Walker, Bull. Am. Phys. Soc. 7, 281 (1962). The crucial sentence should be corrected (private communication, A. R. Erwin) to read "The ρ appears to have a double peak in Reaction (a)."

[7]J. Bernstein and G. Feinberg, Brookhaven National Laboratory Report BNL-6122, 1962 (to be published). The M_{+-} distribution was calculated from their Eq. (29). Initial pure states of ρ and ω are assumed.

[8]After correcting for background events, we obtain a cross section for ρ^0 production in the present experiment of approximately 3.1 mb. Our cross section for the process $\pi^-+p\rightarrow\pi^-+\pi^++2$ or more neutrals gives an upper limit of $3.8\pm.2$ mb to the cross section for ω production, $\pi^-+p\rightarrow\omega+n$. This cross section cannot be measured in this experiment, since for the three-pion decay there are two neutrals in the final state. Toohig et al., preprint (report to the International Conference on High Energy Physics at CERN, Geneva, Switzerland, 1962) give a cross section of 1.4 ± 0.4 mb for ω production in the charge-symmetric $\pi^++n\rightarrow\omega+p$. At our higher energy, a somewhat higher cross section would be expected, and so we have assumed the ρ- and ω-production cross sections to be equal. We have also assumed this equality to apply in the momentum transfer $\Delta^2<0.30\ (\mathrm{BeV}/c)^2$. The calculation is not sensitive to the exact ratio of the cross sections for ρ and ω production.

STUDY OF PION-PION INTERACTIONS FROM PION PRODUCTION BY PIONS*

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Recent experiments on pion production have shown the presence of a strong pion-pion interaction in the isotopic-spin-one state.[1-3] These experiments have established the existence of a resonance (the ρ meson) at an ω value of 750 MeV, where ω is the total energy of the two pions in their barycentric frame. The full width at half maximum of the resonance is approximately 130 MeV.[2] In these experiments, pions were scattered from protons in a hydrogen bubble chamber. Of the two groups of reactions,

$$\pi^{\pm}+p\rightarrow\pi^{\pm}+\pi^0+p \tag{a}$$

and

$$\pi^{\pm}+p\rightarrow\pi^{\pm}+\pi^++n, \tag{b}$$

(a) has received the most attention because a measurement of the recoil proton fixes ω^2 and Δ^2 (where Δ is the four-momentum transfer from the initial- to the final-state nucleon) for an event of the desired type. An extrapolation procedure for analyzing these experiments, suggested by Chew and Low,[4] involves the study of those collisions in which Δ^2 is small. It was postulated that, for small Δ^2, the one-pion exchange interaction would predominate. To date, several experimenters[3,5] have reported some success in analyzing the pion-pion interaction by the extrapolation method.

In the experiment reported here, we study Reactions (b) with an incident pion momentum of 1.75 BeV/c. We find evidence of a pion-pion interaction in the $(\pi^+\pi^-)$ system (which contains isotopic-spin components 0, 1, and 2), although the extrapolation method of analysis appears to fail. The $(\pi^+\pi^+)$ system (pure isotopic-spin 2) shows no comparable resonant state. There is strong evidence for processes other than the one-pion exchange even at relatively low momentum transfers.

Negative and positive pion beams were produced from a beryllium target placed in an almost field-free region of the Bevatron primary beam. They were formed into an external beam and focussed onto a 10-cm-thick liquid-hydrogen target. Detection apparatus consisted of plastic scintillation counters and their associated equipment. The counters were arranged in two groups. The main group consisted of 84 trapezoidal prisms arranged to fit on a section of the surface of a sphere of 160-cm radius with the hydrogen target at its center. Looking from the target at the counter array, one would see the elements grouped in a series of seven concentric rings. Each ring subtended a polar-angular interval of 8 deg. The array extended from 4 to 60 deg. The rings were divided into 12 elements, each with a 30-deg azimuthal angle. All the counters were 15 cm thick in order to be effective in detecting neutrons by recoil protons and inelastic reactions on carbon. Each counter element was coupled to a photomultiplier tube by a hollow aluminum light guide. The second group of counters was 1.0 cm thick and was designed to detect pions that emerged at scattering angles from 60 to 145 deg. This group of counters was placed close to the hydrogen target. The angular resolution for these counters was 20 deg in polar angle and 30 deg in azimuthal angle.

The spatial coordinates of the two pions and the neutron were determined within the resolutions mentioned above. The time of flight of the neutrons from the hydrogen target to the main counter array was measured by comparing the time of arrival of the pions ($\beta \approx 1$) with the time of arrival of the neutrons. The entire time-of-flight range was divided into seven intervals (time bins) with mean energies and rms widths as cited in Table I. Also listed are the corresponding values of the variable p^2 which is related to the neutron laboratory kinetic energy T_{2L} by the equation,[6]

$$p^2 = 2M_n T_{2L}.$$

The quantity p^2 is a relativistic invariant for the system and is connected to the four-momentum transfer squared, Δ^2, by

$$\Delta^2 = (M_p/M_n)p^2 - (M_n - M_p)^2.$$

The subsequent analysis of our experimental data will be in terms of p^2 rather than Δ^2.

Data were handled and processed electronically. Whenever a pion was scattered from the incident beam and a delayed pulse came within the neutron time-of-flight interval, the output of each counter was recorded on magnetic tape. The tapes thus produced were analyzed by using an IBM-709 computer with a program that selected only those events having two fast ($\beta \approx 1$) charged particles and one slow particle. To discriminate against background from neutral pions, a $\frac{1}{4}$-in.-thick sheet of lead was placed across the faces of both counter arrays. There was a high probability

Table I. Values of mean energy and mean $(p/\mu)^2$ for seven time-of-flight intervals.

Time bin	Mean energy T_{2L} and rms width (MeV)	Mean $(p/\mu)^2$ and rms width
τ_1	54 ± 14	5.2 ± 1.3
τ_2	38 ± 12	3.6 ± 1.1
τ_3	29 ± 10	2.8 ± 1.0
τ_4	23 ± 8	2.2 ± 0.8
τ_5	18 ± 7	1.7 ± 0.7
τ_6	13 ± 5	1.3 ± 0.5
τ_7	10 ± 5	1.0 ± 0.5

($\approx 95\%$) that at least one of the two gamma rays resulting from decay of a π^0 would be converted in either the lead or plastic scintillator and thus be rejected by the above selection criterion.

Each two-pion, one-neutron event was thus characterized by seven quantities: the polar angles θ and the azimuthal angles ϕ for the three particles, and the time of flight τ of the neutron. The beam-bending magnets determined the momentum of the incident pions. The efficiency for detecting neutrons was measured in a separate experiment at the Lawrence Radiation Laboratory's 184-inch cyclotron.[7] Five measurements are required to determine completely the kinematics. Since seven measurements were made, two consistency checks were available for a kinematic fit of the data to further discriminate against background.

Measurements were made with two target conditions—flask full and flask empty—and two delay conditions—normal and abnormal. To achieve the abnormal delay conditions, we added sufficient delay to the neutron channels so that any slow particles detected would have had to traverse the flight path with $\beta > 1$ to be correlated with the two charged pions. This condition gave a measurement of the purely accidental neutron background. In terms of the four possible target and delay conditions—full-normal, empty-normal, full-abnormal, and empty-abnormal—the net partial cross sections are given by

$$\frac{d^2\sigma}{d(p^2)d(\omega^2)} = \frac{d^2\sigma_{\rm FN}}{d(p^2)d(\omega^2)} - \frac{d^2\sigma_{\rm EN}}{d(p^2)d(\omega^2)} - \frac{d^2\sigma_{\rm FA}}{d(p^2)d(\omega^2)} + \frac{d^2\sigma_{\rm EA}}{d(p^2)d(\omega^2)}.$$

The neutron-counting efficiency was taken into account in the calculation of $d(p^2)d(\omega^2)$.

In Fig. 1(a) we present the results of our calculation of $d\sigma/d(\omega^2)$, which is obtained from the measured double distributions [Eq. (1)] by using

$$\frac{d\sigma}{d(\omega^2)} = \int_{p_{\rm min}^2(\omega^2)}^{p_{\rm max}^2} \frac{d^2\sigma}{d(p^2)d(\omega^2)}\, d(p^2),$$

where $p_{\rm max}^2 = 6\mu^2$ corresponds to the maximum neutron energy detected. For fixed ω^2 the lower limit $p_{\rm min}^2(\omega^2)$ is determined from the kinematics of the process. This distribution confirms the presence of a resonance in the $(\pi^+\pi^-)$ system at $\omega = 750$ MeV. Our curves show a full width at half maximum of about 220 MeV; however, a correction for the finite resolution of the apparatus leads to an estimate of 190 MeV for the width of the resonance. The $(\pi^+\pi^+)$ distribution is considerably smaller in magnitude and relatively flat.

If we assume that the contribution from the one-pion exchange process dominates in the region of low momentum transfers, we can obtain $\sigma_{\pi\pi}$ from the equation given by Chew and Low,[4]

$$d^2\sigma/d(p^2)d(\omega^2) = (f^2/\pi)(M_n/M_p)^2\frac{(p^2/\mu^2)}{(p^2+\mu^2)^2} \times[(\omega^4/4)-\omega^2\mu^2]^{1/2}(1/q_{1L})^2\sigma_{\pi\pi}(\omega^2), \qquad (1)$$

by an integration over p^2. Figure 1(b) gives the results of these integrations.

Our attempts to obtain the pion-pion cross section by the extrapolation method are shown in Fig. 2. The plotted extrapolation function, $F(p^2, \omega^2)$, is constructed through the equation,

$$F(p^2,\omega^2) = \frac{\pi}{f^2}\left(\frac{M_p}{M_n}\right)^2 \frac{q_{1L}{}^2(p^2+\mu^2)^2}{(\frac{1}{4}\omega^4-\mu^2\omega^2)^{1/2}}\frac{d^2\sigma}{d(\omega^2)d(p^2)}.$$

In the limit, as p^2/μ^2 approaches (-1), the quantity $F(p^2, \omega^2)$ should become $-\sigma_{\pi\pi}(\omega^2)$. It will be

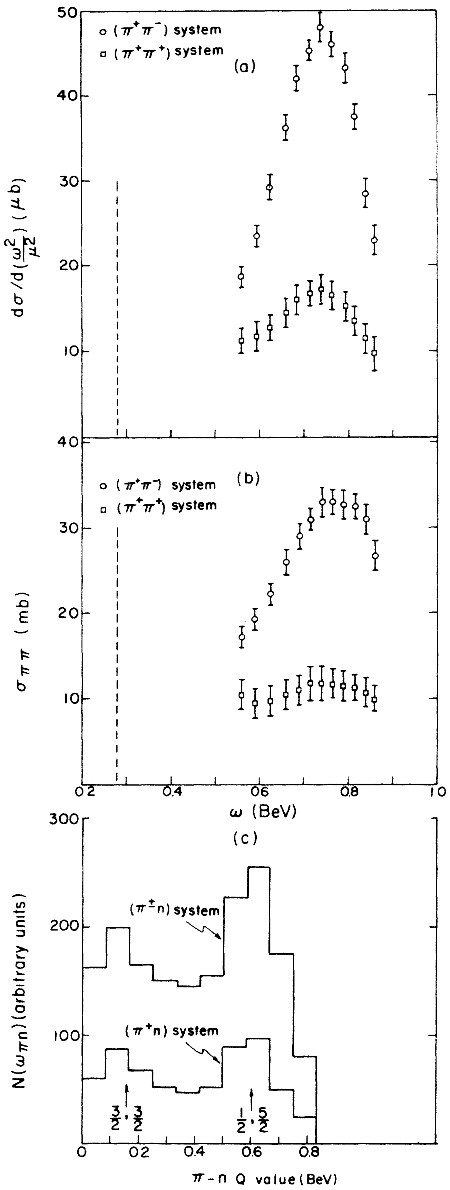

FIG. 1. (a) Differential cross section, $d\sigma/d(\omega^2/\mu^2)$, as a function of ω in BeV. (b) $\sigma_{\pi\pi}$ obtained by integration of Eq. (1) over p^2 in the physical region. (c) The distribution $N(\omega_{\pi n})$ as a function of the Q value in the pion-nucleon system. The positions of the $(\frac{3}{2}, \frac{3}{2})$ and $(\frac{1}{2}, \frac{5}{2})$ pion-nucleon resonances are indicated by arrows.

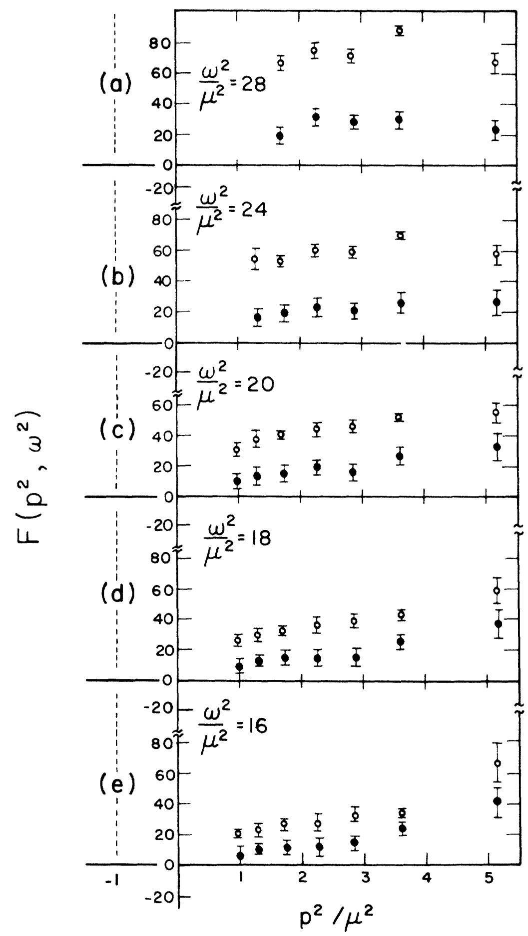

FIG. 2. Extrapolation plots of the function, $F(p^2, \omega^2)$, where the lower curves are for the $(\pi^+\pi^+)$ system and the upper for the $(\pi^+\pi^-)$ system.

seen from Fig. 2 that no simple extrapolation (i.e., linear or quadratic fits) can lead to consistent and reasonable pion-pion cross sections.

We have also analyzed the data for correlations in the two possible final-state pion-nucleon systems. Letting $\omega_{\pi n}$ be the invariant total energy of the neutron and one of the final-state pions in their own barycentric frame, we calculated $\omega_{\pi_1 n}$ and $\omega_{\pi_2 n}$ for each event. Since we cannot distinguish the charge of the final-state pions, each event was plotted twice to give the histogram $N(\omega_{\pi n})$ of Fig. 1(c). This distribution shows a marked peaking in the vicinity of the $(\frac{3}{2}, \frac{3}{2})$ and $(\frac{1}{2}, \frac{5}{2})$ pion-nucleon resonances, and may indicate that the difficulties encountered in the extrapolation method are due to pion-nucleon interactions.

Recently Yang and Treiman have proposed a method of testing the validity of the one-pion exchange model.[8] In the rest frame of the incident pion, the distribution of the plane defined by the final-state pion momenta, $\vec{p}_{\pi_1}$ and $\vec{p}_{\pi_2}$, must be isotropic about $\vec{q} = \vec{p}_n - \vec{p}_p$, if a single pion is exchanged. Our $(\pi^-\pi^+)$ data (see Fig. 3) show a marked anisotropy for the highest momenta measured. For lower momentum transfers, the distribution appears flat within the statistics. For the $(\pi^+\pi^+)$ system, the anisotropy is less pronounced and may not be statistically significant. These anisotropies indicate that the high momentum measurements should not be interpreted as solely due to one-pion exchange.

This experiment confirms the position and approximate width of the resonance in the two-pion system corresponding to the ρ meson. However, we feel that the result of the Yang-Treiman test, the peaking of $N(\omega_{\pi n})$ around the pion-nucleon resonances, may contribute to the failure of the extrapolation method and make it impossible to infer from our data any further details of the pion-pion interaction based on the one-pion exchange model of analysis.

We wish to thank Professor G. F. Chew for many valuable discussions of the theory of pion-pion interactions, and Professor Emilio Segrè for his continued interest and encouragement throughout the experiment.

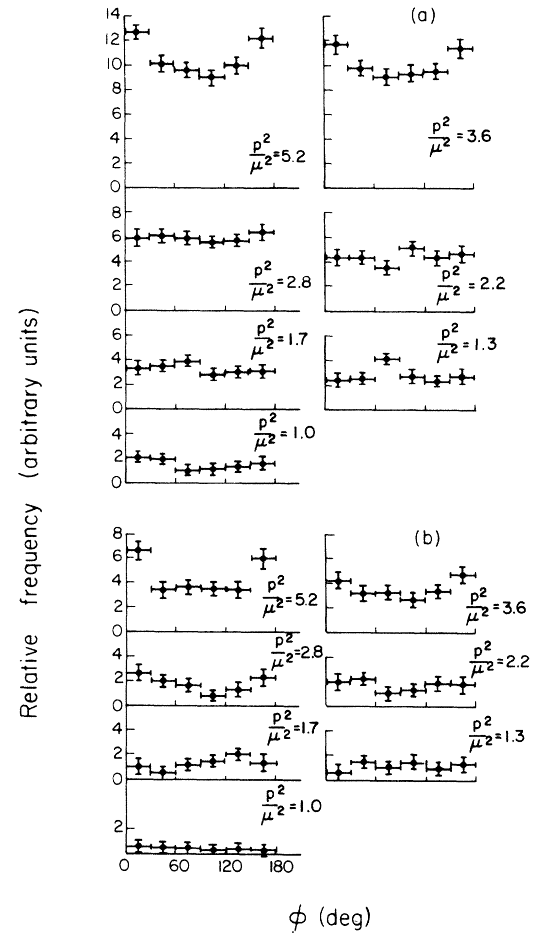

FIG. 3. Relative frequency of occurrence of the separation angle ϕ between the normals, $\vec{k}_{pn}$ and $\vec{k}_{\pi\pi}$, where $\vec{k}_{pn}$ is the unit normal to the proton-neutron scattering plane, and $\vec{k}_{\pi\pi}$ is the unit normal to the plane of the two final-state pions. All quantities are defined in the rest frame of the incident pion. (a) The $(\pi^+\pi^-n)$ system. (b) The $(\pi^+\pi^+n)$ system.

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[1]A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters 6, 628 (1961).

[2]E. Pickup, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters 7, 192 (1961).

[3]J. Anderson, V. Bang, P. Burke, D. Carmony, and N. Schmitz, Phys. Rev. Letters 6, 365 (1961).

[4]G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[5]D. Duane Carmony and Remy T. Van de Walle, Phys. Rev. Letters 8, 73 (1962).

[6]Throughout this article we use the notation of Chew and Low (reference 4): μ is the pion mass, M_p and M_n are the proton and neutron masses, q_{1L} is the laboratory momentum of the incident pion, and $f^2 = 0.08$ is the pion-nucleon coupling constant.

[7]Clyde E. Wiegand, Tom Elioff, William B. Johnson, Leonard B. Auerbach, Joseph Lach, and Thomas Ypsilantis, Rev. Sci. Instr. 33, 526 (1962).

[8]S. B. Treiman and C. N. Yang, Phys. Rev. Letters 8, 140 (1962).