Gaussian Superconducting Fluctuations, Thermal Transport, and the Nernst Effect

Iddo Ussishkin, S. L. Sondhi, and David A. Huse
Department of Physics, Princeton University, Princeton, New Jersey 08544
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We calculate the contribution of superconducting fluctuations to thermal transport in the normal state, at low magnetic fields. We do so in the Gaussian approximation to their critical dynamics which is also the Aslamazov-Larkin approximation in the microscopics. Our results for the thermal conductivity tensor and the transverse thermoelectric response are new. The latter compare favorably with the data of Ong and collaborators on the Nernst effect in the cuprates.

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The study of fluctuations in superconductors [1] had a marked revival after the discovery of the cuprate superconductors. It was realized early that their short coherence lengths produce a large regime of strong fluctuation phenomena [2]. More recently, the "pseudogap" region of the cuprate phase diagram has come into sharp focus, and one line of thought attributes its features to strong superconducting fluctuations [3,4]. Among the most striking experimental findings that plausibly support this link are recent measurements by Ong's group [5] where the Nernst effect becomes sizable far above T_c in the underdoped regime of the high-temperature superconductors. As the Nernst effect is small in ordinary metals and large in the vortex state of superconductors, the *a priori* case for crediting superconducting fluctuations is strong.

In this Letter, we calculate the contribution of Gaussian superconducting fluctuations to the transverse thermoelectric response above T_c in the low magnetic field limit—the simplest computation that can be used for a quantitative comparison with these experimental results. This is already a nontrivial exercise as it requires a proper subtraction of *bulk* magnetization currents that enter naive expressions [6]. We also give results for the thermal conductivity tensor, thus presenting a full picture for Gaussian thermal transport in the system.

Our result for the transverse thermoelectric response depends only on the superconducting coherence length ξ, making it particularly suitable for comparison with experiment. We present a comparison with data from different samples of $La_{2-x}Sr_xCuO_4$ (LSCO). We find that the Gaussian Nernst effect, using the actual T_c, is quantitatively comparable to the measured signal in the optimally doped and overdoped samples. For the underdoped sample, the measured signal in the pseudogap region is larger and requires postulating a suppression of the actual T_c from the mean-field T_c, consistent with the superconducting fluctuations scenario, in order to achieve an understanding of its magnitude, as we illustrate by a self-consistent Hartree computation.

Before proceeding, we note that the traditional description of the Nernst effect is of thermally driven vortices producing a transverse voltage via phase slips. In a fluctuation regime, this description is convenient only if the vortices provide an effective parametrization, as we expect will be the case near a two-dimensional Kosterlitz-Thouless transition. At higher temperatures, where the vortices and antivortices are strongly overlapping, a different simplification involving Gaussian fluctuations becomes available and we will take recourse to that in the present paper.

Gaussian computations.—Our calculations can be carried out in two equivalent ways. First, they involve following Aslamazov and Larkin [7] and keeping the Feynman diagrams that now bear their names and arise in a Gaussian treatment of the quantum functional integral. Second, as was noted, e.g., in the case of paraconductivity [1], one can keep Gaussian fluctuations in a stochastic time-dependent Ginzburg-Landau equation (TDGL) that is designed to recover the equilibrium Ginzburg-Landau free energy. At this level, with the coefficients of the TDGL derived from BCS microscopics near the mean-field T_c, the two computations are identical. The TDGL, however, has a second interpretation—as a model of the critical dynamics traditionally assumed for superconductors (model A) [8] which should have a wider validity, with the coefficients no longer constrained by BCS microscopics. In the following, we will mostly use the TDGL description, while appealing to the microscopics to justify the form of the current operators [9].

The stochastic TDGL is defined by the Ginzburg-Landau free energy,

$$\mathcal{F} = \int d\mathbf{x}\left[a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{\hbar^2}{2m^*}\left|\left(\nabla - i\frac{e^*}{\hbar c}\mathbf{A}\right)\psi\right|^2\right], \tag{1}$$

where $a = a_0(T - T_c)$, and the time evolution

$$(\tau + i\tau')\left(\frac{\partial}{\partial t} + i\frac{e^*}{\hbar}\phi\right)\psi = -\frac{\delta\mathcal{F}}{\delta\psi^*} + \zeta. \tag{2}$$

Equation (2) describes the decay of the order parameter configuration towards its minimal free energy, interrupted by thermal fluctuations introduced through the white noise ζ with correlator

$$\langle \zeta^*(\mathbf{r}, t)\zeta(\mathbf{r}', t')\rangle = 2T\tau\,\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'). \quad (3)$$

The relaxation time of the order parameter is τ, and τ' leads to the precession of the order parameter during the relaxation process (in this convention, $\tau/\hbar$ and $\tau'/\hbar$ are dimensionless). In the presence of particle-hole symmetry $\tau' = 0$.

To calculate the response of the system, Eq. (2) is to be solved in the presence of the appropriate driving field (electric field or temperature gradient). Note that a temperature gradient appears in the TDGL both in the parameter a and in the noise correlator, Eq. (3). At the level of Gaussian fluctuations, the TDGL is a linear equation, with a solution of the form $\psi(\mathbf{r}, t) = \int d\mathbf{r}'dt'G(\mathbf{r}, t; \mathbf{r}', t')\zeta(\mathbf{r}', t')$, where G is the Green function of the TDGL. This expression may then be used to calculate the electric and heat currents,

$$\mathbf{j}^e = -i\frac{e^*\hbar}{2m^*}\left\langle \psi^*\left(\nabla - i\frac{e^*}{\hbar c}\mathbf{A}\right)\psi\right\rangle + \text{c.c.}, \quad (4)$$

$$\mathbf{j}^Q = -\frac{\hbar^2}{2m^*}\left\langle\left(\left(\frac{\partial}{\partial t} - i\frac{e^*}{\hbar}\phi\right)\psi^*\left(\nabla - \frac{ie^*}{\hbar c}\mathbf{A}\right)\psi\right\rangle + \text{c.c.} \quad (5)$$

Ullah and Dorsey [10] used precisely the same model.

Currents and fields.—As model A has no conservation laws, the traditional route to identifying currents is not available. While it is possible to construct hydrodynamic arguments that justify the forms in (4) and (5) [11], for our purposes it is sufficient to appeal to the microscopics of the Aslamazov-Larkin contribution, and show how the heat current arises from the appropriate vertex in the microscopic theory [12].

The microscopic theory is perhaps best recast in a functional integral approach. By means of the Hubbard-Stratonovich decoupling of the interaction, the expectation value of a current operator is expressed as an imaginary time functional integral over the pairing field,

$$\langle \mathbf{j}\rangle = \frac{\int D\psi D\bar{\psi}\langle \mathbf{j}\rangle_{\psi\bar{\psi}\phi}\, e^{-S_{\text{eff}}(\psi,\bar{\psi},\phi)}}{\int D\psi D\bar{\psi} e^{-S_{\text{eff}}(\psi,\bar{\psi},\phi)}}, \quad (6)$$

where $S_{\text{eff}}(\psi, \bar{\psi}, \phi)$ is the effective action for the pairing field ψ, in the presence of a potential ϕ, and $\langle \mathbf{j}\rangle_{\psi\bar{\psi}\phi}$ is the current of a noninteracting Fermi gas driven by electric and pairing fields. Ignoring the ϕ dependence of $\langle \mathbf{j}\rangle$ is the Aslamazov-Larkin approximation; the expectation value is the vertex in their diagram and its long wavelength, low frequency form is the TDGL current. For the electric current this was done in the original calculation [7]. Using the microscopic heat current operator, one obtains a relation between heat current and electric current vertices, $\mathbf{J}^Q = -\omega\mathbf{J}^e/2e$, independent of disorder. This corrects a factor of 2 in the calculation of Reizer and Sergeev [13]. The heat current in the TDGL, Eq. (5), is now immediate by this result.

To obtain a response to a temperature gradient in the microscopic theory, one calculates the response to a "gravitational field," coupled to the energy density in the Hamiltonian, and then uses an Einstein relation to obtain the response to a temperature gradient [14]. The microscopic calculation of the Aslamazov-Larkin diagrams leads to the same results obtained using the TDGL approach, Eqs. (1)–(5), where the response to a temperature gradient is obtained directly. Ignored here are the Maki-Thompson and density of state corrections to the normal state response [1,15].

The transport coefficients are defined by the standard linear response relations,

$$\begin{pmatrix}\mathbf{j}^e_{\text{tr}}\\ \mathbf{j}^Q_{\text{tr}}\end{pmatrix} = \begin{pmatrix}\sigma & \alpha\\ \tilde{\alpha} & \kappa\end{pmatrix}\begin{pmatrix}\mathbf{E}\\ -\nabla T\end{pmatrix}. \quad (7)$$

The off-diagonal thermoelectric tensors obey the Onsager relations $\tilde{\alpha} = T\alpha$. Below, we calculate the contribution of Gaussian superconducting fluctuations to the thermoelectric and the thermal conductivity tensors (α^{SC} and κ^{SC}). We do not consider here the electrical conductivity tensor σ^{SC}, which has been studied extensively [1].

Before proceeding, we note that particle-hole symmetry ($\tau' = 0$) implies that $\sigma^{\text{SC}}_{xy} = \alpha^{\text{SC}}_{xx} = \kappa^{\text{SC}}_{xy} = 0$. In the following, we break particle-hole symmetry only when considering these coefficients. In addition, we calculate the longitudinal coefficients in the absence of magnetic field B, and the transverse coefficients to linear order in B (valid for $B\xi^2 \ll hc/e^*$). The final results in both two and three dimensions are presented in Table I. The generalization of these results to a layered superconductor using a Lawrence-Doniach model is straightforward.

The thermoelectric response α^{SC}_{xx} may be calculated in two ways, either as the heat current response to an electric field or as the electric current response to a temperature gradient. In this way Onsager relations are verified. The result has a logarithmic divergence at T_c in two dimensions [10,13,16].

The calculation of the transverse thermoelectric coefficient α^{SC}_{xy} raises the issue of equilibrium magnetization currents. To illustrate this point, consider the result of calculating both the heat current response to an electric

TABLE I. Contribution of Gaussian fluctuations to thermoelectric and thermal transport coefficients. Here, $\ell_B = (\hbar c/eB)^{1/2}$ is the magnetic length, $\xi = \hbar/(2m^*a)^{1/2}$ is the Ginzburg-Landau coherence length, T_Λ is a cutoff temperature, and $e^* = -2e$ is used. Exact coefficients are given for diverging terms only (for nondiverging contributions, singular behavior is indicated without specifying prefactors).

	Two dimensions	Three dimensions
α^{SCa}_{xx}	$\frac{1}{2\pi}\frac{e}{\hbar}\frac{\tau'}{\tau}\ln(\frac{T_\Lambda}{T-T_c})$	$c_0 - c_1\sqrt{T-T_c}$
α^{SC}_{xy}	$\frac{1}{6\pi}\frac{e}{\hbar}\frac{\xi^2}{\ell_B^2} \propto \frac{1}{T-T_c}$	$\frac{1}{12\pi}\frac{e}{\hbar}\frac{\xi}{\ell_B^2} \propto \frac{1}{\sqrt{T-T_c}}$
κ^{SC}_{xx}	$c_0 - c_1(T-T_c)\ln(\frac{T_\Lambda}{T-T_c})$	$c_0 - c_1(T-T_c)^{3/2}$
κ^{SC}_{xy}	$\frac{1}{4\pi}\frac{eB}{m^*c}\frac{\hbar\tau'}{\tau^2}\ln(\frac{T_\Lambda}{T-T_c})$	$c_0 - c_1\sqrt{T-T_c}$

[a]Previously considered in Refs. [10,13,16].

field and the electric current response to a temperature gradient (in two dimensions),

$$\frac{j_y^Q}{E_x} = -\frac{1}{16\pi}\frac{e^{*2}BT}{m^* c a_0}\frac{1}{T-T_c}, \tag{8}$$

$$\frac{j_y^e}{(-\nabla T)_x} = \frac{1}{48\pi}\frac{e^{*2}B}{m^* c a_0}\left(\frac{2T}{(T-T_c)^2} - \frac{3}{T-T_c}\right). \tag{9}$$

Equation (8) agrees with the calculations of Ullah and Dorsey [10]. However, the two results, Eqs. (8) and (9), give a different answer for the transverse thermoelectric response, and are clearly incompatible with Onsager relations. The reason for this apparent discrepancy is that Eqs. (4) and (5) give the *total* currents in the system, containing contributions from both *transport* and *magnetization* currents. In the context of superconductivity, this issue was raised in Refs. [17,18], but was later ignored in works regarding fluctuation contributions [10,19,20]. Here we follow Cooper, Halperin, and Ruzin [6], who considered the general problem of magnetization currents in magnetothermoelectric transport in detail (they encountered it in the context of the quantum Hall effect).

In the presence of a magnetic field, the system has magnetization currents in equilibrium. These currents are divergence free, and as a consequence do not make any contribution to the net current flows that are measured in a transport experiment. The total currents [calculated by Eqs. (4) and (5)] are thus a sum of transport and magnetization parts,

$$\mathbf{j}^e = \mathbf{j}^e_{\rm tr} + \mathbf{j}^e_{\rm mag}, \qquad \mathbf{j}^Q = \mathbf{j}^Q_{\rm tr} + \mathbf{j}^Q_{\rm mag}. \tag{10}$$

In the present case, following the arguments of Ref. [6], the magnetization electric and heat currents which contribute to the total currents in Eqs. (8) and (9) are

$$\mathbf{j}^e_{\rm mag} = c\frac{\partial \mathbf{M}}{\partial T} \times (-\nabla T), \qquad \mathbf{j}^Q_{\rm mag} = c\mathbf{M} \times \mathbf{E}, \tag{11}$$

where $\mathbf{M}$ is the equilibrium magnetization. The fluctuation contribution to the magnetization is found either by thermodynamics [1] or by calculating the current flowing in the system in equilibrium in the presence of a varying system parameter (such as the magnetic field or T_c).

By subtracting the magnetization currents, Eq. (11), we obtain the result for $\alpha_{xy}^{\rm SC}$ (see Table I), which diverges as the conductivity, $\alpha_{xy}^{\rm SC} \propto \sigma_{xx}^{\rm SC} \propto (T-T_c)^{(d-4)/2}$. As expected, Onsager relations are recovered by this subtraction. We note that the correction due to magnetization currents is not small: In response to an electric field, the magnetization current accounts for two-thirds of the total heat current in the weak magnetic field limit. In response to a temperature gradient, the magnetization current is more singular than the transport current at T_c.

Finally, we consider the thermal conductivity tensor κ. The longitudinal thermal conductivity $\kappa_{xx}^{\rm SC}$ does not diverge in any dimension, although it is singular. (In Ref. [21] a diverging result was obtained due to the wrong form of the heat current vertex.) The transverse thermal conductivity $\kappa_{xy}^{\rm SC}$ has a logarithmic divergence at T_c in two dimensions (similar to $\alpha_{xx}^{\rm SC}$). In contrast with an electric field, a temperature gradient does not give rise to a magnetization heat current in the present case.

Cuprate data.—We now compare our results for $\alpha_{xy}^{\rm SC}$ with experiment. Consider the measurement of the Nernst effect: The sample is placed in a temperature gradient $(-\nabla T) \parallel \hat{\mathbf{x}}$ in the presence of a magnetic field $\mathbf{B} \parallel \hat{\mathbf{z}}$. The transverse electric field E_y is measured in the absence of any transport electric current. Imposing the condition $\mathbf{j}^e_{\rm tr} = 0$ in Eq. (7) gives the Nernst signal,

$$\nu = \frac{E_y}{(-\nabla T)B} = \frac{1}{B}\frac{\alpha_{xy}\sigma_{xx} - \alpha_{xx}\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}. \tag{12}$$

Taking the values for σ and α from the Gaussian contributions, we find that in this approximation the Nernst effect tends to a constant at T_c, $\nu(T_c) = \alpha_{xy}^{\rm SC}/\sigma_{xx}^{\rm SC}B$. However, for comparison with experiment, the fluctuation contributions, $\sigma^{\rm SC}$ and $\alpha^{\rm SC}$, should be added to the normal state contribution, σ^n and α^n. Moreover, we will consider temperatures which are not too close to T_c, such that $\sigma^{\rm SC} \ll \sigma^n \approx \sigma$. Equation (12) may then be written in the low magnetic field limit as

$$\nu \approx \nu^n + \frac{1}{B}\frac{\alpha_{xy}^{\rm SC}}{\sigma_{xx}}, \tag{13}$$

The normal state Nernst effect ν^n is generally small due to a cancellation between the two terms in Eq. (12).

Equation (13) is now used for a comparison with experiment in the low magnetic field limit, presented in Fig. 1 for data measured on three samples of LSCO [22]. The experimental data shows $\sigma_{xx}(\nu - \nu^n)$, where ν and σ_{xx} was measured at each temperature, while ν^n is deduced from an extrapolation from the high-temperature regime. As ν^n is very small in this material, this extrapolation is expected to introduce a relatively small error.

On the theoretical side, the result for $\alpha_{xy}^{\rm SC}$ (see Table I) is generalized for a layered superconductor using the Lawrence-Doniach model,

$$\alpha_{xy}^{\rm SC} = \frac{1}{6\pi}\frac{e}{\hbar}\frac{\xi_{ab}^2}{l_B^2 s}\frac{1}{\sqrt{1+(2\xi_c/s)^2}}. \tag{14}$$

Here, s is the interlayer spacing, and ξ_{ab} (ξ_c) is the coherence length in the direction parallel (perpendicular) to the planes. Note that the only fitting parameters are the coherence length $\xi_{ab}^{(0)}$ [$\xi_{ab} = \xi_{ab}^{(0)}\sqrt{T_c/(T-T_c)}$] and the anisotropy $\gamma = \xi_{ab}/\xi_c$. In Fig. 1, $\alpha_{xy}^{\rm SC}/B$ is plotted using $\xi_{ab}^{(0)} = 30$ Å and $\gamma = 20$ (cf., e.g., Ref. [23]).

As this comparison suggests, Gaussian superconducting fluctuations are sufficient to explain the magnitude of the observed signal above T_c in the low magnetic field limit in the optimally doped and overdoped samples. Two questions arise immediately: What is special about the cuprates and why is the fluctuation contribution to the Nernst signal dominant at such high temperatures? From

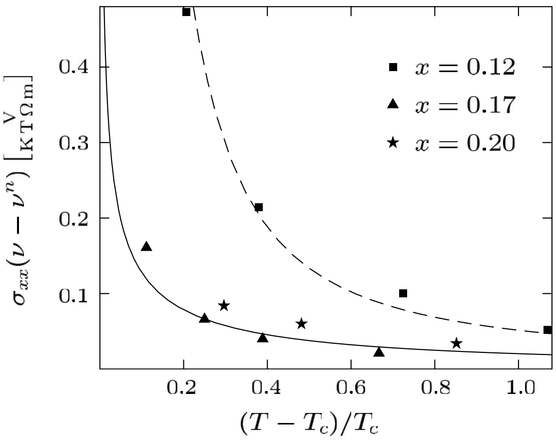

FIG. 1. Points are $\sigma_{xx}(\nu - \nu^n)$ for different samples of $La_{2-x}Sr_xCuO_4$ [22], with $x = 0.12$ (underdoped, $T_c = 29$ K), $x = 0.17$ (near optimal doping, $T_c = 36$ K), and $x = 0.2$ (overdoped, $T_c = 27$ K). The solid line is the theoretical value of α_{xy}^{SC}/B, using $\xi_{ab}^{(0)} = 30$ Å and an anisotropy of $\gamma = 20$. The dashed line is obtained using a Hartree approximation (see text).

our results, the answer to the first appears to lie in their anisotropy and in their smaller conductivity near T_c which together boost ν by 2 orders of magnitude from values we would predict for bulk Al or Nb, despite their larger coherence lengths. In this context, it would be interesting to study low temperature superconductors with high resistivities, such as the Nb films studied for fluctuation effects by Hsu and Kapitulnik [24], where our formula would indicate a Nernst signal comparable to optimally doped LSCO. In answer to the second question, we note that the Nernst signal is particularly suitable for a fluctuation measurement due to the small background signal in the normal state.

In the underdoped sample, the measured signal is larger and cannot be explained by Gaussian fluctuations if one uses the actual T_c. However, this is the pseudogap region and, in the interpretation considered in this paper, the actual T_c is suppressed from the mean-field transition temperature T_c^{MF} by non-Gaussian fluctuations and so a naive fit is not justified; indeed, there is presumably a small effect of this kind even at optimal doping which is unimportant in the region considered in our fit.

The increase in fluctuations with underdoping can be modeled by a growing quartic term in the Ginzburg-Landau functional as well as by increased two dimensionality while the increase in T_c^{MF} needs to be put in directly. To get a sense of what these would do, we treat the resulting problem via a self-consistent Hartree approximation (see, e.g., Ref. [10]), which should be valid to some extent below T_c^{MF}. This amounts to replacing a by the self-consistent solution of $\tilde{a} = a + b\langle|\psi|^2\rangle$. That this is plausible is demonstrated in Fig. 1, where we fit the data for the underdoped sample (using a two-dimensional Hartree approximation, with $T_c^{MF} = 49$ K, $T_\Lambda = 200$ K, $b/a_0 = 0.63\hbar^2/m^*$, and $\xi_{ab}^{(0)} = 30$ Å).

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