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TUNNELING INTO SUPERCONDUCTORS*

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In a recent Letter, Cohen, Falicov, and Phillips[1] have discussed tunneling of electrons through a thin insulating layer between a normal and a superconducting metal on the basis of an effective Hamiltonian,

$$H = H_n + H_S + H_T, \tag{1}$$

where H_n and H_S are exact Hamiltonians for the normal and superconducting metals, respectively, and H_T is an operator which transfers electrons from one to the other:

$$H_T = \sum_{k,q,\sigma} (T_{kq} c_{k\sigma}^* c_{q\sigma} + T_{kq}^* c_{q\sigma}^* c_{k\sigma}). \tag{2}$$

Here k is a quantum number describing states in the normal metal, q refers to states in the superconductor, σ is the spin, and the c's are creation and destruction operators for normal quasi-particle states in both metals. By making use of the equations of motion, they derived an expression for the time rate of change of number of electrons in the superconductor $\langle N_S \rangle$ and thus the tunneling current. They find that the ratio of tunneling currents in superconducting and normal states depends only on the density of states in energy in the superconductor, as indicated by the experiments.[2]

We would like to discuss their derivation from a somewhat different point of view, which we feel brings out a little more clearly the connection with the semiconductor model of a superconductor and also with an earlier discussion of tunneling by the present author.[3] In the semiconductor model, one assumes that there is a set of normally occupied quasi-particle states below the gap and a set of normally unoccupied states above the gap, in one-to-one correspondence with those of the normal metal. At $T = 0°$K, states above are all unoccupied, those below occupied, but at a finite temperature electrons may be thermally excited to states above the gap, leaving holes in the normally occupied band. Electrons may be transferred from the normal to the superconducting metal into unoccupied states above the gap or into holes below the gap. Correspondingly, transfer in the reverse direction occurs from occupied states above the gap or from one of the filled states below the gap, leaving holes behind. It is the occupied states above the gap and the holes below which correspond to quasi-particle excitations of the superconductor.

What the author showed in his earlier Letter is that if there is a one-to-one correspondence between the quasi-particle excitations in normal and superconducting states, the only significant factor in the tunneling current is given by the density of states in energy. However, justification for the one-to-one correspondence and the definition of the quasi-particle states from microscopic theory was not given.

The quasi-particle states in a superconductor are usually defined by the Bogoliubov-Valatin transformation,[4]

$$\gamma_{q\uparrow}^* = u_q c_{q\uparrow}^* - v_q c_{-q\downarrow}, \tag{3a}$$

$$\gamma_{-q\downarrow}^* = u_q c_{-q\downarrow}^* + v_q c_{q\uparrow}, \tag{3b}$$

where $u_q^2 = 1 - v_q^2 = \frac{1}{2}(1 + \epsilon_q/E_q)$; $E_q = (\epsilon_q^2 + \Delta^2)^{1/2}$, and $-q$ indicates the time-reversal conjugate of q. These operators do not conserve particle number and are designed to operate on wave functions which are linear combinations of states with different numbers of particles. However, one can, as was done in the original paper of Cooper, Schrieffer, and the author, discuss states with definite particle number and define quasi-particle operators $\gamma_{q\uparrow}^* = u_q c_{q\uparrow}^* - v_q c_{q\downarrow} p^*$, which do conserve particle number.[5] The operator p^* adds a ground-state pair. Thus, operating by $\gamma_{q\uparrow}^*$ definitely adds an electron to the superconductor. Similarly, the operator $\gamma_{q\uparrow} = u_q c_{q\uparrow} - v_q c_{-q\downarrow}^* p$ removes an electron from the superconductor, with c_{-q}^* adding an electron in $-q\downarrow$ and p removing a ground-state pair to conserve particle number.

As pointed out by Cohen, Falicov, and Phillips, in the tunneling problem one must consider the degeneracy of states with $\epsilon_q = \epsilon_{q'}$, where q is above the Fermi surface, q' is below and q and q' are neighboring states such that $T_{kq} \simeq T_{kq'}$. The superconducting quasi-particle energy $E_q = E_{q'}$ is the same for $\gamma_{q\sigma}^*$ and $\gamma_{q'\sigma}^*$, and the appropriately defined quasi-particle states for the tunneling problem are linear combinations of these.

If one uses particle conserving operators, one may calculate the tunneling current by expressing H_T in terms of the $\gamma_{q\sigma}^*$ and then applying the golden rule. We introduce new quasi-particle operators defined by

$$\xi_{q\sigma} = u_q \gamma_{q\sigma} + u_{q'} \gamma_{q'\sigma}, \quad q > q_F; \tag{4a}$$

$$\xi_{q'\sigma} = v_q \gamma_{q\sigma} + v_{q'} \gamma_{q'\sigma}, \quad q' < q_F; \tag{4b}$$

so that

$$c_{q\uparrow} + c_{q'\uparrow} = \xi_{q\uparrow} + p\,\xi_{-q'\downarrow}^*, \tag{5a}$$

$$c_{-q\downarrow} + c_{-q'\downarrow} = \xi_{-q\downarrow} - p\,\xi_{q'\uparrow}^*. \tag{5b}$$

Thus H_T becomes

$$H_T = \sum_{k,\sigma,q>q_F} c_{k\sigma}^* \xi_{q\sigma} T_{kq} + \sum_{k,q'<q_F} (c_{k\uparrow}^* p\,\xi_{-q'\downarrow}^* - c_{k\downarrow}^* p\,\xi_{-q'\uparrow}^*) T_{kq'} + \text{comp. conj.} \tag{6}$$

We may interpret $\xi_{q\sigma}^*$ in the semiconductor model as adding an electron to an unoccupied state above the gap, $p^*\xi_{q'\sigma}$ as filling and thus destroying a hole in $q'\sigma$ below the gap. In both cases, an electron is definitely added to the superconductor. The matrix elements for these quasi-particle excitations are the same as those of the corresponding transitions in the normal state. Thus expression (6) for H_T gives justification for use of the semiconductor model.

We see that $\xi_{q\sigma}^*$ is a linear combination of two ways one may add an electron to a superconductor to create an excitation of energy $E_q = E_{q'}$. Starting from the ground state, one may add an electron to a configuration in which the pair $(q\uparrow, -q\downarrow)$ is unoccupied, creating an excitation $\gamma_{q\uparrow}^*$, or one may add a particle to $q'\uparrow$ in an unoccupied pair $(q'\uparrow, -q'\downarrow)$ below the sea, leaving a hole excitation in $-q'\downarrow$. This latter quasi-particle excitation is defined by $\gamma_{q'\uparrow}^*$. The excitation created by tunneling is the linear combination, $u_q\gamma_{q\uparrow}^* + u_{q'}\gamma_{q'\uparrow}^*$. If T_{kq} and $T_{kq'}$ were different, the quasi-particle operators would be defined differently and the square of the matrix element would be $|T_{uq}|^2 u_q^2 + |T_{kq'}|^2 u_{q'}^2$.

While use of the semiconductor model is, in general, justified, there is a significant difference. Suppose that electrons are added to states above the gap. In a semiconductor, one can get charge compensation only by adding a corresponding number of holes below the gap. In a superconductor, compensation can occur by supercurrent flow which changes the number of electrons in ground-state pairs. Further, in a superconductor there is not a sharp distinction between excitations corresponding to particles above and holes below the gap; one type of excitation gradually changes to the other as the Fermi surface is crossed. After the energy of the particles originally added above the sea becomes thermalized, there will be in the superconductor equal numbers of excitations which one would describe in the semiconductor model as conduction electrons and holes. In other words, an excitation can change by scattering alone from one corresponding to a particle above to one corresponding to a hole below the gap, with no sharp change in character of the excitation as the Fermi surface is crossed. Of course, the total number of quasi-particle excitations can be changed only by a recombination process of the type discussed by Schrieffer and Ginsberg.[6]

Note added in proof: In a recent note, Josephson[7] uses a somewhat similar formulation to discuss the possibility of superfluid flow across the tunneling region, in which no quasi-particles are created. However, as pointed out by the author (reference 3), pairing does not extend into the barrier, so that there can be no such superfluid flow.

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[5]J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957). The use of p^* in the definition of γ^* is a symbolic way of writing the wave functions given explicitly in Eqs. (3.3) to (3.5). The particle-conserving operators defined with p^* obey the same equations of motion as the Bogoliubov-Valatin operators, (3a) and (3b). In linearizing the equations of motion, a term c^*c^*c is written $\langle c^*c^*p\rangle p^*c$, instead of $\langle c^*c^*\rangle c$, as in the Bogoliubov theory. Since the quasi-particle operators are to $O(1/N)$ independent of the number of ground-state pairs, and since the commutator $[H-\mu N, p^*]=0$, one may treat p as a c number in most calculations.

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[7]B. J. Josephson, Physics Letters 1, 251 (1962).

OPTICAL CONSTANTS OF ALUMINUM IN VACUUM ULTRAVIOLET

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We have obtained the values of the frequency-dependent complex dielectric constant $\epsilon(\omega)$ of aluminum in the photon energy region of 12-17 eV from the characteristic electron energy loss spectra.[1] The values were obtained by utilizing the correlation between the intensity of the electron absorption spectrum of a substance with its optical properties[1,2] from recently obtained inelastic electron scattering data.[3] The importance of this method, applied for the first time in this investigation, is that in this photon energy region the usual optical methods of obtaining the frequency-dependent complex dielectric constant are fraught with many difficulties such as inconvenient light sources, vacuum problems, and surface effects, etc.[4] An advantage of the inelastic electron scattering experiments is that one can in most cases separate the surface effects from the bulk effects by varying the thickness of the material to be studied and/or varying the incident energy of the electrons. The electron energy absorption spectrum arising from the interaction of the incident electrons with the "bulk" of the medium is characterized by $\mathrm{Im}[1/\epsilon^*(\omega)]$, where $\epsilon^*(\omega)$ is the complex conjugate of the frequency-dependent $\epsilon(\omega)$. The Kronig-Kramers[5] dispersion relations were then applied to the electron absorption spectrum, which ranged from 11.1 to 18.3 eV, to calculate $\mathrm{Re}[1/\epsilon^*(\omega)]$ and, hence, $\epsilon(\omega)$ is obtained. The input data were scaled by means of the sum rule,[6] $\int_0^\infty \omega\,\mathrm{Im}[1/\epsilon^*(\omega)]d\omega=(\pi/2)\omega_p^2$, where the plasma frequency ω_p is related to the number of electrons in the medium available to interact with the incident electron beam. In the integration, we take the integrand to be vanishingly small beyond the recorded absorption region. An estimate[7] of the influence of neglecting absorption processes outside the recorded absorption spectrum was made by adding a hypothetical contribution,

$$\int_{\Delta E_0-\delta}^{\Delta E_0+\delta}\omega\,\mathrm{Im}\frac{1}{\epsilon^*(\omega)}d\omega=f\int_{\Delta E_1}^{\Delta E_2}\omega\,\mathrm{Im}\frac{1}{\epsilon^*(\omega)}d\omega,$$

to the sum rule, where ΔE_1 and ΔE_2 are the limits of the recorded absorption spectrum, and $f=5\%$ at $\Delta E_0=7$ eV and $f=10\%$ at $\Delta E_0=30$ eV. Figure 1 shows the calculated optical properties with and without the hypothetical contribution added separately to the Kronig-Kramers integration.

In this investigation, we found that the optical constants of aluminum can be approximated by a two-parameter Drude-like model (solid curve in Fig. 1) with N, the number of "free" electrons per atom, as 2.6 and τ, the relaxation time, as 1.1×10^{-15} sec. The density N was calculated from $\omega_p^2=4\pi e^2N/m$, where the plasma frequency ω_p was approximated as the value of ω at the inelastic electron absorption peak and m is the free electron mass. The relaxation time τ was obtained from the integrated half-width of the absorption line. These two parameters compare quite favorably with the values obtained from optical data[8] in the region between 2200 Å to 5 μ,