

ber group at Lawrence Radiation Laboratory and, in particular, the support of Professor Luis Alvarez. Without the cooperation of the Bevatron personnel under the direction of Dr. E. J. Lofgren this work could not have been carried out. We are indebted to Professor N. Byers for many stimulating discussions and for clarification of

questions regarding the Ξ^* spin. Last, but not least, we wish to acknowledge the painstaking and patient work of our scanners.

*Work supported in part by the U. S. Atomic Energy Commission.

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THEOREM ON LOCAL ACTION OF LEPTON CURRENTS*

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(Received June 26, 1962)

It is generally supposed that in any reaction involving a lepton pair the individual lepton fields have a common space-time point as their respective argument in the effective interaction density for the reaction. This idea is here called the local action of lepton currents. It may apply to decay reactions where both leptons appear in the final state as well as to scattering processes, one lepton in, one lepton out.

This absence of smearing out effects reflects the following dynamical assumptions. (1) There exist no other forces between (e, ν) or (μ, ν) than local weak couplings. (2) These local weak couplings are to be treated only to lowest order. Such assumptions can be entertained irrespective of whether or not intermediate bosons mediate weak interactions.

In recent times there has developed a growing interest in the question of higher order weak interaction effects, in particular as to their high-frequency behavior. One way in which such effects could show up is in violation of the local action of lepton currents. This is one of the reasons why it is interesting to know what local action implies in practice.

This problem was first studied¹ in connection with K_{l3} decays (l stands for either μ or e). The following result was obtained. Let $W(p, \cos\theta)$ be the decay distribution in the K rest system, where p is the π momentum and θ the angle between the pion and the neutrino. Then we have

$$K(p, \cos\theta)W(p, \cos\theta) = A(p) + B(p) \cos\theta + C(p) \cos^2\theta. \quad (1)$$

The kinematic factor K is a known function of p and $\cos\theta$. The functions A , B , and C , on the other hand, can only be determined from the complete dynamics of the problem which we do not master.

However, we do know that they depend on p only. The assumption of local action now says that the product KW is, in general, a quadratic function of $\cos\theta$.

Equation (1) was derived before it was even known that in ordinary β decay we have a $V-A$ interaction. It was, in fact, one of the aims of the investigation to find out what one can say about the structure functions, A , B , and C , depending on the choice among the usual five covariants. It turned out that there are criteria of practical interest for the case where one may neglect the lepton mass as in K_{e3} decay. In this special case these criteria have already led to useful experimental information.^{2,3} It should be emphasized, however, that the general form (1) is rigorous and does not depend on the value of the lepton mass.

Let us now gladly accept that K_{l3} decay goes via V coupling. Then, for finite lepton mass the number of structure functions is reduced to two independent ones.¹ It may be noted that further refined measurements on both K_{e3} and $K_{\mu3}$ could tell us something about local action. Even if deviations from local action were to exist, one would not expect them to show up sizably in these decays, however, as the momentum transfers are not very high.

It is often useful⁴ to take the lepton energy ω as a distribution variable instead of θ . We have

$$\omega = \frac{(m - E)^2 + p^2 + \mu^2 + 2p(m - E) \cos\theta}{2(p \cos\theta + m - E)}. \quad (2)$$

E is the pion energy, m and μ are the K and lepton mass, respectively. It follows from Eqs. (1) and (2) that

$$K'(p, \omega)W'(p, \omega) = a_0(p) + a_1(p)\omega + a_2(p)\omega^2. \quad (3)$$

Here W' is the distribution in p and ω , K' is again a known kinematic factor. The local action principle says that $K'W'$ is a quadratic function of ω , and this is again true in the presence of the lepton mass.

Recently the consequences of local action have been studied in connection with high-energy neutrino experiments.⁵⁻⁷ Differential cross sections have been expressed in terms of structure functions. It is the purpose of this note to derive by simpler means than used earlier a theorem which applies to leptonic decays as well as to (elastic and inelastic) neutrino scattering. It will be shown that with suitable definitions Eqs. (1) and (3) are valid in all these cases. The theorem is valid without neglect of the lepton mass and it casts the consequences of local action in a relatively simpler form for practical purposes.

Consider the prototype reactions

$$T \rightarrow F + l + \nu, \quad (4)$$

$$\nu + T \rightarrow F + l. \quad (5)$$

T is the decaying system in (4), the target in (5). We work in the T -rest system. l and ν denote the (anti)lepton and (anti)neutrino.⁸ F is the final state and consists, in general, of an assembly⁹ of strongly interacting particles. We use the following notation for the respective energy-momentum 4-vectors: $P_\lambda^0 = (0, im_0)$ for T , $P_\lambda^l = (\vec{p}^l, i\omega)$ for l , $P_\lambda^\nu = (\vec{p}^\nu, ip^\nu)$ for ν , and $P_\lambda = (\vec{p}, iE)$ for F . P_λ refers to the total energy-momentum of F . Whenever F is an assembly of particles, $P_\lambda^2 = -m^2$ is to be considered⁷ as an independent variable along with $p = |\vec{p}|$. We perform an averaging over all intrinsic variables of F , and also over the lepton spin. θ will denote the angle between \vec{p}_ν and \vec{p} . We will denote by W either the decay distribution of Eq. (4) or the differential cross section of Eq. (5).

Theorem. Apart from a given kinematic factor, the local action of the lepton current implies that W is a quadratic function in each of the three variables $\cos\theta$, ω , and p_ν ,

$$KW(p, m, \cos\theta) = \alpha_0(p, m) + \alpha_1(p, m) \cos\theta + \alpha_2(p, m) \cos^2\theta, \quad (6)$$

$$K'W'(p, m, \omega) = \beta_0(p, m) + \beta_1(p, m)\omega + \beta_2(p, m)\omega^2, \quad (7)$$

$$K''W''(p, m, p_\nu) = \gamma_0(p, m) + \gamma_1(p, m)p_\nu + \gamma_2(p, m)p_\nu^2. \quad (8)$$

Here the K 's denote the kinematic factors. The coefficients α , β , and γ depend on p and m only.

The proof of one of these relations implies that the other two hold as well, because of¹⁰

$$p_\nu = \pm \frac{m^2 + m_0^2 - \mu^2 - 2m_0E}{2(m_0 - E + p \cos\theta)}, \quad \omega = \mp p_\nu + m_0 - E. \quad (9)$$

We derive Eq. (7). The transition probabilities for the reactions (4) and (5) are found by taking the appropriate matrix element of the space integral of¹¹

$$J_\lambda(x) \cdot i\bar{\psi}_l(x) \gamma_\lambda (1 + \gamma_5) \psi_\nu(x) + \text{H.c.}, \quad (10)$$

where $J_\lambda(x)$ is the current density of the heavy (strongly interacting) particles. Average the absolute square of the matrix element over the lepton spin. Because the lepton current is local, it follows that this average is of the form $A_{\alpha\beta} P_\alpha^\nu P_\beta^l$. Apart from a kinematic factor $\omega^{-1} p_\nu^{-1}$, $A_{\alpha\beta}$ depends on the heavy-particle variables only, and after performing all averages described above $A_{\alpha\beta}$ depends on P and P^0 only. The dependence on the (l, ν) variables is therefore as follows: Either we get terms containing $P_\alpha^l P_\alpha^\nu$ which are independent of ω , or else we have to multiply each of the two factors

$$P_\alpha^0 P_\alpha^\nu = \pm m_0(E + \omega - m_0),$$

$$P_\alpha P_\alpha^\nu = \pm \frac{1}{2}(2m_0\omega + m^2 - m_0^2 - \mu^2), \quad (11)$$

with either of the factors

$$P_\alpha^0 P_\alpha^l = -m_0\omega,$$

$$P_\alpha P_\alpha^l = m_0(m_0 - \omega - E) + \frac{1}{2}(m^2 + \mu^2 - m_0^2).$$

Under any circumstance we therefore get a quadratic function in ω . The scalar coefficients still depend on the residual independent heavy-particle variables p and m .

Thus Eq. (7) has been verified. The extremely simple form of the relations (6)-(8) is due to the choice of coordinate system and of the most suitable variables in that system.

Remarks. (1) As has been noted earlier^{1,7} the proof of such relations does not depend on the validity of T invariance, nor⁷ on the validity of the $|\Delta\vec{I}| = 1$ rule for S -conserving weak processes. (2) There exists a deviation of local action due to the electromagnetic coupling of l with either T or

F or both.¹² This effect should be quite small especially at high energies. (3) Equations (6)-(8) also apply to the reactions $l+T \rightarrow F+\nu$ provided that θ is again the angle between \vec{p}_ν and \vec{p} . (4) If one considers the reactions (4) and (5) with specified lepton helicity, one obtains an expression with five structure functions.⁷ A subsequent summation over the lepton helicities (for finite lepton mass) then leads to Eqs. (6)-(8) by an amazing collapse of these five functions into three with the desired properties.¹³

The two-body reactions

$$\nu+n \rightarrow p+l^-, \quad (12)$$

$$\bar{\nu}+p \rightarrow n+l^+, \quad (13)$$

have been studied in some theoretical detail.⁷ In

the notation of reference 7 we have here

$$\begin{aligned} \langle p | J_\lambda | n \rangle \\ = (i/\sqrt{2}) \bar{u}_p [\gamma_\lambda (g_V + g_A \gamma_5) + i(n_\lambda + p_\lambda)(f_V + f_A \gamma_5) \\ + i(n_\lambda - p_\lambda)(h_V + h_A \gamma_5)] u_n. \end{aligned}$$

We have found that the relation (7) for the partial cross sections of the reactions (12) and (13) takes the following form:

$$d\sigma = [\beta_2(p)\omega^2 + \beta_1(p)\omega + \beta_0(p)] \frac{m d^3 p}{4\pi^2 \omega E p_\nu (dE_{\text{total}})}, \quad (14)$$

where the three structure functions β are given by¹⁴

$$\frac{1}{2}\beta_2 = |g_A|^2 + |g_V|^2 - 2mf_V|^2 + 2m(E-m)(|f_V|^2 + |f_A|^2), \quad (15)$$

$$\begin{aligned} \frac{1}{2}\beta_1 = (E-m)|g_A \pm g_V|^2 - (\mu^2/2m)(|g_A|^2 + |g_V|^2) \\ + 2m[E-m-\mu^2/2m][(E+m)|f_V|^2 + (E-m)|f_A|^2 - 2\text{Re}f_V^* g_V] \\ - \mu^2 \text{Re}[h_V^* g_V - (E+m)f_V^* h_V - (E-m)f_A^* h_A], \end{aligned} \quad (16)$$

$$\begin{aligned} \beta_0 = (E-m)(E-m-\mu^2/2m)|g_A \pm g_V|^2 + m(E-m+\mu^2/2m)(|g_A|^2 - |g_V|^2) \\ - [2m^2(E-m) + \frac{1}{2}\mu^2(3E-m) - \mu^4/4m][(E+m)|f_V|^2 + (E-m)|f_A|^2 - 2\text{Re}f_V^* g_V] \\ + \frac{1}{2}\mu^2(E-m+\mu^2/2m)[(E+m)|h_V|^2 + (E-m)|h_A|^2] \\ - \mu^2(E-m) \text{Re}[h_V^* g_V + h_A^* g_A - (E+m)f_V^* h_V - (E-m)f_A^* h_A] \\ + (\mu^4/2m) \text{Re}[h_V^* g_V - h_A^* g_A - (E+m)f_V^* h_V - (E-m)f_A^* h_A]. \end{aligned} \quad (17)$$

In Eqs. (16) and (17) the upper (lower) sign refers to the $\nu(\bar{\nu})$ reaction. We see from Eq. (15) that the difference $d\sigma_\nu - d\sigma_{\bar{\nu}}$ is (apart from a kinematic factor) a linear function in ω . This difference depends⁷ on the one structure function $\text{Re}g_A^* g_V$ only. The expressions for the β 's simplify if both T invariance and the $|\Delta I| = 1$ rule are valid, in which case $f_A = h_V = 0$.

Equation (8) takes the following form for the reactions (12) and (13). Replace in Eq. (14) the square bracket by $[\gamma_2(p)p_\nu^2 + \gamma_1(p)p_\nu + \gamma_0(p)]$ with

$$\begin{aligned} \gamma_2(p) = \beta_2(p), \quad \gamma_1(p) = 2(m-E)\beta_1(p) + \beta_0(p), \\ \gamma_0(p) = (m-E)^2\beta_2(p) + (m-E)\beta_1(p) + \beta_0(p). \end{aligned} \quad (18)$$

This form is well suited to perform averages over a (known) incident neutrino spectrum for fixed p .

I wish to thank Dr. G. Feinberg for discussions on lepton problems.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

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⁸For any combination satisfying lepton conservation.

⁹An example where this is the case for Eq. (4) is the K_{e4} decay.

¹⁰In Eqs. (9) and (11) the upper and lower signs refer to the reactions (4) and (5), respectively.

¹¹The validity of the relations (6)-(8) does not depend on the fact that the lepton covariant is $V-A$, but only on the circumstance that this covariant does not contain space-time derivatives.

¹²See, for example, reference 1, Fig. 1(b).

¹³In the notation of reference 7, what happens is the following: It is seen from Eqs. (59) and (60) that after summation over lepton spin, one has the three structure functions $\{A_+ + x^{-1}y^{-1}B_-\}[x+y+v_l(x-y)]$, $\{B_+ + x^{-1}y^{-1}A_-\} \times [x+y-v_l(x-y)]$, and $2v_l C$. It is then shown that these three expressions have a factor v_l in common, which goes into the "kinematic factor." Finally it is seen that the remaining three functions each are quotients of functions linear in ω which leads to Eq. (7).

¹⁴These expressions are in general not all positive, as is the case for the structure functions in reference 7.

$3\pi^0$ TO $\pi^+\pi^-\pi^0$ BRANCHING RATIO OF A 0^{-+} η MESON*

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(Received June 18, 1962)

The assignment of zero spin, odd spatial parity, and even G parity (0^{-+}) to the recently discovered η meson¹ has been suggested by several authors.² The purpose of this note is to consider, under certain assumptions which will be specified in the following discussion, evidence in favor of this assignment based on a comparison of the Fabri-Dalitz plot for the 3π decay modes of η and K^+ , K_2^0 , and to point out that the $3\pi^0$ to $\pi^+\pi^-\pi^0$ branching ratio of η is uniquely related to the π^0 energy spectrum in the decay $\eta \rightarrow \pi^+ + \pi^- + \pi^0$.

It was shown by Sawyer and Wali³ that as a consequence of the $\Delta T = \frac{1}{2}$ selection rule and invariance under CP , K^+ and K_2^0 decay into a final state of 3π 's in isotopic spin $T=1$ state. A single scalar function of three variables $f(\omega_1, \omega_2, \omega_3)$, where $\omega_1, \omega_2, \omega_3$ are the energies of the pions, then determines the decay amplitude into various charged and uncharged modes of the final state 3π 's. Further, without any dynamical assumptions regarding the specific form of the function $f(\omega_1, \omega_2, \omega_3)$, one can conclude that the $\pi^+(\pi^0)$ energy spectrum in the τ' -decay mode of the K^+ ($K^+ \rightarrow \pi^0 + \pi^0 + \pi^+$) should be the same as the π^0 (π^+ or π^-) spectrum in the decay $K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0$. Experimentally the data⁴ on $K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$ indicate that a power series expansion in $\omega_1, \omega_2, \omega_3$ for the decay amplitude with the retention of only the linear terms agrees reasonably well with the data. If such a "linear approximation"^{5,6} is assumed, then only one parameter is needed to fit the shapes of the various spectra in both K^+ and K_2^0 decays. The relevant point of interest to our discussion is the prediction that the number of decays $K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0$ as a function of π^0 energy should decrease with

the increase in the π^0 energy. This and similar predictions have been confirmed by experiments.⁷ Therefore, it is interesting to note that the π^0 spectrum in the charged mode of decay of η ($\eta \rightarrow \pi^+ + \pi^- + \pi^0$) has the same qualitative features as the π^0 in $K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0$. Since η is assigned even G parity, it decays into 3π 's by violating charge independence. If we consider the nonvanishing lowest order in the electromagnetic interaction, invariance under C requires that the final state of the 3π 's be in the isotopic spin-one state.

Given that both K and η decay into $T=1$ states of three pions, there is a very general dynamical assumption under which the spectra may be related. This is the assumption that in both cases the deviation of the matrix element from a constant is due to final-state pion-pion interactions. In either case one begins, in the absence of final-state interactions, with a constant matrix element and the completely symmetric $T=1$ state; the spectra induced by rescattering will differ only by the effect of the small K, η mass difference.⁸

Following reference 3, therefore, we write the Lorentz invariant decay amplitude, $\vec{A}(\omega_1, \omega_2, \omega_3)$, in the $T=1$ state, which satisfies Bose statistics for the pions as

$$\begin{aligned} \vec{A}(\omega_1, \omega_2, \omega_3) = & (\vec{\varphi}_1 \cdot \vec{\varphi}_2) \vec{\varphi}_3 f(\omega_1, \omega_2, \omega_3) \\ & + (\vec{\varphi}_2 \cdot \vec{\varphi}_3) \vec{\varphi}_1 f(\omega_2, \omega_3, \omega_1) \\ & + (\vec{\varphi}_3 \cdot \vec{\varphi}_1) \vec{\varphi}_2 f(\omega_3, \omega_1, \omega_2), \end{aligned} \quad (1)$$

where $f(x, y, z) = f(y, x, z)$. The $\vec{\varphi}$'s are the isotopic spin vectors of the pions. The matrix elements for the various decay modes can then be written down from (1). We assume an expansion