because of the steep rise of the two-body potential.[8] The hard core of the Sutherland model provides an automatic cutoff for w_{123} when $r_{ij} < \sigma$.

By expanding the exponentials in Eq. (2) the integral can be evaluated analytically for the Sutherland model. The first two terms of the expansion of $\exp[-u(r)/kT]$ when $r > \sigma$, i.e., $1 + (\mu/kT)r^{-6}$, give a better approximation to the exponential expression for the (12, 6) potential than the unexpanded formula. The correction to the third virial coefficient, obtained analytically in "corresponding states" form, is

$$\Delta C^* = (15\alpha^*/4T^*)[1 + (2.1067 - 0.0693\alpha^*)/T^*], \quad (3)$$

where C^* is the reduced value of C, T^* the reduced temperature, and α^* the reduced polarizability. We set $\mu = 4\epsilon\sigma^6$ and $b_0 = 2\pi N\sigma^3/3$. Then $C^* = C/b_0^2$, $T^* = kT/\epsilon$, and $\alpha^* = \alpha/\sigma^3 \approx 0.05$ for most noble gases. The Lennard-Jones parameters σ and ϵ can be approximately identified with the Sutherland parameters σ and ϵ as defined. When adjusted to fit $B(T)$ data, they have nearly the same numerical values.

Figure 1 shows a plot of C^* vs T^* for the (12, 6) potential assuming additivity. The correction ΔC^*, obtained from Eq. (3), is also plotted and it is seen that the experimental points tend to agree with the corrected curve. Thus Eq. (3), which contains no adjustable constant, removes most of the disagreement at low temperatures. The error incurred in using the Sutherland model for ΔC^* is a small correction on a correction. We conclude that the third virial coefficients of the noble gases provide evidence for the predicted three-body dispersion forces.

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ENHANCED DIFFUSION IN AN rf DISCHARGE IN THE PRESENCE OF A MAGNETIC FIELD*

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In a great number of plasmas such as arcs, positive columns, PIG discharges, etc., it has been observed that the diffusion in a direction transverse to a static magnetic field B is anomalous in that it does not decrease monotonically[1-8] like B^{-2}. In plasmas of the type just mentioned, one generally assumes the existence of a static longitudinal electric field $\vec{E}$ and a direct current $\vec{I}$, both of which are parallel to B; however, in other plasmas the existence of the directed current $\vec{I}$ has not been demonstrated with certainty. Kadomtsev and Nedospasov[9] have recently proposed an explanation for anomalous diffusion which is based on the existence of a directed current $\vec{I}$. Our purpose is to show that anomalous diffusion seems also to occur in plasmas where a directed current $\vec{I}$ does not exist; namely, in an rf discharge.

Glass tubes of length ~60 cm and of diameter ϕ varying from 1.25 cm to 5 cm were placed in a solenoid of length $L \sim 40$ cm. The magnetic field was variable from zero to approximately 1000 gauss. The electrodes to which the rf voltage was applied were separated by approximately 40 cm, and consisted of strips of copper foil wrapped around the outside surface of the glass cylinder. A leak valve was used to vary the pressure and type of gas in the glass tube. The working pressure was in the range of 20 microns to 200 microns of Hg and was measured with a MacLeod gauge. Hydrogen and argon were the two gases utilized in the experiment. In Fig. 1, the experimental setup is presented.

The applied rf voltage is of the order of 200 volts peak to peak and is maintained constant. It is furnished by a 0.1-kW rf push-pull oscillator operating at 23 Mc/sec. The plasma density is

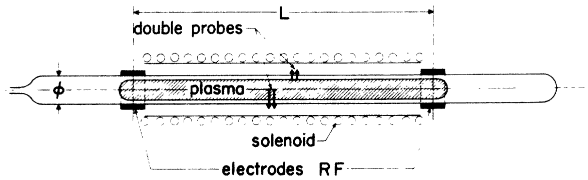

Fig. 1. Arrangement used to study diffusion in an rf discharge.

of the order of 10^8 cm^{-3} and the rf field penetrates the entire plasma.

The possibility of electron diffusion due to the short-circuit effect[2] was avoided, because not only was the ratio of the tube length to diameter, L/ϕ, generally large, but also the magnetic field lines intercepted the walls of the tube well outside the plasma.

Assuming an electron temperature between one and four electron volts,[10] σ_{eN}, the electron-neutral cross section, is found to vary from approximately 3×10^{-16} cm^2 to 20×10^{-16} cm^2. For gas pressures between 20 μ and 200 μ Hg, N_0, the neutral gas density number, is between approximately 7×10^{14} cm^{-3} and 7×10^{15} cm^{-3}. The electron-neutral collision frequency $\nu_{eN}=N_0\sigma_{eN}v^-$ lies between 1.8×10^7 sec^{-1} and 170×10^7 sec^{-1} and exceeds the rf frequency. Also, the electron-neutral mean free path $\lambda_{eN}=v^-/\nu_{eN}$ is small compared to the length of the discharge. Under these conditions, the discharge mechanism at equilibrium is rather well known and one is able to calculate the current carried by the electron cloud (see reference 10, p. 86),

$$j^-\simeq n^-e^2V_{\rm rf}\sin(\omega t+\varphi)/m^-L(\nu_{eN}^{\;2}+\omega^2)^{1/2},$$

where $\omega=2\pi\nu_{\rm rf}$ and $\tan\varphi=\omega/\nu_{eN}$. The oscillatory displacement of electrons about an average position is given by

$$x\simeq eV_{\rm rf}\cos(\omega t+\varphi)/m^-\omega L(\nu_{eN}^{\;2}+\omega^2)^{1/2}.$$

In our experiment x is always less than 1 cm. As a result, the possibility of an electron current existing in one single direction along the axis of the tube is eliminated. A radial electric field is the only obvious directed macroscopic electric field which exists inside the solenoid. (However, this does not exclude the possibility that the azimuthal electric field may have local components.) Therefore, this field is essentially localized in the sheath, between the wall and the plasma. The nonluminous portion of the sheath has a thickness of a few millimeters and is of the same order of magnitude as a Debye length. The Larmor radius of the electrons is always smaller than the dimensions of this sheath (except for very weak values of the magnetic field B) which is not necessarily the case for the ion Larmor radius.

It has been observed that while increasing B, one passes through a critical value of magnetic field B_c above which the flux of ions across the magnetic field no longer decreases but actually increases for a while. Since the rf production of ions can hardly be affected by a magnetic field oriented along the rf electric field (which has good penetration), one would conclude that diffusion must be enhanced by some unknown mechanism. The onset of this anomalous rise in diffusion rate is generally accompanied by the commencement of rf noise of unknown origin. The value of magnetic field at which this noise is first observed corresponds roughly to the value of B_c. It has likewise been observed that the product of the critical magnetic field and the diameter of the discharge tube is roughly constant, all other things being equal. Finally, the value of B_c increases nonlinearly with pressure.

The variation of the perpendicular ion flux in our experiment was measured by a method similar to that used by Bonnal et al.[11] In this method the ion densities in the interior of the plasma and at the outside edge of the plasma were determined by measuring the ion saturation currents with two sets of double probes. Each set of double probes consisted of a flat probe of area ~9 mm^2, the plane of which was parallel to B, and a very narrow cylindrical probe which was separated from the flat probe by a distance of ~1 mm. The flat probe was biased negatively with respect to the cylindrical probe in such a way that we were operating on the horizontal branch of the double probe current-voltage characteristic. The entire double probe assembly floated in the plasma.

Assuming that the ion temperature is independent of B for the range of B considered, the ion saturation currents measured by the external and internal double probes, $j_{\rm ext}^+$ and $j_{\rm int}^+$, are proportional to the corresponding densities $n_{\rm ext}^+$ and $n_{\rm int}^+$. Therefore, the variation of the ratio $j_{\rm ext}^+/j_{\rm int}^+$ as a function of magnetic field strength gives one an idea of the variation of the perpendicular ion flux and, consequently, allows one to estimate the critical field B_c. We note the position of the exterior probe ought to be within ±1 mm of the boundary between the luminous plasma and the nonluminous sheath. Close to the value of B_c, the onset of an rf noise was detected by a floating probe in the plasma which was connected to the differential input of an oscilloscope. This provided another method of determining B_c. It has been observed by Schlüter[12] that the loading of an rf oscillator used to produce a plasma varies with B, and in particular that the grid voltage of the oscillator reaches an extremum for a certain value of B. However, he did not establish with certainty a correspondence between this value of B and the value B_c corresponding to the onset of enhanced diffusion. We have utilized this technique and have observed that this extremum V_g corresponds to nearly the same value of magnetic field B_c at which the rf noise commences. Thus, the monitoring of the grid voltage of the rf oscillator as B is varied is a very practical way of determining B_c without introducing into the plasma a pickup probe which sometimes perturbs the plasma. In addition, this proves that the unknown mechanism producing the rf noise manifests itself as a very sensitive loading of the rf oscillator.

In Fig. 2 an example is given which illustrates how j_{ext}^+, j_{int}^+, j_{ext}^+/j_{int}^+, and V_g vary with B. We have also indicated there the values of B corresponding to the onset of rf noise. The crosses indicate the range of magnetic field strength corresponding to the critical magnetic field B_c as determined by the three diagnostic methods.

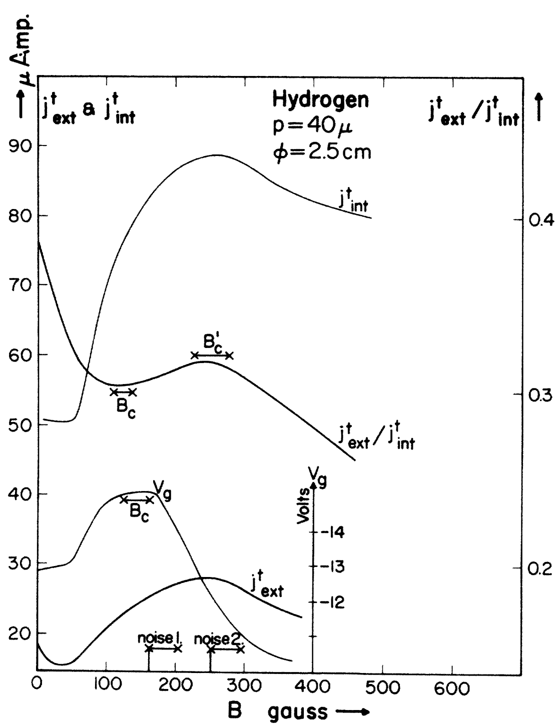

Fig. 2. Plot of j_{ext}^+, j_{int}^+, j_{ext}^+/j_{int}^+, and V_g vs magnetic Field.

In general, these values are reproducible to within ±10%. Under certain experimental conditions V_g varies in an incoherent fashion, and hence B_c cannot be deduced. This often happens when the probes draw a large ion current which represents an additional loading of the oscillator. In general the onset of the noise is very clearly defined at low pressures, but its amplitude diminishes as the pressure increases. A second region of critical magnetic field strengths was also observed and is denoted by B_c' in Fig. 2. The noise level corresponding to B_c' was found to be larger than that corresponding to B_c, and when B_c and B_c' are very close, it is difficult to determine the precise value of a magnetic field strength corresponding

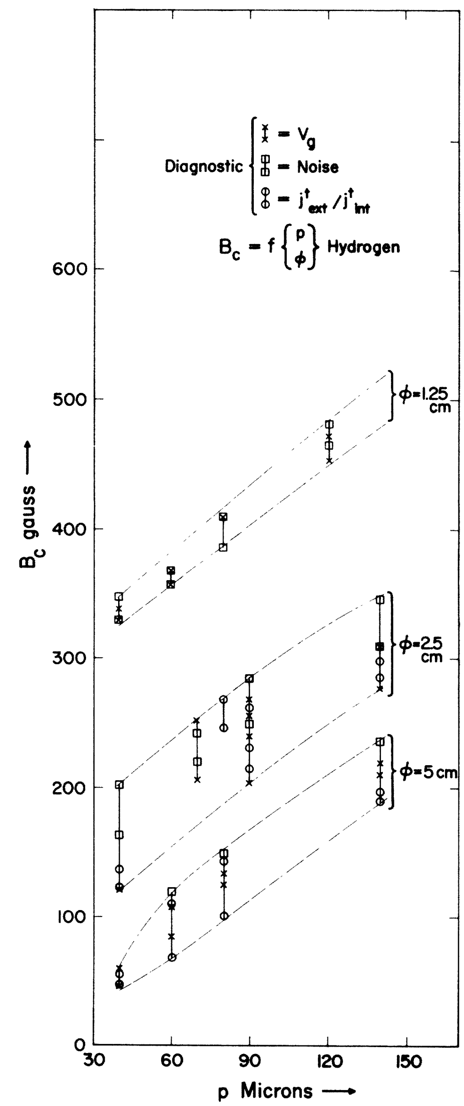

Fig. 3. Plot of the critical magnetic field, B_c, in hydrogen vs pressure for various discharge tube diameters.

to the onset of the noise. In Figs. 3 and 4, the value of the critical magnetic field B_c in hydrogen and argon as determined by the three diagnostic techniques are plotted for different pressures and different diameters of the discharge tube.

We note that in spite of the dispersion of experimental results, the product $B_c\phi$ is almost constant, a fact which has intrigued many researchers dealing with enhanced diffusion. The variation of experimental results may be due to several reasons. First we note that in Figs. 3 and 4, ϕ represents the diameter of the glass tube; the diameter of the discharge is clearly smaller, especially for the case of ϕ = 1.25 cm since the sheath occupies a fairly large portion of the tube cross section, and the sheaths formed around the double probes are such that they hinder the discharge created between the two rf electrodes. Therefore, the probes are an important perturbing factor. Note also that the electron temperature could not be controlled in our experiment, and we kept only the rf field constant. A detailed study taking into account this parameter should be undertaken. Perhaps the theory of Hoh[13] will then be applicable. Next we note that the noise picked up by the floating probe is a "grass" type of noise and implies microinstabilities rather than an organized orderly movement of a magnetohydrodynamic nature.[1,11] Finally, let us also mention that a Russian team[14] has announced the observation of an unexpected variation of the potential difference between the axis and the wall of an rf discharge tube when B was varied, and that their value of B_c in argon is quite compatible with our results.

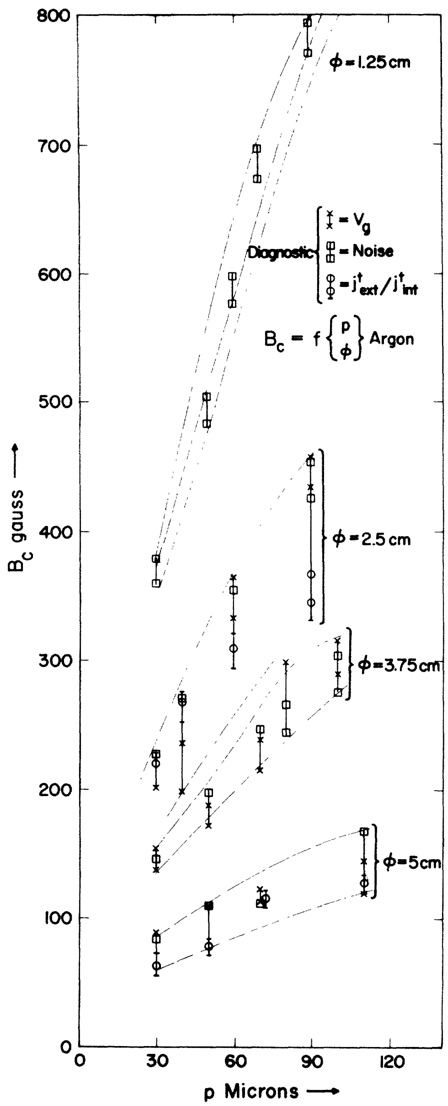

Fig. 4. Plot of the critical magnetic field, B_c, in argon vs pressure for various discharge tube diameters.

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