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FARADAY ROTATION NEAR THE BAND EDGE OF SILICON

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It has been only recently that a renewed interest in the Faraday effect occurring with infrared radiation at wavelengths corresponding to the energy gap has become evident,[1] and as a consequence a meager amount of empirical data exists. Hartmann and Kleman[2] reported on measurements of the room-temperature Verdet coefficients for an intrinsic germanium sample near the edge. The results indicated that both indirect and direct transitions contributed to the dispersion. Intrinsic silicon is an excellent material for studying the effects of indirect transitions on the Faraday rotation, because the direct transitions require such a high photon energy that they should have little or no effect on the dispersion at wavelengths beyond the edge. Hence one should see Faraday rotation phenomena associated only with the indirect transitions when the sample is irradiated with polarized radiation of wavelengths approaching those corresponding to the intrinsic edge.

We have observed Faraday rotation in intrinsic silicon at infrared wavelengths from 1.05 microns to 5 microns at 297°K and at 77°K. The experimental technique is a conventional one and is a modification of that used by Austin in studying Bi_2Te_3.[3] The Verdet coefficients of an n-type silicon sample (30-Ω cm) at a magnetic-field strength of 19.1 kilogauss is shown in Fig. 1. We propose that the dispersion effects responsible for these curves are associated only with the mechanism corresponding to indirect transitions.

Lax and Nishina[4] have developed an expression for the Faraday rotation corresponding to indirect transitions based upon a quantum mechanical

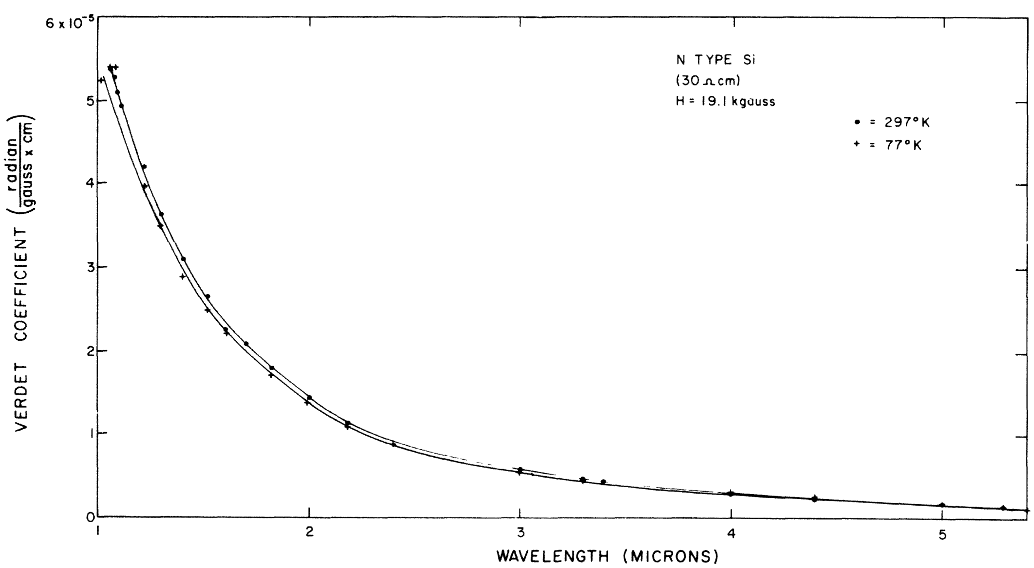

FIG. 1. Verdet coefficients for n-type silicon at 77°K and 297°K. Magnetic-field strength 19.1 kG.

treatment of virtual interband transitions leading to magneto-optical absorption effects. Their result for the Faraday rotation θ_i is given by the following expression:

$$\theta_i = (-KB/x)\{\ln(1-x^2) + x\ln[(1+x)/(1-x)]\},$$

$$x = \omega/\omega_g. \qquad (1a)$$

This expression holds for weak magnetic fields and for frequencies well below that corresponding to the energy gap ω_g. K is a coefficient, which should be independent of ω and T, and B is the magnetic induction. For ω much less than ω_g, this expression becomes the following:

$$\theta_i \cong -KBx(1 + x^2/6). \qquad (1b)$$

Thus one can see that this expression predicts that the Faraday rotation will approach zero as λ^{-1} at very long wavelengths. The data of Fig. 1 replotted in Fig. 2, however, indicate that the Faraday rotation approaches zero very nearly as λ^{-2}. This indicates that instead of the Faraday rotation associated with indirect transitions being described by Eq. (1b), it would be of the form

$$\theta_i = K_1 B(\omega^p/\omega_g)(1 + ax^m), \qquad (1c)$$

where the exponent p is very close to two.

Assuming (1b) or (1c) and for small values of $x = \omega/\omega_g$, the temperature variation of the Faraday rotation can be written in the following form:

$$\theta^{-1}d\theta/dT = -(1 + \tfrac{1}{3}x^2)\omega_g^{-1}d\omega_g/dT. \qquad (1d)$$

Equation (1d) indicates that, for wavelengths much longer than that corresponding to the indirect gap, the Faraday rotation should vary with temperature as the function, $\omega_g^{-1}d\omega_g/dT$, with a correction term which vanishes at very long wavelengths. McLean and co-workers[5] have determined the value of $\omega_g^{-1}d\omega_g/dT$ over the range 77°K-300°K to be approximately -1.6×10^{-4}/°K. Using this value in (1d), we get a value of $\theta^{-1}d\theta/dT$ at 1.5 microns of 1.92×10^{-4}/°K compared with a measured value of $(2.1 \pm 0.2) \times 10^{-4}$/°K.

Equation (1c) is very similar to the form that one gets if one considers a collection of classical oscillators having a resonant frequency of ω_0. For the very special case[6] of the single bound oscillators, Faraday rotation is given by the following expression:

$$\theta = \omega_p^2\omega_c\omega^2/2nc(\omega_0^2 - \omega^2)^2, \qquad (2a)$$

and the refractive index n by

$$n^2 = 1 + \omega_p^2/(\omega_0^2 - \omega^2), \qquad (2b)$$

where ω_p is the plasma frequency and ω_c is the cyclotron frequency ($\omega_c = eB/m^*$). Expression (2a) can also be written as

$$\theta = (\omega_c/2c)\lambda dn/d\lambda \qquad (2c)$$

or

$$\theta = [(n^2-1)^2/2nc]\omega_c\omega^2/\omega_p^2. \qquad (2d)$$

An expression such as (2d) describes the curve in Fig. 2 very well over a wide range of wavelengths, because n is not varying rapidly with λ. If one considers Eqs. (2a) and (2b), one can obtain

$$\theta^{-1}d\theta/dT = [(3n^2+1)/(n^2-1)]n^{-1}dn/dT. \qquad (2e)$$

Lukeš[7] has measured the temperature variation of the refractive index of silicon, and reports it to be $(5.6 \pm 0.2) \times 10^{-5}$/°K at 1.5 microns. Using this value and a value of $n = 3.5$ in (2e), we get $\theta^{-1}d\theta/dT = (1.9 \pm 0.1) \times 10^{-4}$/°K.

The temperature dependence of the Faraday rotation associated with indirect transitions is evidently closely related to the temperature dependence of the energy gap and the refractive index in agreement with Moss's quasi-empirical law $n^4\omega_g$ = constant.[8]

The concept of single classical oscillator is not

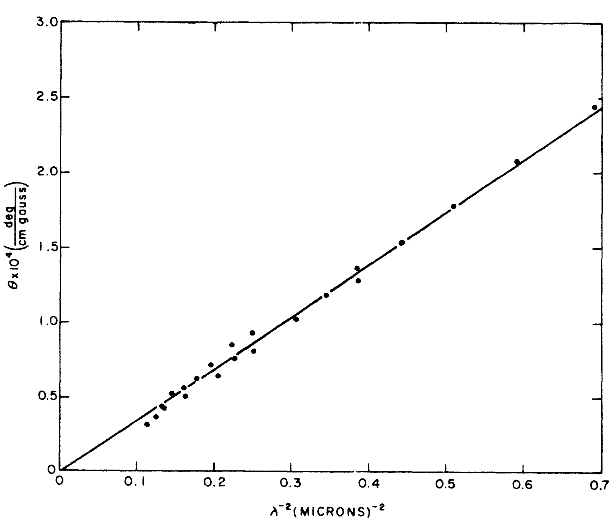

FIG. 2. The Verdet coefficient as a function of λ^{-2} for n-type silicon. Data taken at 297°K.

adequate in itself to account for the interband Faraday rotation, because the contribution to the rotation of the virtual transitions must be summed over the bands. A more detailed quantum mechanical treatment of dispersion gives results very similar to Eq. (2b).[6] This close similarity explains why Eq. (2b) is in such good agreement with experimental observations on dielectrics. Korovin[9] used the classic concepts when calculating the index of refraction, using the one-electron approximation. The electron from the valence band is represented as a classical oscillator, whose natural frequency ω_0 is equal to the difference between the energies in the valence band and the conduction band.

The complex refractive index N is given in this case by

$$N = 1 + (8\pi e^2/3m^2\hbar)\sum_k |p(k)|^2/(\omega_0^2 - \omega^2 + j\omega'\omega)\omega_0, \tag{3a}$$

where $p(k)$ is the matrix element of momentum for transitions of an electron with a given wave vector k from the valence band to the conduction band, ω is the frequency of the radiation, and ω' describes the damping of the oscillator, which corresponds to the finite lifetime of the virtual states of electrons in the conduction band.

It is very interesting that the experimental results of the indirect rotation follow very closely the expressions (2a) to (2e) which are derived from the single classical oscillator. This leads to the suggestion that the quantum mechanical treatment of the Faraday rotation should result in a similar frequency dependence for $\omega < \omega_g$, but with constants determined by the distributed transition energies.[10]

It may very well be that the interband Faraday-rotation theory must also take into account additional effects such as the Coulomb interaction between hole-electron pairs (excitons). For the case of direct transitions, Suffczynski has suggested, in the case of germanium, the possibility that excitons can contribute, to a significant extent, to the Faraday rotation at frequencies less than that of the direct gap.[11]

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$$\theta_i = (D\gamma_i H/2c)\{(\omega_{gi}^2/\omega^2)\ln[\omega_{gi}^2/(\omega_{gi} - \omega^2)] + (\omega_{gi}/\omega)\ln[(\omega_{gi} - \omega)/(\omega_{gi} + \omega)] + 1\},$$

where D is a constant, involving matrix elements, electron charges, and masses, and

$$\omega_{gi} = \omega_g \pm \omega_{ph}.$$

($\hbar\omega_g$ is the gap energy, $\hbar\omega_{ph}$ the phonon energy, and 1 and 2 designate the valence and conduction bands, respectively.) The above expression gives, for $\omega/\omega_g < 1$, an expression of the form (1c) with $p = 2$, $m = 2$, and $a = \frac{2}{3}$. We thank B. Lax for supplying this information.

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