squared distributions of the $\Xi\pi$ and $K\bar{K}$ systems by means of the χ^2 test. The probabilities that the observed distributions originate from their corresponding phase-space distributions are <0.0001 for the $\Xi\pi$ system and <0.01 for the $K\bar{K}$ system. The large χ^2 values arise mainly from the single peaks appearing in each curve.

[4]For both the $\Xi\pi$ and $K\bar{K}$ systems the mass resolution is about ±3 MeV. This resolution has been estimated from the distribution of Λ masses found when the pion and proton from the Λ decay have been fitted to the production vertex.

[5]No subtraction of background has been made in the derivation of these ratios.

REGGE POLE MODEL FOR HIGH-ENERGY p-p AND $\bar{p}$-p SCATTERING

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The p-p data from 3 to 30 GeV/c seem to fit the assumption of a single vacuum Regge pole trajectory[1] (which we label P) and have been analyzed in these terms.[2-5] However, two more trajectories must in fact be important in this region: the ω trajectory to give the big difference between p-p and $\bar{p}$-p scattering, and a second vacuum trajectory (lablled P') to keep the p-p total cross section roughly constant.[6] (Igi[7] has already established the presence of P' from π-N dispersion relations.) Other known trajectories seem less important; the ρ trajectory gives differences between p-p and n-p scattering, but these are small.[2]

It is important to know if these extra poles spoil the one-pole analysis. We present here a realistic model of p-p and $\bar{p}$-p scattering, using the P, P', and ω trajectories, which fits many aspects of the data. This model suggests that one-pole analyses need certain corrections but remain qualitatively valid.

We first assume the scattering is dominated by the spin-averaged amplitude; this is reasonable, at least in the diffraction peak, and leaves a scalar problem. (A complete spinor treatment[8,9] introduces some modifications to be discussed later.) The amplitude contains terms of the usual form

$$T = \beta(t) P_\alpha(\cos\Theta_t)[1 \pm \exp(-i\pi\alpha)]/\sin\pi\alpha, \qquad (1)$$

where $\alpha(t)$ is the corresponding trajectory, t is the invariant momentum transfer squared, P_α is the Legendre function, Θ_t is the scattering angle in the crossed channel, and $\beta(t)$ is the residue function. The signature ± is + for P and P', and - for ω. The p-p and $\bar{p}$-p amplitudes have the forms

$$T\begin{pmatrix} p\text{-}p \\ \bar{p}\text{-}p \end{pmatrix} = T_P + T_{P'} \pm T_\omega \qquad (2)$$

in an obvious notation.

From $\beta(t)$ we factor out the statistical weight $(2\alpha+1)$, a factor α to remove[8] "ghost" singularities at $\alpha = 0$ for P and P', and the threshold dependence[8,10] $(t-4m^2)^\alpha/(4m^2)^\alpha$ where m is the nucleon mass; any remaining t dependence is ignored. The choice of units to make the threshold term dimensionless is not trivial, for it affects the t dependence. We note that the factor $(2\alpha+1)$ also serves to remove a singularity in P_α at $\alpha = -\frac{1}{2}$. The ω term has no ghost at $\alpha = 0$, but we keep the factor α for symmetry; it also helps to fit the data, as explained later.

We now use the asymptotic form of P_α, write $\cos\Theta_t = (2s-4m^2+t)/(t-4m^2)$ where s is the invariant energy squared, and neglect t compared to s. Each pole term becomes

$$T = B\alpha(2\alpha+1)2^\alpha \frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)} \frac{1 \pm \exp(-i\pi\alpha)}{\sin\pi\alpha} \left(\frac{s-2m^2}{2m^2}\right)^\alpha, \qquad (3)$$

where B is a constant and Γ is the gamma function. Note that $(s-2m^2)/2m = E_L$, the total proton laboratory energy.

We assume the P trajectory is a straight line between $\alpha_P = 1$ at $t = 0$ and $\alpha_P = 0$ at $t = -1$ (GeV/c)^2, as suggested by the one-pole analysis; we restrict ourselves to this range of t. Since the P' and ω contributions to the p-p total cross section are to cancel, their trajectories and residues are equal at $t = 0$; we continue this symmetry for $t < 0$ by choosing these trajectories to coincide on a straight line parallel to the P trajectory. The intercept at $t = 0$ is taken to be 0.5 from Igi's work[7]; this value also seems consistent with the variations of total cross sections in the N-N, $\bar{N}$-N, π-N, K-N, and $\bar{K}$-N systems. The numerical coefficients are chosen in the ratio $B_P:B_{P'}:B_\omega = 0.45:1:1$ for an approximate fit to the p-p and $\bar{p}$-p total cross sections.[2-5] The model is now completely specified.

For a comparison with experiment, we write the differential cross section as

$$d\sigma/dt = (d\sigma/dt)_{\rm opt} F(s,t)(E_L/m)^{2\alpha_P(t)-2}, \qquad (4)$$

where the subscript "opt" indicates the minimum forward cross section found from the optical theorem. If only the P trajectory contributes, F is a function of t alone; this is then essentially the formula used to fit p-p data in reference 3. The predictions of our model are shown in Fig. 1; $F(s,t)$ is plotted against t for various fixed values of s, and is compared with $F(t)$ deduced from p-p data between 3 and 30 GeV using the one-pole formula.[2-5]

We see that the model agrees qualitatively with p-p data, by giving the initial fall of F as $|t|$ increases. It does not give the subsequent flattening or rise, indicated by a one-pole analysis[2-5] over the whole range 3-30 GeV/c and shown in Fig. 1; however, if the range is restricted to 9-30 GeV/c, the one-pole analysis no longer shows the above marked effect.[11] The model also agrees qualitatively with $\bar{p}$-p data,[12] which

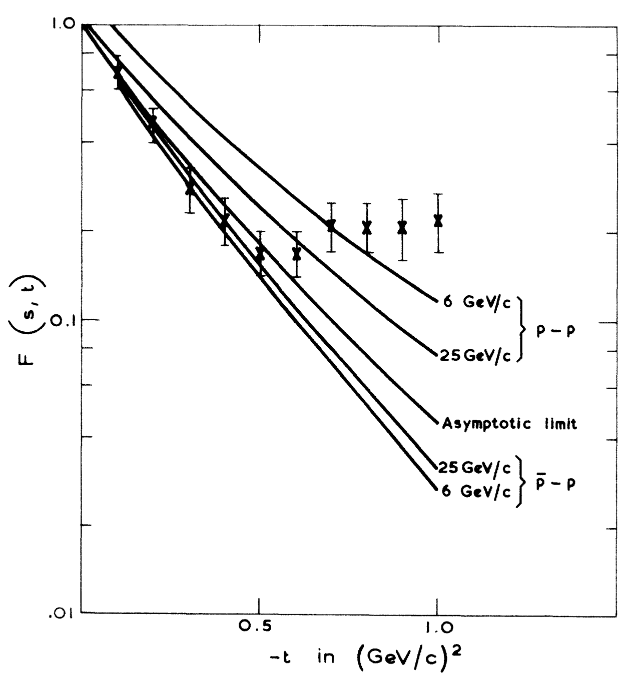

FIG. 1. Predictions of the three-pole model for various fixed values of s: the p-p and $\bar{p}$-p cases have the same asymptotic limit. "Experimental" results,[2-5] from using the one-pole formula on data from 3-30 GeV/c, are also shown.

show a much narrower diffraction peak than in the p-p case. At $E_L = 3$ GeV and $t = -0.5$ (GeV/c)^2 the $\bar{p}$-p peak has fallen by an extra factor 5: our model gives a factor 3 (though it is not really designed for such low energies). We notice a consequence of the model, that as s increases, the shape of $F(s,t)$ shrinks for the p-p case and expands for the $\bar{p}$-p case: both reach the same asymptotic limit, given by the P term alone.

Two arbitrary features of the model can now be discussed. First it is clear that a different choice of units for the threshold dependence of $\beta(t)$—which in turn determines the units for s in Eq. (3)—would lead to different shapes for $F(s,t)$. The rough agreement with experiment suggests that our choice, originally made for convenience, may have a deeper significance. Secondly, we can see how the factor α gratuitously included in the ω term helps to fit experiment. For since the cross section for $\bar{p}$-p is bigger than that for p-p at $t=0$, and since it falls off much faster for $t<0$, the two curves must intersect somewhere; at this point the ω term must vanish, in the three-pole model. The extra factor α_ω insures this vanishing, at $t = -0.5$ (GeV/c)^2 where $\alpha_\omega = 0$ with our choice of ω trajectory.

Now consider the plot of $\ln[(d\sigma/dt)/(d\sigma/dt)_{\rm opt}]$ against $\ln(E_L/m)$ at fixed t. The one-pole assumption gives a straight line. The one-pole analysis is made by fitting a straight line to this plot of the data at each value of t; $\alpha(t)$ and $F(t)$ are found from the slope and the intercept at $\ln(E_L/m) = 0$. Our model, however, gives a curved line, concave upward for the p-p case,[13] suggesting that the one-pole analysis tends to overestimate both $F(t)$ and $(1-\alpha)$ for the P trajectory.

To illustrate this point, we plot the model predictions for the p-p case between 6 and 25 GeV/c, with nominal 10% uncertainty, and make straight-line fits to these synthetic data, as shown in Fig. 2. The curvature is scarcely visible in this momentum range and might escape detection even with measurements to ±1% (we include some 3-GeV/c points to show the curvature is there, but these are not considered in the fit). Nevertheless, the curvature between 25 GeV/c and ∞ is enough to make appreciable corrections to $\alpha(t)$ as shown in Table I. The determination of $F(t)$ involves a large extrapolation and the corrections here are consequently bigger.

A complete spinor treatment for N-N scattering is formulated by Gell-Mann.[8,9] With a one-

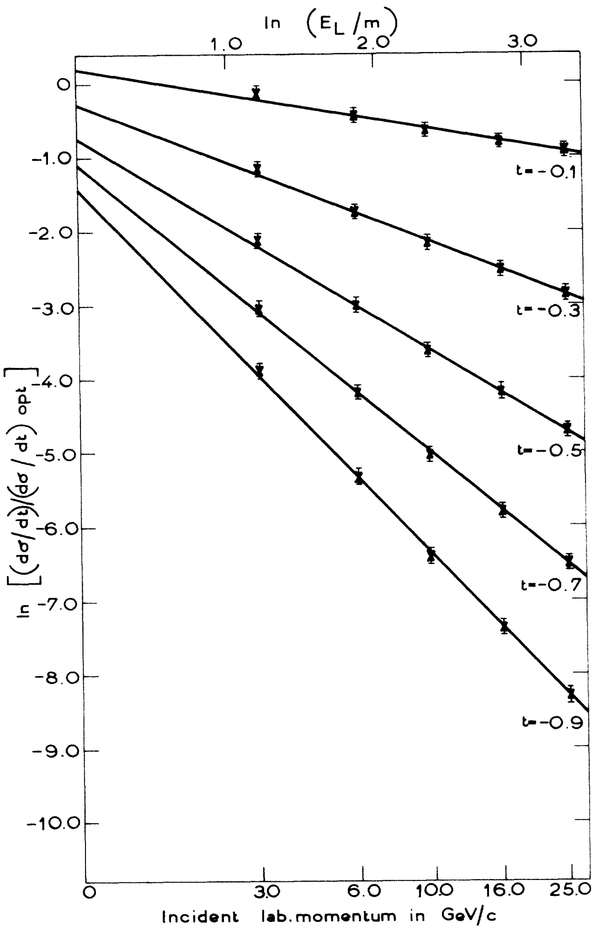

FIG. 2. One-pole fit to the three-pole model, between 6 and 25 GeV/c, for p-p case. Plot of $\ln[(d\sigma/dt)/(d\sigma/dt)_{\text{opt}}]$ against $\ln(E_L/m)$ at fixed t is fitted by straight lines. Points at 3 GeV/c are shown, but not used in the fit.

pole model there are now two independent residue functions η_1 and η_2 (occurring quadratically) in place of one. It seems impossible to determine both independently from unpolarized differential cross-section measurements alone. However, only η_1 affects the total cross section. If we make assumptions about η_1 equivalent to those we made about $\beta(t)$, and put $\eta_2 = 0$, the cross section differs from the corresponding scalar model by a factor $[1 - t/4m^2]^2$, essentially because of spin-dependent terms. If instead we put $\eta_2 = c\eta$, where c is a constant, the additional t dependence asymptotically has the form $(1 - |1 - c|^2 t/4m^2)^2$. Unless c is much larger than 1, the one-pole curve in Fig. 1 is little altered.

With a three-pole spinor model there is much arbitrariness. However, if the three-pole terms

Table I. Results of the one-pole analysis, applied to our three-pole model. Comparison of true and apparent values of α and F.

$-t$ $(\text{GeV}/c)^2$	$\alpha_P(t)$ True	$\alpha_P(t)$ Apparent	$F(t)$ True	$F(t)$ Apparent
0.1	0.9	0.83	0.68	1.2
0.3	0.7	0.61	0.34	0.74
0.5	0.5	0.40	0.18	0.47
0.7	0.3	0.18	0.10	0.33
0.9	0.1	-0.04	0.06	0.23

differ qualitatively only in signature and we choose the residue functions suitably, we can recover the results of the scalar model simply multiplied by a t-dependent factor as above. Thus a scalar model is not unreasonable for $|t| \ll 4(\text{GeV}/c)^2$.

This provides a limit on the model's range of validity in t. The limits of validity in energy are hard to estimate. However, we are reluctant to take the model seriously below, say, $E_L = 6$ GeV, if only because the forward p-p amplitude develops a large real part in this region, contrary to experiment. At 10 GeV, however, the forward cross section is only 16% above the "optical" value, which agrees with the experimental[11] figure, 20 ± 20%. In our model the $\bar{p}$-p forward amplitude is always purely imaginary.

To summarize, our model uses the following assumptions:

(i) The problem is essentially scalar.

(ii) The P, P', and ω trajectories dominate.

(iii) The P' and ω terms are symmetrical, so that in the p-p amplitude their imaginary parts cancel, not only at $t = 0$, but also for $t < 0$.

(iv) The residue functions have the minimum known t dependence (apart from an extra factor in the ω term, needed for symmetry).

(v) The threshold dependence of residues introduces an arbitrary scale in each pole term, affecting the t dependence; this scale is chosen the same for all three terms.

(vi) The common trajectory of P' and ω remains parallel to the P trajectory; we take the latter to be a straight line passing through 0 at $t = -1$ $(\text{GeV}/c)^2$.

We reach the following conclusions:

(i) The model fits qualitatively what is known about high-energy p-p and $\bar{p}$-p scattering in its range of validity, $|t| \ll 4(\text{GeV}/c)^2$.

(ii) Fitting data in the range 6-25 GeV/c by a one-pole formula is likely to overestimate both $[1 - \alpha(t)]$ and $F(t)$ for the P-trajectory, to the extent shown in Table I. On the other hand, the fit will appear deceptively good.

(iii) Since the curvature in Fig. 2 is so hard to see, the prospects for separating different pole contributions in the region $t<0$ are poor, at least with presently accessible energies.

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[13]This curvature is another expression of the shrinking of the $F(s,t)$ pattern. For the $\bar{p}-p$ case the curvature has the opposite sign, and $F(s,t)$ expands.

THEORY OF NONLEPTONIC HYPERON DECAY

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Several theories[1-11] of nonleptonic hyperon decay have been proposed, but only one, due to Pais,[1] is consistent with all of the observed relations[12-14]

$$\alpha_+ \approx \alpha_- \approx 0, \quad (1)$$

$$\alpha_\Lambda \approx -\alpha_0, \quad (2)$$

$$\alpha_\Xi \approx -\alpha_\Lambda, \quad (3)$$

between the asymmetry parameters in Σ, Λ, and Ξ decay. Pais uses the baryon doublet approximation[15] and Relation (1) to predict (2). Instead of (3), however, he obtains the weaker prediction:

$$|\alpha_\Xi| \approx |\alpha_\Lambda|. \quad (4)$$

Here we propose another theory in which (1), (2), and (3) are derived from time-reversal invariance, the $\Delta T = \frac{1}{2}$ rule,[12,16] and three doublet symmetries. Since the electromagnetic interaction satisfies two of these symmetries, we are able to make predictions about the weak electromagnetic decays $\Sigma^+ \to p+\gamma$ and $\Xi^- \to \Sigma^- + \gamma$.

In the doublet approximation,[1,15] the isotopic spin $\vec{T}$ is split into a doublet spin $\vec{I}$ and a K-spin $\vec{K}$,

$$\vec{T} = \vec{I} + \vec{K}, \quad (5)$$

and the baryons are grouped into four $I=\frac{1}{2}$ doublets:

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix}, \quad N_2 = \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix}, \quad N_3 = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix}, \quad N_4 = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix};$$

$$Y^0 = 2^{-1/2}(\Lambda^0 - \Sigma^0), \quad Z^0 = 2^{-1/2}(\Lambda^0 + \Sigma^0). \quad (6)$$

N_2 and N_3 form a $K=\frac{1}{2}$ doublet:

$$N_{23} = \begin{pmatrix} N_2 \\ N_3 \end{pmatrix}, \quad (7)$$

with $K_z = +\frac{1}{2}, -\frac{1}{2}$, respectively, and N_1 and N_4 have zero K spin. The assignments for π and K mesons are $(I=1, K=0)$ and $(I=0, K=\frac{1}{2})$, respectively. To supplement this scheme, we introduce a hypercharge spin $\vec{U}$, with z component

$$U_z = \tfrac{1}{2}(B+S), \quad (8)$$

and observe that N_1 and N_4 form a $U=\frac{1}{2}$ doublet