in the potential. We have also applied the correction factor to the traditional test case Cu^+, which was studied by Pratt[9] and by Hartree.[10] The new potential is closer than the old to the Hartree-Fock self-consistent potential seen by a $3d$ electron. While this fact has no particular quantitative significance, it suggests that the screened potential, applied in atomic calculations, will not give results vastly different from "self-consistent" calculations.

We have considered in some detail the justification, within the context of the RPA, of the use of the Thomas-Fermi dielectric function. Neglecting the plasmon contribution, we find that for densities of interest here the total interaction energy computed exactly within the RPA agrees, to within 1%, with that obtained using the simpler Thomas-Fermi screening.

We have also considered the possibility that our correction overestimates the effect of correlation, since the T-F screening is too strong for low densities at small q. We might modify Eq. (3) in such a way that $\epsilon(q)$ reduces to some empirical dielectric constant κ at $q \to 0$:

$$\epsilon(q) = 1 + k_S^2/(q^2 + q_S^2), \qquad (9)$$

with $q_S^2 = k_S^2/(\kappa - 1)$. It is found that in practice any $\kappa \geq 4$ yields an effective exchange potential much closer to the screened potential $(\kappa \to \infty)$ than to the unscreened potential $(\kappa = 1)$; this is illustrated in Fig. 2.

We emphasize that our procedure, like that of Slater, is based on a uniform-electron-gas approximation for local regions. Although its validity in regions of rapidly varying density is limited,[10,11] it is likely to be less susceptible to criticism on these grounds because the particle-particle interactions involved are screened. We conjecture, then, that our approximate potential is never poorer than the original Slater potential; it has a much shorter range and incorporates a correction which is known to be important in the uniform electron gas.

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LOW-MASS ANOMALY IN PHOTOPRODUCTION OF PION PAIRS

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Abashian, Booth, and Crowe[1] have observed an anomaly in the production of pion pairs in the reaction

$$p + d \to He^3 + 2\pi.$$

This anomaly (referred to hereafter as "ABC") occurs in the isotopic spin state $T = 0$ of the two pions and may be interpreted as being caused by a final-state interaction of the two pions or as being caused by the production of some particle. The most popular explanation of the ABC at the present time is that it is caused by a strong S-wave π-π interaction which can be characterized by a scattering length.[1-3] For convenience, the ABC will be referred to in the following as if it were a particle.

Several previous experiments[4-6] have attempted to observe the ABC in photoproduction from hydrogen. Bernardini et al.[4] used photons of about 750 MeV, and detecting only the recoil proton, observed no effect. They gave an upper limit to the cross section of 0.3×10^{-30} cm^2/sr. Gomez et al.[5] searched for a particle which decays into a π^0 and a gamma ray and also found nothing. Their upper limit on the cross section was 3.2 $\times 10^{-32}$ cm^2/sr. Berkelman, Cortellessa, and Reale[6] searched for a state which decayed into a π^+ and a π^- meson. They too found no effect and gave an upper limit to the c.m. differential cross section of 5×10^{-32} cm^2/sr.

This experiment was an attempt to observe the ABC in the reaction

$$\gamma+p\rightarrow p+\text{ABC},$$

without any restriction on the possible decay modes of the ABC if it is a particle. In addition, the kinematics were arranged so that the ABC would be produced at a c.m. angle of about 90° so that interference terms between several angular momentum states could not reduce the cross section too much. The photon energy was chosen so that the π^0 background was small and so that the ABC would be produced with a c.m. momentum near that of the experiment of Abashian, Booth, and Crowe.

The yield of protons at a given angle and momentum was measured as a function of the peak energy of the incident bremsstrahlung spectrum. The experimental arrangement is shown in Fig. 1. The electron beam from the Stanford Mark III linear accelerator passed through a copper radiator 0.029 radiation length thick. The resulting bremsstrahlung beam and the degraded electron beam passed through a liquid-hydrogen target and into a large Faraday cup which monitored the electron beam. The recoil protons produced in the target passed through a $1\frac{1}{2}$-in. thick tungsten collimating slit and, on leaving the scattering chamber, entered a 180° double-focusing magnetic spectrometer. The protons were detected by two plastic scintillators at the exit focus of the spectrometer. Whenever a fast coincidence between the two counters was observed, the pulse height of the output of the second counter was recorded on a 256-channel pulse-height analyzer. The pulse-height distribution allowed one to reject those coincidences caused by the smaller pulses due to π^+ and e^+ which traversed the counters.

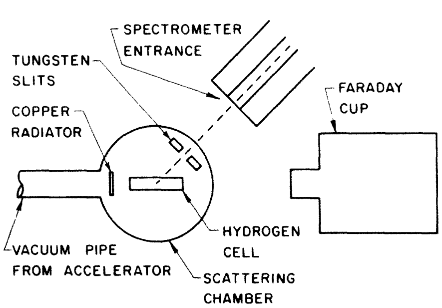

FIG. 1. Experimental arrangement.

Recoil protons can be produced by the following reactions:

$$\gamma+p\rightarrow p+\pi^0, \qquad e^-+p\rightarrow e^-+p+\pi^0,$$
$$\gamma+p\rightarrow p+2\pi, \qquad e^-+p\rightarrow e^-+p+2\pi,$$
$$\gamma+p\rightarrow p+\text{ABC}, \qquad e^-+p\rightarrow e^-+p+\text{ABC},$$
$$e^-+p\rightarrow e^-+p.$$

In order to observe the detailed shape of the yield curve above the threshold for π-pair production, one must subtract the contributions from π^0 production and electron scattering. The shape of the π^0 yield curve was calculated by computing the thick-target bremsstrahlung spectrum using the Bethe-Heitler thin-target equation for the Cu radiator and the Alvarez[7] thick-target computer program. The straggled electron spectrum was then folded with the Wheeler-Lamb[8] bremsstrahlung equations for hydrogen to obtain the gamma-ray spectrum from the relatively long hydrogen target. The contribu-

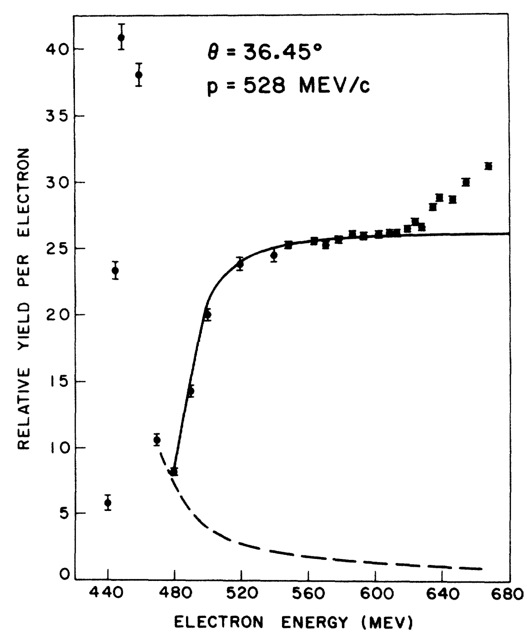

FIG. 2. Relative proton yield per electron vs incident electron energy. The dashed curve is the calculated electron-scattering contribution. The solid curve is the sum of the electron-scattering and the computed π^0 yield curves. The computed π^0 yield was normalized to fit the data between 549 and 580 MeV. The π^0 threshold is at 486 MeV.

tion to the yield from electroproduction was computed by folding the effective gamma-ray spectrum in electroproduction, as defined by Dalitz and Yennie,[9] with the straggled electron spectrum. The relative contributions of the three processes to π^0 production near the π-pair threshold were 54 % from bremsstrahlung in copper, 19 % from bremsstrahlung in hydrogen, and 27 % from electroproduction. The total effective gamma-ray spectrum was then folded with the spectrometer resolution function to give the expected shape of the meson production yield. The electron-scattering background was computed from the straggled electron spectrum and the area under the electron-scattering peak below π^0 production threshold. Figure 2 shows a plot of all the data at one proton angle and momentum.

The pion-pair production cross section, assuming no final-state pion-pion interaction, was calculated by Bjorken[10] using the Cutkowsky-Zachariasen[11] model which assumes the photoproduction of one pion in an S state and the other in a P state relative to the proton. The cross section is given by

$$d^2\sigma/d\Omega\, dp = (0.07)^2 \alpha p \beta (M + 2k)(S^2 - 4\mu^2)^{3/2}(1 + \epsilon)$$

$$\times \left\{ \pi\omega^2\mu^4 S \left[\left(1 - \frac{\omega}{2.2\mu}\right)^2 + \frac{0.01(\omega^2 - \mu^2)^3}{\omega^2\mu^4} \right] \right\}^{-1};$$

$$\epsilon = \left(\frac{5 - 3\cos^2\theta^*}{16}\right)\left[\frac{4\omega^2 - S^2}{S^2 - 4\mu^2} + \frac{4}{3}\,\frac{4\omega^2 - S^2}{S^2}\right],$$

where $\hbar = c = 1$; α is the fine-structure constant; 2ω is the total energy of the two pions in the total c.m. system; S^2 is the square of the effective mass of the two pions; θ^* is the proton c.m. angle; M is the proton mass; μ is the pion mass; p and β are the proton momentum and velocity in the laboratory system; and k is the incident photon energy. Since the photon energy range covered by this experiment is below that required to produce two pions both of which are at the 3, 3 resonance relative to the proton, and the change in the available phase space dominates the energy dependence of the cross section, the Bjorken calculation should be a fairly good approximation to the actual energy dependence of the cross section.

The electron scattering and π^0 production contribution to the yield were fitted to the data below the threshold for pion pair production, and then subtracted from all the data. A least-squares analysis of the subtracted yield was made assuming the yield was of the form

$$Y = \alpha_{2\pi} F_{\pi\pi} + \alpha_{\mathrm{ABC}} F_{\mathrm{ABC}}$$

where $F_{\pi\pi}$ was obtained by folding the pion-pair production cross section with the bremsstrahlung spectrum and the spectrometer resolution function, and F_{ABC} was computed in the manner described above for determining the shape of the π^0 yield curve. The least-squares analysis was done for several values of the threshold energy for ABC production. The absolute normalization of the coefficients $\alpha_{2\pi}$ and α_{ABC} was made by using the π^0 photoproduction cross sections of Berkelman and Waggoner.[12] The error on the π^0 cross section listed by them is 6 %, and therefore a further 6 % uncertainty should be included in the absolute normalization. Table I shows the results of the least-squares analysis. Figure 3 shows the high energy part of the data with the π^0 and electron scattering contribution subtracted out.

While this analysis has been made assuming the production of a particle, the effect could equally well be caused by a resonant type of π-π interaction with a finite width. The spectrometer resolution function in terms of a particle production threshold has a full width at half-height of 20 MeV. A resonant interaction with a width of up to about 20 MeV could therefore also fit

Table I. Least-squares fit to the data of Fig. 3 assuming that the proton yield is due to a combination of π-pair production and single-particle production, as a function of the particle threshold energy. The first column gives the assumed threshold energy for single-particle production. The second column gives the corresponding particle mass. The third and fourth columns give, respectively, the least-squares adjusted values for π pairs relative to the theoretical π-pair yield, and the center-of-mass differential cross section for single-particle production. The last column gives the value of χ^2 derived using the numbers in columns 3 and 4.

k (MeV)	m (MeV)	$a_{2\pi}$	$(d\sigma/d\Omega)_{\mathrm{c.m.}} \times 10^{30}$	χ^2
---	---	1.30 ± 0.23	0	32.2
625	313	0.33 ± 0.33	0.74 ± 0.25	26.9
630	318	0.53 ± 0.17	0.64 ± 0.14	17.8
635	322	0.58 ± 0.20	0.75 ± 0.19	15.9
640	327	0.75 ± 0.17	0.61 ± 0.18	19.4
645	331	0.96 ± 0.13	0.53 ± 0.30	23.8

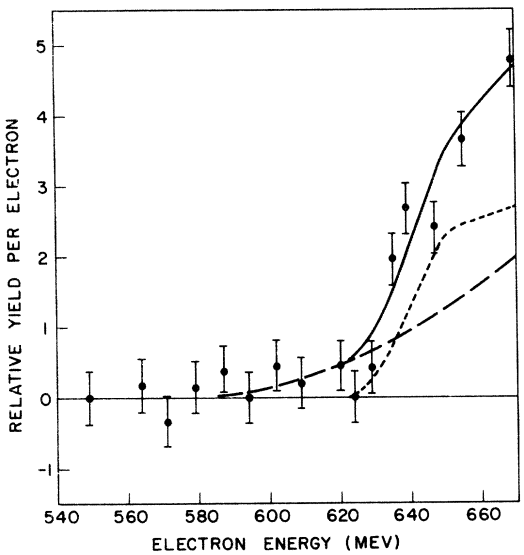

FIG. 3. Proton yield per electron vs electron energy with the π^0 and electron-scattering yields subtracted. The dashed curve is the least-squares value of the π-pair yield; the dotted curve is the least-squares value of the ABC yield for a mass of 322 MeV; and the solid curve is their sum. The $\pi^+-\pi^-$ threshold is at 590 MeV.

the data.

The best fit to the data occurs at an ABC mass of 322 MeV. At this value of the mass, the coefficient $\alpha_{2\pi}$ is given by

$$\alpha_{2\pi} = (0.58 \pm 0.20)(1 \pm 0.06),$$

where the last factor comes from the normalization error. If the pion pair cross section used in this analysis were in exact agreement with nature, $\alpha_{2\pi}$ should equal one. The lack of exact agreement is not too surprising since the calculation is based on a relatively simple model and includes no π-π interaction. Its energy dependence is, however, a better approximation than simple phase space. The conclusion of this experiment will not be markedly changed by the use of any smoothly varying pion-pair production cross section. The sharp increase in the proton yield at about 635 MeV (see Fig. 3) can be accounted for only by a relatively narrow bump in the cross section as a function of photon energy, or by an experimental mistake.

In summary, the properties of the ABC as measured by this experiment are

$$\text{mass} = 322 \pm 8 \text{ MeV},$$

$$\text{width} \leq 20 \text{ MeV (full width at half maximum)},$$

$$(d\sigma/d\Omega)_{\text{c.m.},\,90^\circ} = (0.75 \pm 0.20) \times 10^{-30} \text{ cm}^2/\text{sr}.$$

The error in the mass was determined by the change in photon energy required to reduce the χ^2 probability by a factor of 3.

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