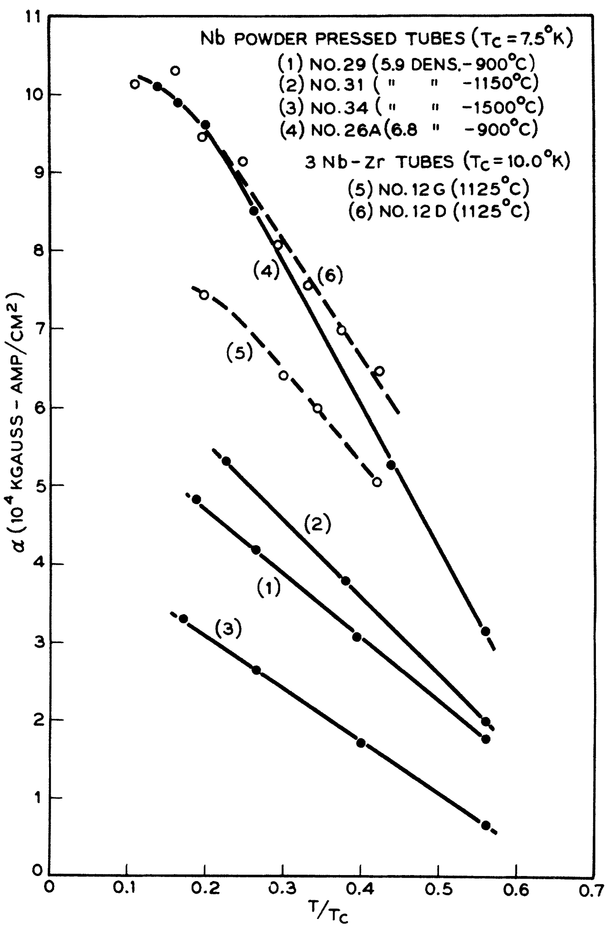

FIG. 2. α as a function of T/T_C for various tube samples. The solid circles are experimental values for Nb powder pressed samples of different densities, and sintered at different temperatures as shown. The open circles are for 3Nb-Zr samples annealed at temperatures as indicated.

over α, but little effect on T_C.[9] This agrees with earlier observations[4] that the current-carrying capacity of HSC is very sensitive to the physical structure of the material. Although α values vary markedly among samples, their temperature dependences remain linear down to $T/T_C \simeq 0.2$. For the sintered Nb powder samples, the lines converge toward a point on the abscissa at ~0.75. For the 3Nb-Zr samples, the point of convergence is 0.85. Thus, for the temperature range of this investigation, the empirical relation describing α is

$$\alpha = (1/d)(a - bT). \tag{8}$$

The constants a and b appear to satisfy $a/b \lesssim T_C$, while d will depend strongly on the physical microstructure of the material.

For all the HSC tested to date (Nb_3Sn, 3Nb-Zr, and Nb powder), the relation (5) holds up to $B \sim 15 B_0$. When $B \gg B_0$, (5) implies $JB = \alpha = \text{const}$ indicating that the Lorentz force plays a crucial role in determining the critical current density. This suggests that the usual free energy density difference between the normal and superconducting states will be reduced by an amount $JBd = \alpha d$, where d is a distance probably of the order of a filament diameter. By combining (5) and (8), we arrive at an empirical relation,

$$[a - J(B_0 + B)d]/T = b, \tag{9}$$

a form that suggests a thermally activated process

Anderson has developed a theory (see following Letter[10]) in parallel with the present experiments in which the empirical relation (9) is accounted for by a process of "flux creep." Furthermore, his theory predicts that the persistent currents should decay linearly in $\log t$, where t is time. These changes in current should be directly reflected in changes in H', so we plotted H' vs $\log t$. Two typical plots are shown in Fig. 3. In these experiments, H was quickly changed by a small amount in such a direction as to move from a critical state condition slightly into the "normal region." The heat pulse which may arise from the induced normal current is quickly carried away, and we then follow the return of H' to a new critical state. The external field H is held constant (to about one part in 10^4) during the runs, one of which was continued for three hours. Dozens of such runs, starting at various points on the critical state curve, indicate that the change in H' is always proportional to $\log t$. At lower temperatures we generally measure slower decay rates, as expected from Anderson's theory.[10] At 4.2°K, the rate predicted by Eq. (13) of Anderson's Letter has been qualitatively verified. Quantitative aspects of the dependence of the decay rate upon temperature are currently being investigated. These measurements indicate that there is no precise "critical state," but the magnetization curve $H'(H)$ represents only quasi-equilibrium states.

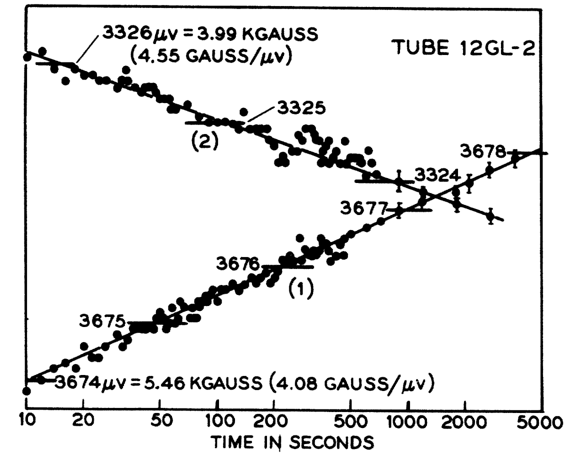

FIG. 3. Decay rate measurements of "persistent" currents induced in a 3Nb-Zr tube. Digital voltmeter readings of H' probe are displayed as a function of $\log t$. The vertical flags on data points indicate the ranges of H' for 20 consecutive (2-sec interval) readings. The data in (1) were taken at a point on the shielding portion of the magnetization curve (see Fig. 1), and (2) on the trapping portion.

The fastest decay rate yet observed is $dH'/d(\log t) = 10$ gauss per decade, from an initial state of $M = H' - H = 1000$ gauss. If this rate of decay continues indefinitely, we estimate that the persistent current in this HSC sample will die out after 3×10^{92} years. In any practical sense then, the persistent current is persistent. But the result is significant in that no theory has been able to explain conclusively a truly persistent current.[11] Previous attempts[12-18] to search for this decay gave negative results. However, the previous measurements with one exception[18] were made with soft superconductors, in which the mechanism governing the persistent currents may be entirely different from that in HSC. Nevertheless, the fact that we can measure a finite decay rate only at the very beginning of the critical state may be pertinent to this question. If the decay rate is measured, for example, at the 90% level of critical current, seven years will be required to detect a decay of one part in 10^4.

We are indebted to Miss P. Watson for her collaboration in the experiments and to R. Epworth and J. Hasiak for their technical assistance. We are grateful to P. W. Anderson and T. H. Geballe for helpful discussions throughout the course of the present experiments.

*On leave of absence from the University of Washington, Seattle, Washington.

[1]Y. B. Kim, C. F. Hempstead, and A. R. Strnad (to be published).

[2]K. Mendelssohn, Proc. Roy. Soc. (London) A152, 34 (1935).

[3]D. Shoenberg, Superconductivity (Cambridge University Press, New York, 1960).

[4]J. E. Kunzler, Revs. Modern Phys. 33, 1 (1961).

[5]J. J. Hauser and H. C. Theuerer (to be published).

[6]A. A. Abrikosov, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1442 (1957) [translation: Soviet Phys. – JETP 5, 1174 (1957)].

[7]B. B. Goodman, Phys. Rev. Letters 6, 597 (1961).

[8]C. P. Bean, Phys. Rev. Letters 8, 250 (1962).

[9]Transition temperatures of the samples used in the present experiments were measured by G. W. Hull, Bell Telephone Laboratories, Murray Hill, New Jersey.

[10]P. W. Anderson, following Letter [Phys. Rev. Letters 9, 309 (1962)].

[11]M. R. Schafroth, in Solid State Physics, edited by F. Seitz and D. Turnbull (Academic Press, Inc., New York, 1960), Vol. 10, p. 293.

[12]H. K. Onnes, Leiden Comm. 140b, 141b (1914).

[13]V. P. Grassman, Physik. Z. 37, 569 (1936).

[14]Collins, quoted by J. W. Crowe, IBM J. Research Develop. 1, 294 (1957).

[15]R. T. Broom, Nature 190, 992 (1961).

[16]D. J. Quinn, III, and W. B. Ittner, III, J. Appl. Phys. 33, 748 (1962).

[17]J. E. Mercereau and T. K. Hunt, Phys. Rev. Letters 8, 243 (1962).

[18]C. F. Hempstead and Y. B. Kim, Bull. Am. Phys. Soc. 7, 309 (1962). An upper limit of resistivity of 10^{-16} ohm-cm in Nb_3Sn quoted in this abstract was later improved to 2×10^{-19} ohm-cm for Nb-Zr.

THEORY OF FLUX CREEP IN HARD SUPERCONDUCTORS

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received September 12, 1962)

A major difficulty in understanding hard superconductors has been the appreciable temperature dependence of critical currents and fields at temperatures as low as 0.1 T_c. None of the properties of the bulk superconducting state vary noticeably at this temperature.

Bean[1] has introduced the notion of the "critical state" of the hard superconductor, and Kim et al.[2,3] have shown that it may be defined, in their experiments, in terms of constants α and B_0:

$$\alpha(T) = J_{\mathrm{cr}}(B_{\mathrm{cr}} + B_0). \qquad (1)$$

If the current density J or the field B is increased beyond the critical values in (1), the process we shall call "flux creep" sets in, and flux leaks through the material and returns it to the critical state (or, occasionally, complete breakdown occurs). $\alpha(T)$ is a structure-sensitive constant of the material, and a typical temperature dependence is shown in Fig. 1.

We shall show that this behavior of α can be explained by assuming that the mechanism of flux creep is thermally activated motion of bundles of flux lines, aided by the Lorentz force $\vec{J}\times\vec{B}$, over free energy barriers coming from the pinning effect of inhomogeneities, strains, dislocations, or other physical defects. This theory also explains the constant B_0, and predicts time relaxation behavior strikingly similar to those of magnetic aftereffect and some forms of plastic creep, which are explained in a very similar way.[4] This behavior has been verified by Kim et al.[3]

First, we must define, if somewhat imperfectly,

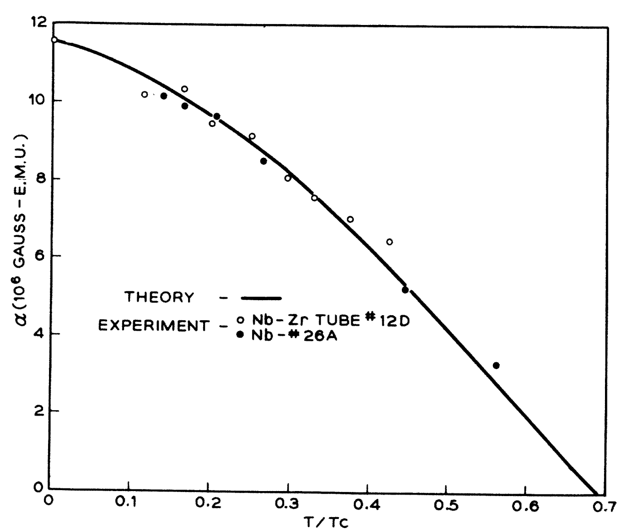

FIG. 1. The critical state constant α as a function of T/T_c. The line is a theoretical fit; the points are experimental ones for two different tubes.