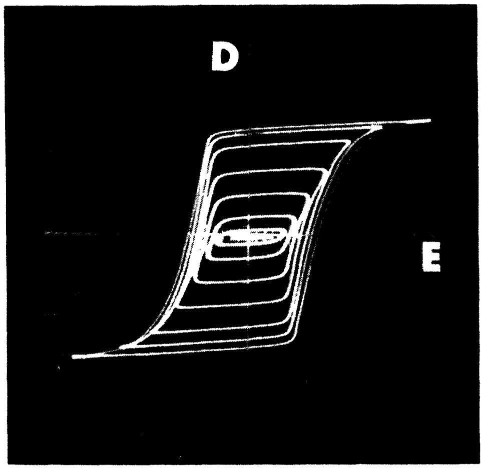

FIG. 2. D vs E characteristic of KNO_3 as a function of driving voltage.

but a hysteresis loop has been observed at frequencies up to 20 kc/sec which is currently the limit of our instrumentation.

A great deal of work still has to be done on understanding this effect. For example, it has been reported[3] that phase III can exist stably at room temperature but only under high pressure, whereas our samples were essentially prepared in a stress-free state. Therefore, our work is now directed toward understanding the variables affecting the phase transformations, especially III to II, in potassium nitrate.

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MAGNON RENORMALIZATION IN FERROMAGNETS NEAR THE CURIE POINT*

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As pointed out by Kittel,[1] Dyson's results[2] extrapolated to high temperatures suggest that magnon-magnon interactions in ferromagnets may be relatively weak even at the Curie temperature: The mean free path of a magnon of energy k_BT_C in an assembly of magnons in thermal equilibrium at T_C is about 50 lattice constants in a simple cubic lattice for spin $\frac{1}{2}$. It appears not entirely unreasonable to explore the possibility of treating a ferromagnet as a gas of weakly interacting magnons, up to the Curie point. A quasi-particle model of magnons has been studied with the exchange Hamiltonian limited in its expansion to terms of fourth order in magnon variables. The energy of the quasi-magnons is derived as a solution of an implicit equation. The remarkable result is found that no solution of the implicit equation exists above a maximum temperature which is within several percent of the Curie temperature calculated[3] by the methods of Bethe-Peierls-Weiss (BPW) and Kramers-Opechowski (KO). The temperature dependence of the renormalized magnon energy and of the magnetization is given from absolute zero to t_{max}. To the order considered, all calculations are carried out exactly: Full account is taken of the finite extent of the Brillouin zone, in the dispersion relation and in the limits of integration.

The diagonal part of the exchange Hamiltonian is

$$H=\sum_{\vec{k}} n_{\vec{k}}\epsilon_{\vec{k}} - JN^{-1}\sum_{\vec{k},\vec{k}'} n_{\vec{k}}n_{\vec{k}'}(\gamma_0+\gamma_{\vec{k}-\vec{k}'}-\gamma_{\vec{k}}-\gamma_{\vec{k}'}), \tag{1}$$

where

$$\epsilon_{\vec{k}}=2JS(\gamma_0-\gamma_{\vec{k}}), \qquad \gamma_{\vec{k}}=\sum_{\vec{\delta}}\exp(i\vec{k}\cdot\vec{\delta}); \tag{2}$$

here $\vec{\delta}$ denotes the vectors to the nearest neighbors. For a simple cubic lattice, H can be rewritten exactly as

$$H=\sum_{\vec{k}} n_{\vec{k}}\epsilon_{\vec{k}} - (24JNS^2)^{-1}\sum_{\vec{k},\vec{k}'} n_{\vec{k}}n_{\vec{k}'}\epsilon_{\vec{k}}\epsilon_{\vec{k}'}. \tag{3}$$

The free energy is

$$F=U-TS=\sum_{\vec{k}}\langle n_{\vec{k}}\rangle\epsilon_{\vec{k}} - (24JNS^2)^{-1}\sum_{\vec{k},\vec{k}'}\langle n_{\vec{k}}\rangle\langle n_{\vec{k}'}\rangle\epsilon_{\vec{k}}\epsilon_{\vec{k}'} + k_BT\sum_{\vec{k}}[\langle n_{\vec{k}}\rangle\ln\langle n_{\vec{k}}\rangle - (\langle n_{\vec{k}}\rangle+1)\ln(\langle n_{\vec{k}}\rangle+1)], \tag{4}$$

using the standard result for the entropy of a boson gas. The free energy is an extremum with respect to the occupancy $\langle n_{\vec{k}}\rangle$ if

$$\partial F/\partial\langle n_{\vec{k}}\rangle = 0 = \epsilon_{\vec{k}}[1-(12JNS^2)^{-1}\textstyle\sum_{\vec{k}'}\langle n_{\vec{k}'}\rangle\epsilon_{\vec{k}'}] + k_B T \ln[\langle n_{\vec{k}}\rangle/(\langle n_{\vec{k}}\rangle+1)]. \qquad (5)$$

This equation is satisfied if

$$\langle n_{\vec{k}}\rangle = \{\exp[\bar{\epsilon}_{\vec{k}}(T)/k_B T]-1\}^{-1}, \qquad (6)$$

where

$$\bar{\epsilon}_{\vec{k}}(T) = \epsilon_{\vec{k}}[1-(12JNS^2)^{-1}\textstyle\sum_{\vec{k}'}\langle n_{\vec{k}'}\rangle\epsilon_{\vec{k}'}] \qquad (7)$$

is the quasi-magnon energy. This result had been derived previously, but only in the long-wavelength approximation, by Keffer and Loudon,[4] and it also follows from Brout and Englert's[5] treatment of a ferromagnet. Combination of Eqs. (6) and (7) leads to an implicit equation for the ratio,

$$\alpha(T) = \bar{\epsilon}_{\vec{k}}(T)/\epsilon_{\vec{k}}. \qquad (8)$$

Here $\alpha(T)$ is independent of $\vec{k}$; this result appears to be a special property of the simple cubic lattice. We expect a k-dependent relation for other lattices and higher orders in the interaction. The implicit equation was solved with the aid of a computer, IBM 704. Results are shown as a function of the reduced temperature $t = 3k_B T/2\pi JS$ in Figs. 1 and 2 for spin $\frac{1}{2}$ and 1. In the range of temperature 0°K$-t_{max}$, two roots are found for α, of which the larger corresponds in the low-temperature limit to the standard results of the spin-wave theory.[2] The maximum temperature t_{max} at which a solution for α can be found is within 4 to 7.5 percent of the KO Curie temperature for $S=\frac{1}{2}$ and within 0.7 percent for $S=1$. The reduced magnetizations $m_\alpha(t)$ and $m_1(t)$ are also shown. The value of $m_\alpha(t)$ follows immediately from the knowledge of $\alpha(t)$, and $m_1(t)$ is the reduced magnetization for noninteracting magnons with full account taken of the effects of the Brillouin zone's boundaries. For $S=\frac{1}{2}$, $m_1(t)$ is zero at 1.84 times the KO Curie temperature, whereas $m_\alpha(t)$ has dropped at t_{max} to about 20 percent of its value at absolute zero. If $U(t)$ is the internal energy of the spin system referred to complete disorder, the ratio $U(t)/U(0)$ which measures the amount of exchange energy in the system at temperature t is shown to be equal to $[\alpha(t)]^2$, and has approximately the value 0.45 for spin $\frac{1}{2}$ and 0.40 for spin 1 at t_{max}. The model obviously has to be improved in the neighborhood of the Curie point. We are extending the present method to higher orders, to get a better understanding of the state of the ferromagnet and the nature of the transition.

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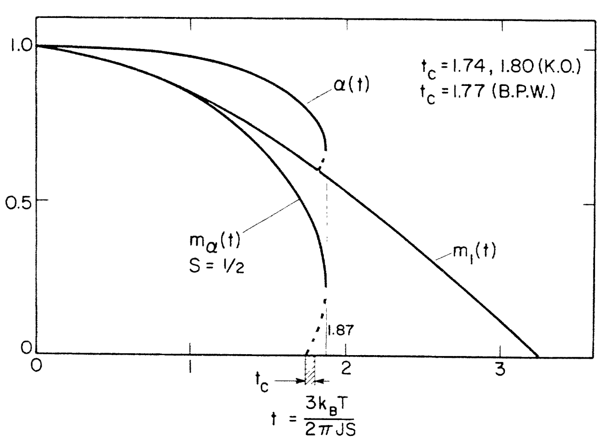

FIG. 1. Temperature dependence for spin $\frac{1}{2}$ of the ratio $\alpha = \bar{\epsilon}_{\vec{k}}(T)/\epsilon_{\vec{k}}$ and of the reduced magnetization m_α for $\alpha = 1$ and $\alpha(T)$.

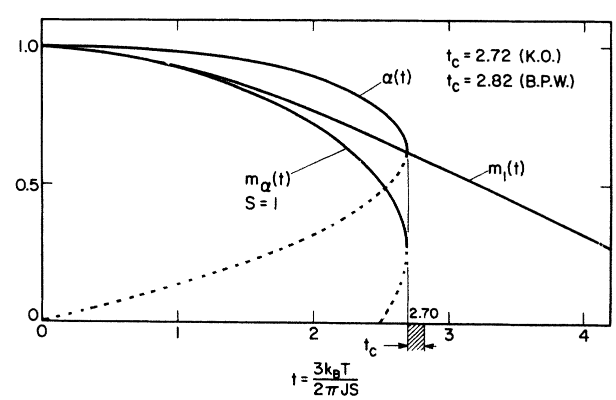

FIG. 2. Temperature dependence for spin 1 of the ratio $\alpha = \bar{\epsilon}_{\vec{k}}(T)/\epsilon_{\vec{k}}$ and of the reduced magnetization m_α for $\alpha = 1$ and $\alpha(T)$.