sistent.

It is remarkable that if the above crude formulas for f_{11} are used in the low-energy physical region, we have

$$f_{11}(\omega) \approx \frac{\frac{16}{9}\gamma_{33}}{\omega_{33}+\omega}\left(-\frac{\omega_{33}}{\omega}\right)+4C_\rho \frac{1}{4k^2}\ln\left(1+\frac{4k^2}{m_\rho^2}\right) \text{ for } \omega \lesssim 2, \tag{14}$$

dropping the small contribution from nucleon exchange. Thus the prescription of merely adding the ρ term, as proposed by Bowcock, Cottingham, and Lurié,[6] seems roughly correct even when the nucleon is treated as a bound state. However, if one were to include in a better approximation to (4) terms odd in ω, or to improve the approximation (12), the ρ effects would not be simply additive. Formula (14) with $\gamma_{33} = 0.11$ and $C_\rho = 0.4$ predicts that δ_{11} should start off negative at threshold and change sign at $\omega \approx 2$, a behavior not in disagreement with experimental knowledge, as discussed in reference 6.

It is reasonable to hope that a relativistic version of the bootstrap calculation outlined here, taking due account of the Regge asymptotic behavior,[8] will not require cutoffs and will yield rough values for the nucleon and (3,3) masses as well as absolute values for λ_{11} and λ_{33}. If this goal can be reached, the most striking characteristic of strong-interaction theory will have been demonstrated: It would then be almost certain, even without a detailed treatment of the strange particles, that no arbitrary parameters can be tolerated.

---

*This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1]G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 7, 394 (1961) and 8, 41 (1962), and Phys. Rev. 124, 264 (1961); see also G. F. Chew, The S-Matrix Theory of Strong Interactions (W. A. Benjamin and Co., New York, 1961).

[2]G. F. Chew and S. Mandelstam, Nuovo cimento 19, 752 (1961); F. Zachariasen, Phys. Rev. Letters 7, 112 (1961); F. Zachariasen and C. Zemach (to be published); L. A. P. Balázs, Lawrence Radiation Laboratory Report UCRL-10157, March, 1962 (to be published).

[3]G. F. Chew and F. Low, Phys. Rev. 101, 1570 (1956).

[4]L. A. P. Balázs, Lawrence Radiation Laboratory Report UCRL-10026, January, 1962 (to be published).

[5]S. W. MacDowell, Phys. Rev. 116, 774 (1960); W. Frazer and J. Fulco, Phys. Rev. 119, 1420 (1960); S. Frautschi and D. Walecka, Phys. Rev. 120, 1486 (1960).

[6]J. Bowcock, W. N. Cottingham, and D. Lurié, Nuovo cimento 16, 918 (1960).

[7]S. Frautschi, Phys. Rev. Letters 5, 159 (1960).

[8]T. Regge, Nuovo cimento 14, 951 (1959) and 18, 947 (1960).

[9]G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).

ELECTROMAGNETIC RADIATIVE CORRECTIONS AND THE ELEMENTARY NATURE OF THE PHOTON*

Maurice Lévy†
Department of Physics, Stanford University, Stanford, California
(Received August 13, 1962)

In an interesting paper, Blankenbecler, Cook, and Goldberger[1] have recently raised the question as to whether the photon should be treated as a Regge pole. Their main motivation was tha the effective electromagnetic interaction should decrease at high energy; otherwise it would overtake the strong interactions if the latter are governed by Regge poles (including the so-called "vacuum" trajectory). They also noted that this Regge behavior of electromagnetic interactions might come either from the photon being a composite particle (in the technical sense of not being associated with a quantized wave field) or from the accumulation of many "elementary" photon effects. We wish to point out that the summation of a large number of emissions and absorptions of soft (or "infrared") photons leads, indeed, to a Regge behavior of electron scattering cross sections which is directly connected with the energy spectrum of positronium.

The electromagnetic radiative corrections at high energy were studied in detail by Yennie, Frautschi, and Suura,[2] who showed that they are dominated by the infrared real and virtual photons. They demonstrated in a general way the cancellation of all infrared divergences and proved that the contribution from soft photons can always be separated as an exponential factor multiplying the uncorrected elastic cross section. To be more precise, let us consider as an example the case of electron scattering by a potential (or, equivalently, by protons of infinite mass). The corrected cross section can be written in this case as

$$d\sigma/d\Omega = G(t)(E/\epsilon)^{2[\gamma(t)+\nu]}d\sigma_0/d\Omega, \qquad (1)$$

where $d\sigma_0/d\Omega$ is the usual elastic cross section. Here, t is the invariant square of the momentum transfer of the electron ($t<0$ in the physical region), E the energy of the incident electron, and ϵ the lowest energy of the photons which can be detected by the experiment. $\gamma(t)$ is defined by

$$\gamma(t) = -\nu + \frac{\alpha t}{\pi}\int_0^1 \frac{(1+x^2)dx}{4m^2 - t(1-x^2)}, \qquad (2)$$

where α is the fine-structure constant and m the electron mass. ν is a constant which we have formally added to the exponent in Eq. (1) and subtracted from $\gamma(t)$; its meaning will become clear later. $\gamma(t)$ was first obtained by Schwinger[3] who also conjectured that, if $\delta(E,t)$ is the lowest order radiative correction to elastic scattering, $\exp\{-\delta(E,t)\}$ should be the corresponding sum to all orders. Equation (1) constitutes a partial proof, limited to the infrared part, of this conjecture. $G(t)$ is the product of several factors of which we can isolate at least two:

$$G(t) = \boldsymbol{F}(t)\exp[\rho(t)]G_1(t). \qquad (3)$$

F is the remaining contribution from the emission of real soft photons and has been given by Yennie et al.:

$$\boldsymbol{F} = \frac{1}{2\pi\epsilon}\int_{-\infty}^{+\infty} dz \exp(iz)\exp\left\{2[\gamma(t)+\nu] \times \int_0^1 \frac{du}{u}[1-\exp(-iuz)]\right\}. \qquad (4)$$

In principle, it can be evaluated in terms of known functions. The second term $\exp[\rho(t)]$ comes from the rest of the infrared exponential factor and is given by

$$\rho(t) = \frac{\alpha}{4\pi}\int_0^1 dx \ln\left[1 - \frac{t}{4m^2}(1-x^2)\right]. \qquad (5)$$

[$\rho(t)$ contains also small E-dependent terms of order m/E which we have neglected.] Finally, $G_1(t)$ is the sum of all the uncalculated radiative corrections which could also perhaps add up to an exponential ("magnetic" terms, vacuum polarization by the potential, etc.). One may ask if there are not other powers of E which could appear in Eq. (1) in the limit of large E and finite t, if higher-order corrections were included. {These terms would have to add up to a factor like $\exp[\phi(t)\ln(E/m)]$.} It is possible that the "magnetic" terms, for example, could give such a contribution, as we shall see later. It would correspond in any case to a higher-order correction in α to $\gamma(t)$. Also, it can be said that Eq. (1) becomes exact in the limit of high resolution (i.e., $\epsilon\to 0$).

We would like now to argue that Eq. (1) expresses the contribution to the electron-proton scattering channel ("t channel") of a Regge pole formed in the crossed "s channel," namely, electron-positron scattering (in other words, a Regge pole associated with positronium). This is first suggested by the fact that the same exponential factor appears in a various number of cross sections involving pairs of incoming or outgoing electrons or positrons, as has been shown by Yennie et al. However, the basic confirmation of the interpretation of $\gamma(t)$ as the "positronium trajectory" comes from an analysis of its behavior as a function of t. For $t<4m^2$, $\gamma(t)$ is real and is given by one of the following two expressions (we write $z=|t/4m^2|$): for $-\infty<t<0$ (physical region),

$$\gamma(t) = -\nu - (\alpha/\pi)\{(1+2z)[z(z+1)]^{-1/2} \times \ln[z^{1/2}+(z+1)^{1/2}] - 1\}; \qquad (6)$$

for $0<t<4m^2$ ("positronium region"),

$$\gamma(t) = -\nu - (\alpha/\pi)\{(1-2z)[z(1-z)]^{-1/2} \times \tan^{-1}[z/(1-z)]^{1/2} - 1\}. \qquad (7)$$

For $t\approx 0$, $\gamma(t)$ is continuous and is given by

$$\gamma(t) \approx -\nu + \alpha t/3\pi m^2. \qquad (8)$$

Note that $\gamma'(0)$ is much larger than the slope of the vacuum trajectory $\alpha_P'(0)$ which has been estimated[4] as $\simeq(50\,\mu_\pi^2)^{-1}$:

$$\gamma'(0)/\alpha_P'(0) \simeq [50/3\pi(137)](\mu_\pi/m)^2 \simeq 2.3\times 10^3. \qquad (9)$$

Above threshold, i.e., for $t>4m^2$, $\gamma(t)$ becomes complex:

$$\gamma(t) = -\nu - (\alpha/\pi)\{(2z-1)[z(z-1)]^{-1/2}\ln[z^{1/2}+(z-1)^{1/2}] - 1 - \tfrac{1}{2}i\pi(2z-1)\theta(z-1)/[z(z-1)]^{1/2}\}. \qquad (10)$$

Note that $\mathrm{Im}\gamma(t)$ is always positive, as should be expected.

It is now interesting to interpret the states corresponding to the values of t for which γ is an integer. These occur when t is smaller than, but fairly close to, $4m^2$. Putting $\gamma = l$ and using Eq. (7), we find, by expanding t in powers of α,

$$t^{1/2} \simeq 2m - \alpha^2 m/4(l+\nu)^2 + \tfrac{11}{64}\alpha^2 m/(l+\nu)^4 + \cdots . \quad (11)$$

This is evidently the spin-independent part of the positronium spectrum. It shows that the constant ν is related to the "radial" quantum number p:

$$\nu = p + 1. \quad (12)$$

We have a series of trajectories which are, however, all obtained by translating one of them along the γ axis. (Figure 1 represents Reγ and Imγ as functions of t for $\nu = 1$). It might be worth noting that the α^4 correction in Eq. (11) is the exact spin-independent correction[5] which can be calculated from the Breit equation. Presumably, if the contribution from the "magnetic" or spin-dependent terms to the radiative corrections were included in Eq. (1), each of the above trajectories would split into four, according to whether we are dealing with singlet or triplet states, or to whether the charge conjugation quantum number,[6]

$$c = (2S - 1)(-1)^{l+1}$$

(S is the spin and l the angular momentum), is even or odd. However, each of the trajectories would contain only states corresponding to values of the angular momentum changing by two units at a time (for example, the states ^1S, ^1D, ^1G, etc. would belong to the even singlet trajectory; the states ^3P, ^3F, etc. to the even triplet one, and so on). It is clear that it is the number c which determines here what is called the signature of the trajectory. We believe that the division between the "even" and "odd" contributions to the cross section is probably realized automatically by the function $G(t)$ in Eq. (1).

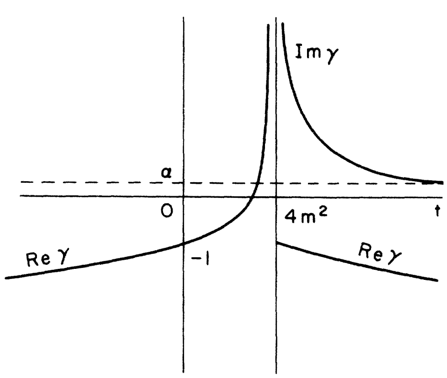

FIG. 1. Real and imaginary parts of the positronium trajectory $\gamma(t)$.

One may wonder why it is the infrared radiative corrections which determine the main part of the positronium trajectory. The answer probably is that the exponential in Eq. (1) represents mainly a vertex part containing the effect of the tail of the electron-positron interaction, i.e., the effect of the Coulomb potential. This also explains why the photon itself is not a member of the γ trajectory. Presumably, if the photon could be transformed into a Regge pole by the summation of many radiative effects, this would only appear in corrections of higher order[7] in α.

A last remark about $\gamma(t)$: One verifies directly on Eq. (2) that, as could be expected, it is analytic in the finite complex plane, except for a cut on the real axis for $4m^2 < t < +\infty$. It has, however, a logarithmic singularity at infinity.

The fact that the infrared part of the radiative corrections induces a Regge behavior in the cross section enables us to draw a certain number of interesting conclusions:

(1) First of all, the value of the Regge approximation at high energy is confirmed since Yennie et al. have given a fairly convincing proof that the bulk of the radiative corrections at high energy is, indeed, given by Eq. (1).[8] On the other hand, this behavior is not something really new in quantum electrodynamics: The general form of Eq. (1) was certainly known to Schwinger in 1949, and the vanishing of the elastic cross section in the limit of infinite resolution ($\epsilon \to 0$) was proved long ago by Bloch and Nordsieck.[9]

(2) If we increase ϵ up to the order of E, so as to include the whole of inelastic scattering, it seems likely that the Regge behavior of Eq. (1) disappears. Perhaps one could conclude that the Regge approximation is valuable only for elastic or nearly elastic processes, and that it is somewhat "washed out" when the inelastic channels become important. Presumably, however, the fine-structure splitting of the even and odd trajectories would remain. This would correspond to residual Regge poles varying much more slowly with t. It could be also that, in addition to $\gamma(t)$, there is a "true" Regge pole associated with the photon, as Blankenbecler et al. have postulated. Since, in principle, radiative corrections to electron scattering can be calculated exactly, one could separate them and see if there is any residual effect. However, because of Eq. (9), it seems difficult to make this separation in the type of experiment which they have suggested [namely, measuring

the ratio $\sigma(E_1, t)/\sigma(E_2, t)$ at a moderate value of t].

(3) The connection which we have found between radiative effects and Regge poles puts the calculation of electromagnetic radiative corrections in a new perspective. For example, it is our knowledge of the fine structure of the positronium spectrum which leads us to believe that there probably are spin-dependent terms which should be summed in an exponential factor in the electron-proton scattering cross section. Also, the main part of the radiative corrections in a given process at high energy can probably be estimated directly from the known energy spectrum of the intermediate states allowed in the crossed channel.

(4) Finally, we would like to make a remark on the question of the origin of the Regge poles which has been widely discussed recently. In the theory of strong interactions, where we do not have a consistent field theory to start with, the question of whether Regge poles are "fundamental" or whether they can be derived by calculations based on a specific Lagrangian is rather a matter of taste (or, as has been said, a "philosophical" question). In quantum electrodynamics, the situation is different, because there exists a field theory enabling us to make predictions which can be tested experimentally. And since a Regge behavior of the cross sections can be obtained by a consistent high-energy approximation to the field equations, we do not see any reason at present to question the elementary nature of the photon.

We would like to thank Professor Hofstadter, Professor Tsai, and Professor Walecka for interesting discussions, and the Physics Department of Stanford University for its kind hospitality.

---

*Work supported in part by the U. S. Air Force Office of Scientific Research.

†Permanent address: Laboratoire de Physique Theorique et Hautes Energies, Boite Postale 12, Orsay, Seine-et-Oise, France.

[1]R. Blankenbecler, L. F. Cook, and M. L. Goldberger, Phys. Rev. Letters 8, 463 (1962).

[2]D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. 13, 379 (1961).

[3]J. Schwinger, Phys. Rev. 76, 790 (1949).

[4]S. Frautschi, M. Gell-Mann, and F. Zachariasen (to be published).

[5]H. A. Bethe and E. E. Salpeter, Handbuch der Physik (Springer-Verlag, Berlin, 1957), Vol. 35, p. 203. We conjecture that Eq. (7), with $\gamma + \nu = n$, gives to all orders the energy levels of a system of two spinless bosons with equal masses and opposite charges interacting relativistically through the electromagnetic field.

[6]L. Michel, Nuovo cimento 10, 319 (1953).

[7]A slope of the order of α^2/m^2 would still, however, be about 200 times larger than that of the vacuum trajectory.

[8]In addition to the exponential Regge behavior, it has been noted by Yennie et al. that the function $F(t)$ in Eq. (2) constitutes also a strong reduction factor for the high-energy cross section.

[9]F. Bloch and A. Nordsieck, Phys. Rev. 52, 54 (1937). It is apparent from Eq. (1), that making $\epsilon \to 0$ is equivalent to increasing $E \to \infty$.

REGGE POLES AND LANDAU SINGULARITIES

V. N. Gribov
A. F. Ioffe Physico-Technical Institute, Leningrad, U.S.S.R.

and

I. Ya. Pomeranchuk
Institute of Theoretical and Experimental Physics, Moscow, U.S.S.R.

(Received July 30, 1962)

A scattering amplitude[1] as a function of energy t and momentum transfer s is known to have singularities at threshold values of these variables and on Landau curves.[2] It is a remarkable property of Landau singularities that the threshold of one of the variables, e.g., t, is a line in the (s, t) plane which, as s increases to infinity, is asymptotic to an infinite number of Landau curves.[3] On the other hand, it has become clear recently that the asymptotic behavior of the scattering amplitude at high s is determined by analytic properties of partial-wave amplitudes $f_l(t)$, as functions of angular momentum l, in the channel where $\sqrt{t}$ is the total energy.[4] The problem is how the accumulation of Landau curves near the threshold value of t is manifested in the behavior of singularities of the amplitudes as functions of angular momentum l. It will be shown that at the energy $t = t_i$ corresponding to any two-particle threshold, the accumulation of an infinite number of poles