ACOUSTO-ELECTRIC EXPLANATION OF NON-OHMIC BEHAVIOR IN PIEZOELECTRIC SEMICONDUCTORS AND BISMUTH

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Figure 1 shows the current-voltage characteristics observed by Smith[1] and McFee[2] in piezoelectric semiconductors, and by Esaki[3] in bismuth in the presence of a strong magnetic field. We propose that both types of departure from Ohm's law may be simply ascribed to the acousto-electric current[4] accompanying a large acoustic flux that is produced by a traveling-wave amplification process. We also propose that this acoustic flux is limited in steady state by a nonlinear loss mechanism arising from the traveling-wave interaction with the carriers.

Traveling-wave acoustic amplification has been observed in piezoelectric semiconductors[5] and has been postulated in bismuth,[6-8] and the departures from Ohm's law have been ascribed to it.[1,7] However, a model of sufficient detail for a physical understanding of the non-Ohmic effect has not yet been presented.

To obtain the current-voltage characteristic, we require the steady-state acoustic flux as a function of the applied fields, and then an expression for the acousto-electric current arising from the acoustic flux and applied fields. We shall consider first a piezoelectric semiconductor with an applied electric field E_d, and shall make use of a one-dimensional simplification considering only currents and acoustic traveling waves in the x direction. In the previous analysis of acoustic gain and loss,[5,9] current density (for extrinsic n-type material) was written as

$$J = \mu q(n+n_s)(E+E_d) + q\mathfrak{D}\partial n_s/\partial x, \qquad (1)$$

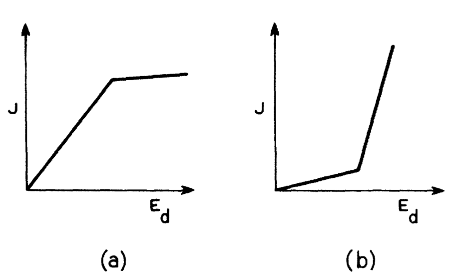

FIG. 1. (a) Current vs drift-field characteristic observed for an extrinsic piezoelectric semiconductor such as CdS or ZnO. The kink occurs when $\mu E_d \geq$ sound velocity. (b) Current-field characteristic observed in bismuth when $\omega_{cyc}\tau \gg 1$. The kink occurs when the drift velocity of carriers in the direction perpendicular to the E_d and B fields exceeds the sound velocity.

where μ is the mobility, q is the electronic charge, n is the average carrier concentration, $\mathfrak{D}$ is the carrier diffusion constant, E is the wave-periodic longitudinal electric field accompanying a traveling wave due to the piezoelectric effect, and n_s is the wave-periodic amplitude of carrier concentration arising from carrier bunching by the wave. A theory for acoustic loss or gain was developed by using Poisson's equation and the equation of continuity to obtain the displacement, D, in terms of E from (1); then substituting this in the equation $D=\epsilon E+eS$ (piezoelectric constant e and strain S) to obtain $E(S)$. This self-consistent electric field was then substituted in $T=cS-eE$ (T = stress) to obtain a complex elastic stiffness constant, c^*, describing the anomalous dispersion of the acoustic wave. The resulting linear (small-signal) theory, in which the terms in the product n_sE are neglected, yields the time constant for decay (or growth) of a wave:

$$2\alpha v = Q\omega = K^2 \frac{(\omega_c/\gamma)}{1+(\omega_c/\gamma\omega)^2(1+\omega^2/\omega_c\omega_D)^2}, \qquad (2)$$

where K = electromechanical coupling constant, $\omega_c=\sigma/\epsilon$, $\omega_D=v^2/\mathfrak{D}$, v = sound velocity, σ = conductivity, and $\gamma = 1-v_d/v$ where $v_d=\mu E_d$ = the Ohmic drift velocity of the carriers. Equation (2) is plotted as a function of ω in Fig. 2. Note that there is a maximum build-up at $\omega^2=\omega_c\omega_D$ and that the gain decreases at higher frequencies due to carrier diffusion. Also shown on Fig. 2 is a linear loss mechanism proportional to ω^2 which is of the Akhiezer[10] type and which has been (very roughly) estimated from the work of Bömmel and Dransfeld[11] on quartz. A build-up of acoustic flux through the application of E_d is then to be expected over a band of frequencies whose upper limit is determined by the intersection of the linear loss and gain curves of Fig. 2.

Steady state for the acoustic flux under amplifying conditions occurs when the amplitudes of the amplified waves are sufficiently great to bring a nonlinear loss mechanism into equality with the gain. (Linear loss mechanisms such as isotope or impurity scattering or the Akhiezer relaxation mechanism clearly cannot limit the acoustic flux.) The loss mechanism which then occurs is one in which interference between a pair of waves of the group which is being amplified produces a third

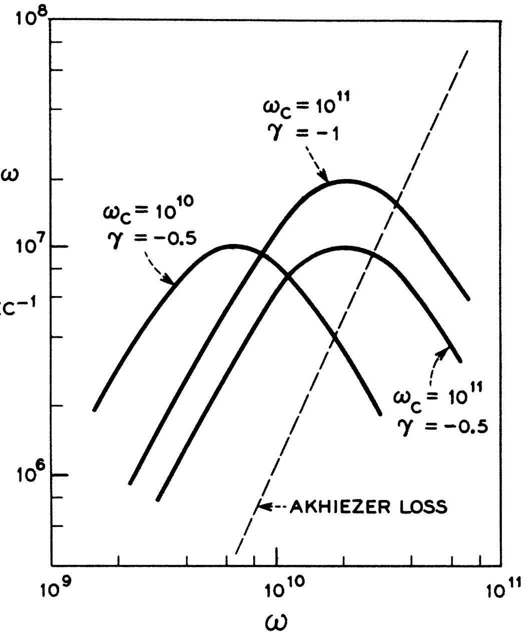

FIG. 2. Three plots of Eq. (2) for piezoelectrically active shear waves in n-type CdS under conditions of acoustic gain (Q negative). An ω^2 loss term (Q positive) of Akhiezer type is also shown. For $\omega_c = 10^{10}$, $\sigma \cong 0.01$ Ω^{-1} cm^{-1}; for $\omega_c = 10^{11}$, $\sigma \cong 0.1$ Ω^{-1} cm^{-1}.

wave at the sum frequency such that the sum frequency is higher than the highest frequency for which linear gain exceeds linear loss (see Fig. 2). Loss from the wave of the sum frequency may then occur by the Akhiezer mechanism whence crystal momentum is lost through Umklapp processes, and acoustic energy is dissipated as heat. This pairwise interference to produce a sum frequency is the familiar three-phonon collision, and the rate of this process may be obtained, using perturbation theory, from the square of the term in the Hamiltonian involving the amplitudes of the three waves and the density (in energy) of the final states which satisfy $\omega_l + \omega_m = \omega_n$ and $k_l + k_m = k_n$, simultaneously. We propose that the nonlinear (n_sE) terms in the amplification theory make the major contribution to the cubic term in the Hamiltonian. In some cases, the density of final states may be enhanced for three waves all of the same branch, because of the anomalous dispersion (ω increasing slightly faster than linearly with k) accompanying the traveling-wave interaction.

The cubic term in the Hamiltonian may be obtained from the energy density term $\int \vec{E} \cdot \vec{D} dx$. From Poisson's equation, the continuity equation, and Eq. (1), we find

$$\frac{\partial^2 D}{\partial x \partial t} = \frac{\partial J}{\partial x} = \mu q \left(n_s \frac{\partial E}{\partial x} + E \frac{\partial n_s}{\partial x} \right) + \mu q n \frac{\partial E}{\partial x} + q \mathfrak{D} \frac{\partial^2 n_s}{\partial x^2}, \tag{3}$$

and since

$$n_s = \sum_i n_i \cos(k_i x - \omega_i t + \varphi_{in}),$$

and

$$E = \sum_i E_i \cos(k_i x - \omega_i t + \varphi_{iE}), \tag{4}$$

the first term on the right of Eq. (3) yields components of D at the sum and difference frequencies proportional to the product of two wave amplitudes which, when multiplied by E and integrated over volume, yields an energy density proportional to the product of three wave amplitudes.

The dc acousto-electric current accompanying the steady-state acoustic flux arises simply from the n_sE term in Eq. (1). Using Eq. (4), it is

$$J_a = \mu q \sum_i n_i \cdot E_i \cos\varphi_i, \tag{5}$$

where φ_i is the phase angle between the bunched carriers and the self-consistent electric field accompanying the ith wave. Using Eq. (5), J_a can be computed from the linear amplification theory. The physically revealing result is that in the acousto-electric component of the current, the carriers flow with the wave when it is being attenuated and against the wave when it is being amplified. Hence, when E_d exceeds the threshold required for amplification, the acousto-electric current which accompanies waves that are being built up, subtracts from the Ohmic current, $J_0 = \mu q n E_d$ [as illustrated in Fig. 3(a)], and causes the "kink" in Fig. 1(a).

In bismuth, at low temperatures and with a large applied magnetic field, elastic-wave amplification commences when the component of drift velocity

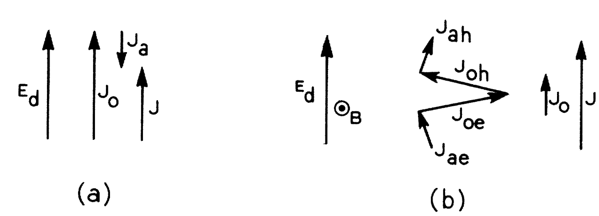

FIG. 3. (a) Vector diagram of Ohmic (o) and acousto-electric (a) currents for an extrinsic piezoelectric semiconductor with $E_d > E_{\text{kink}}$. (b) Vector diagram of Ohmic and acousto-electric currents for electrons (e) and holes (h) in bismuth for $E_d > E_{\text{kink}}$. Equal Hall angles are assumed for electrons and holes.

of electrons and holes in the direction perpendicular to $\vec{E}$ and $\vec{B}$ exceeds the sound velocity.[7,8] Space-charge-free bunching of electrons and holes results from the difference of electron and hole deformation potentials associated with the acoustic strain. As in the semiconductor case, the gain falls off at high frequencies due to carrier diffusion, and there exists a maximum frequency above which linear loss exceeds linear gain. (The important linear loss mechanism for bismuth at low temperature may be mode conversion at the sample surfaces.) Nonlinear losses which yield a steady-state acoustic flux arise in analogous fashion to those in the semiconductor case, i.e., the space-charge bunches of one frequency are acted upon by the deformation potential gradients of a different frequency converting acoustic energy to the sum frequency from which it is quickly dissipated. The acousto-electric currents of electrons and holes are driven by their respective wave-periodic deformation potential gradients acting on the carrier bunches of corresponding wave vector. As in the semiconductor case, under amplifying conditions, the electrons and holes are driven in the direction opposite to the acoustic flux, but they are also acted upon by the magnetic field so that their acousto-electric drift velocities are inclined to the negative acoustic flux direction by their Hall angles. Figure 3(b) gives a vector diagram showing the Ohmic and acousto-electric currents for $E_d > E_{\text{kink}}$. The acousto-electric current can be seen to add to the Ohmic current in agreement with Fig. 1(b).

This model differs from those of Esaki[3] and Miyake and Kubo,[12] in that it does not assume any additional scattering of carriers by phonons of the amplified part of the acoustic branch, because the acoustic wavelengths are long compared with the deBroglie wavelengths of the carriers. The apparent resistance of a sample for $E_d > E_{\text{kink}}$ will depend upon the time of measurement—for times short compared with the build-up time for acoustic flux the resistance will be changed from its small-field value only by field-induced changes of the electron-phonon scattering.[3,12]

It seems probable that the current oscillation observed by Esaki,[3] Smith,[1] and McFee[2] whose period is that of a round trip for a sound wave between the ends of the sample is a relaxation oscillation of the gain mechanism brought about by the acousto-electric current accompanying the acoustic flux reflected from the sample boundary.

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