Radiative Corrections and Parity Nonconservation in Heavy Atoms

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The self-energy and the vertex radiative corrections to the effect of parity nonconservation in heavy atoms are calculated analytically in orders $Z\alpha^2$ and $Z^2\alpha^3 \ln(\lambda_C/r_0)$, where λ_C and r_0 are the Compton wavelength and the nuclear radius, respectively. The sum of the radiative corrections is -0.85% for Cs and -1.41% for Tl. Using these results, we have performed analysis of the experimental data on atomic parity nonconservation. The values obtained for the nuclear weak charge, $Q_W = -72.90(28)_{exp}(35)_{theor}$ for Cs, and $Q_W = -116.7(1.2)_{exp}(3.4)_{theor}$ for Tl, agree with predictions of the standard model. As an application of our approach, we have also calculated analytically the dependence of the Lamb shift on the finite size of the nucleus.

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Atomic parity nonconservation (PNC) has now been measured in bismuth [1], lead [2], thallium [3], and cesium [4]. Analysis of the data provides an important test of the standard electroweak model and imposes constraints on new physics beyond the model (see Ref. [5]). The analysis is based on the atomic many-body calculations for Tl, Pb, and Bi [6] and for Cs [7,8] (see also more recent Refs. [9,10]). The accuracy of both experiments and the theory is best for Cs. Therefore, this atom provides the most important information on the standard model in the low-energy regime. The analysis performed in Ref. [4] has indicated a deviation of the measured weak charge value from that predicted by the standard model by 2.5 standard deviations σ.

In the many-body calculations [6–8], the Coulomb interaction between electrons was taken into account, while the magnetic interaction was neglected. The contribution of the magnetic (Breit) electron-electron interaction was calculated in papers [11,12]. It proved to be much larger than a naive estimate, and it shifted the theoretical prediction for PNC in Cs.

Radiative correction to the nuclear weak charge due to renormalization from the scale of the W-boson mass down to zero momentum had been calculated a long time ago (see Ref. [13]). This correction is always included in the analysis of data. However, another important class of radiative corrections was omitted in the analysis of atomic PNC. This fact has been pointed out in an analysis [14] that demonstrated that there are corrections $\sim Z\alpha^2$ caused by the collective electric field of the nucleus. Here Z is the nuclear charge and α is the fine structure constant. The simplest correction of this type is due to the Uehling potential. It has been calculated numerically in Ref. [15] and analytically in our paper [16]. In that paper [16], we have also analyzed the general structure of the radiative corrections caused by the collective electric field. It has been shown that, as well as the usual perturbative parameter $Z\alpha$, there is an additional parameter $\ln(\lambda_C/r_0)$, where λ_C is the electron Compton wavelength and r_0 is the nuclear radius.

In this Letter, we present the results of calculations of the radiative corrections to the atomic PNC effect in orders $Z\alpha^2$ and $Z^2\alpha^3 \ln(\lambda_C/r_0)$. Details of calculations will be presented elsewhere [17]. Using our results, we reanalyze the experimental data. Agreement with the standard model is excellent. As an application of our approach, we have also calculated the dependence of the Lamb shift on the finite size of the nucleus. Agreement of our analytical formula with results of previous computations [18,19] is perfect.

The strong relativistic enhancement makes PNC radiative corrections different from previously considered radiative corrections to the hyperfine structure. The relativistic enhancement factor is proportional to $R \sim (\lambda_C/(Z\alpha r_0)^{2(1-\gamma)}$, where $\gamma = \sqrt{1-(Z\alpha)^2}$. The factor is $R \approx 3$ for Cs and $R \approx 9$ for Tl, Pb, and Bi [20]. The logarithmic enhancement of radiative corrections mentioned above is closely related to the existence of the factor R. The Feynman diagram for the leading contribution to the PNC matrix element between $p_{1/2}$ and $s_{1/2}$ states, as well as diagrams for radiative corrections, are shown in Fig. 1(a) and Figs. 1(b)–1(f), respectively. Strictly speaking, the diagrams in Fig. 1 are not quite well defined because they describe the matrix element between states with different energies. However, these are states of the external atomic electron, say $6s, 6p, 7s, \ldots$ states in

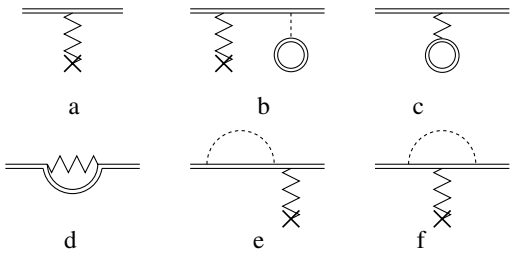

FIG. 1. (a) Leading contribution to the PNC matrix element. (b)–(f) Radiative corrections. The double line is the exact electron Green's function in the Coulomb field of the nucleus; the cross denotes the nucleus; the zigzag and the dashed lines denote Z boson and photon, respectively.

Cs, that have energies of the order of $m\alpha^2$. Therefore the uncertainty in the definition of the diagrams appears only in the order $\alpha^3(Z\alpha)$, which we do not consider in the present work. The diagram Fig. 1(b) corresponds to a modification of the electron wave function due to the vacuum polarization. This correction, calculated analytically in Ref. [16], reads

$$\delta_b = \alpha\left(\frac{1}{4}Z\alpha + \frac{2(Z\alpha)^2}{3\pi\gamma}[\ln^2(b\lambda_C/r_0) + f]\right), \quad (1)$$

where $b = \exp[1/(2\gamma) - C - 5/6]$, $C \approx 0.577$ is the Euler constant, and $f \sim 1$ is some smooth function of $Z\alpha$, independent of r_0. Hereafter, we denote by δ the relative value of the correction. So, Eq. (1) represents the ratio of diagrams Fig. 1(b) and 1(a).

The renormalization of the nuclear weak charge Q_{W} from the scale of the W-boson mass down to $q = 0$ was performed in Ref. [13]. However, as has been pointed out in Ref. [16], atomic experiments correspond to $q \sim 1/r_0 \sim 30$ MeV. The correction due to renormalization from $q = 0$ to $q = 1/r_0$ is described by diagrams Fig. 1(c) and 1(d). It has the form [16]

$$\delta_{cd} = \frac{4\alpha Z}{3\pi Q_{\mathrm{W}}}(1 - 4\sin^2\theta_{\mathrm{W}})\ln(\lambda_C/r_0) \sim -0.1\%, \quad (2)$$

where θ_{W} is the Weinberg angle, $\sin^2\theta_{\mathrm{W}} \approx 0.2230$ (see Ref. [5]).

Diagrams Figs. 1(e) and 1(f) correspond to the contributions of the electron self-energy operator and the vertex operator, respectively. Neither of these diagrams is invariant with respect to the gauge transformation of the electromagnetic field. However, their sum is gauge invariant. It has been demonstrated in Ref. [16] that the correction $\delta_e + \delta_f$ is of the form

$$\delta_{ef} = \delta_e + \delta_f = A\ln(b\lambda_C/r_0) + B, \quad (3)$$

where A and B are functions of $Z\alpha$, and the constant b is defined after Eq. (1). In the leading approximation in the parameter $Z\alpha$, the functions are $A = (a_2/\pi)\alpha(Z\alpha)^2$ and $B = a_1\alpha(Z\alpha)$. In Ref. [16], we have also obtained preliminary estimates for the coefficients a_1 and a_2. The estimates were based on an assumption of analogy between the polarization operator and the self-energy operator. This assumption and, hence, the preliminary estimates proved to be wrong. In the present work, we calculate the coefficients a_1 and a_2 exactly.

The simplest part of the work is the calculation of the $\alpha(Z\alpha)$ contribution to B. It is convenient to use the Fried-Yennie gauge [21] together with the effective operators approach [22] where the corrections under discussion coincide with those for the $\boldsymbol{\sigma}\cdot\mathbf{p}$ structure in the forward scattering amplitude (see Fig. 2). The result of our calculation for the nonlogarithmic term in Eq. (3) reads

$$B = -\alpha(Z\alpha)(\tfrac{7}{12} + 2\ln 2). \quad (4)$$

For calculation of the function A [the logarithmic part in Eq. (3)], we have used the Feynman gauge. There are two contributions to A, the self-energy contribution, A_{SE}, given by the diagram Fig. 1(e), and the vertex contribution A_V given by Fig. 1(f). It is convenient to represent the self-energy operator as a series in powers of the Coulomb field of the nucleus, $\sum = \sum_0 + \sum_1 + \sum_2 + \ldots$ (see Fig. 3), and to perform calculations in momentum space. Distances $r_0 \ll r \ll \lambda_C$, which give rise to the logarithmic terms, correspond to momenta $m \ll p \ll 1/r_0$, where m is the electron mass. Therefore, the mass and the electron binding energy can be neglected in electron propagators. It is necessary to make the ultraviolet regularization of the operators $\sum_0$ and $\sum_1$. Each of the operators depends on the parameter of this regularization. However, the sum $\sum_{01} = \sum_0 + \sum_1$ is independent of the regularization parameter because of the Ward identity. The operator $\sum_2$ does not require any regularization. Solution of the Dirac equation, taking account of the self-energy operator and subsequent calculation of the PNC matrix element, gives the following contributions of $\sum_{01}$ and $\sum_2$ to A_{SE}:

$$A_{01} = -\frac{\alpha(Z\alpha)^2}{\pi}, \qquad A_2 = -\frac{\alpha(Z\alpha)^2}{6\pi}(\pi^2 - 9),$$
$$A_{SE} = A_{01} + A_2 = -\frac{\alpha(Z\alpha)^2}{6\pi}(\pi^2 - 3). \quad (5)$$

Next, we consider the logarithmic contribution of the vertex operator described by Fig. 1(f). The coordinate representation is the most convenient for this part of the problem. We have used an integral representation for the Green's function derived in Ref. [23] exactly in $Z\alpha$. Keeping in mind the contact nature of the PNC interaction and the logarithmic accuracy of the calculation, one can demonstrate that only the angular momentum

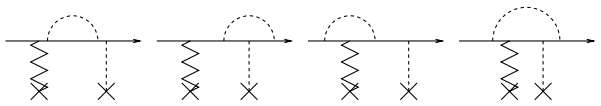

FIG. 2. $\alpha(Z\alpha)$ self-energy and vertex radiative corrections.

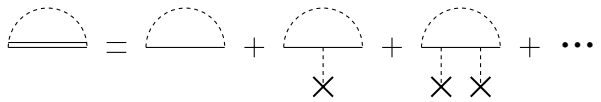

FIG. 3. The self-energy expanded in powers of the Coulomb field. The solid line is the free electron Green's function.

$j = 1/2$ is significant in the partial wave expansion of the electron Green's function in the external Coulomb field. A part of the vertex independent of $Z\alpha$ requires an ultraviolet regularization and, hence, is dependent of the regularization parameter. However, due to the Ward identity, the contribution corresponding to this part exactly cancels out the renormalization of the wave function (see, e.g., Ref. [24]). Another part is dependent on $Z\alpha$ and gives the following contribution to the logarithmic term in Eq. (3):

$$A_V = -\frac{\alpha(Z\alpha)^2}{\pi}\left(\frac{17}{4} - \frac{\pi^2}{3}\right). \tag{6}$$

Together with Eq. (5), this gives the final result for A:

$$A = -\frac{\alpha(Z\alpha)^2}{\pi}\left(\frac{15}{4} - \frac{\pi^2}{6}\right). \tag{7}$$

Thus, according to Eqs. (3), (4), and (7), the total relative correction to the PNC matrix element due to the self-energy and the vertex operators reads

$$\delta_{ef} = -\alpha\left[(Z\alpha)\left(\frac{7}{12} + 2\ln 2\right) + \frac{(Z\alpha)^2}{\pi}\left(\frac{15}{4} - \frac{\pi^2}{6}\right)\ln(b\lambda_C/r_0)\right]. \tag{8}$$

This correction versus the nuclear charge Z (the long-dashed line) is plotted in Fig. 4. The leading unaccounted contribution in (8) is of the order of $\sim Z^2\alpha^3/\pi$. For Cs ($Z = 55$) this gives 5%–10% uncertainty in δ_{ef}. The solid line in Fig. 4 shows the total radiative correction to the PNC effect that includes both δ_{ef} [Eq. (8)] and δ_b [Eq. (1)].

The radiative shift of the atomic energy levels (Lamb shift) depends on the finite nuclear size. This correction has a structure very similar to that of the PNC radiative correction because the effective sizes of the perturbation sources in both cases are much smaller than λ_C. The self-energy and the vertex corrections to the finite-nuclear-size effect (SEVFNS) for the $s_{1/2}$ state have previously been calculated in order $\alpha(Z\alpha)$ in Refs. [25,26]. The corrections for $1s_{1/2}$, $2s_{1/2}$, and $2p_{1/2}$ states have been calculated numerically, exactly in $Z\alpha$, in Refs. [18,27,28]. The structure of higher order $Z\alpha$ corrections and their logarithmic dependence on the nuclear size has not been understood. We have applied our approach to the SEVFNS problem and found the following expression for the

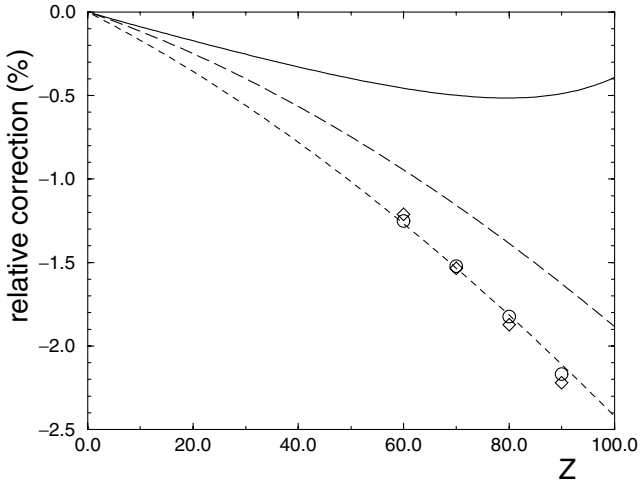

FIG. 4. Relative radiative corrections (%) for the PNC and for the finite-nuclear-size effects versus the nuclear charge Z. The long-dashed line shows the correction δ_{ef} given by Eq. (8) [Figs. 1(e) and 1(f)]. The solid line shows the total radiative correction to the PNC effect that includes both δ_{ef} and δ_b [Eq. (1), Fig. 1(b)]. The dashed line shows the correction Δ_s given by Eq. (9). Results of computations of Δ_s for $1s$ and $2s$ states [18,19] are shown by circles and diamonds, respectively.

$s_{1/2}$-state relative correction:

$$\Delta_s = -\alpha\left[(Z\alpha)\left(\frac{23}{4} - 4\ln 2\right) + \frac{(Z\alpha)^2}{\pi}\left(\frac{15}{4} - \frac{\pi^2}{6}\right)\ln(b\lambda_C/r_0)\right]. \tag{9}$$

The term linear in $Z\alpha$ agrees with results of Refs. [25,26]. The logarithmic term coincides with that in Eq. (8) for the PNC correction. Moreover, the logarithmic term in the SEVFNS correction Δ_p for the $p_{1/2}$ state is also equal to that in Eqs. (8) and (9). The reason for this equality is very simple. The logarithmic terms come from small distances ($r \ll \lambda_C$) where the electron mass can be neglected. When the mass is neglected, the relative matrix elements for the PNC radiative correction and for SEVFNS are equal. The correction Δ_s given by Eq. (9) is shown in Fig. 4 by the dashed line. Results of the computations [18,19] for $1s$ and $2s$ states are shown by circles and diamonds, respectively. The agreement is excellent up to $Z = 90$. This confirms the validity of our approach. The correction Δ_p has a more complex structure than that of δ_{ef} and Δ_s. Results of calculation of Δ_p will be published separately [29].

It has been recently suggested in Ref. [30] that the following "precise relation" $\delta_{ef} = (\Delta_s + \Delta_p)/2$ is valid. Values of the corrections obtained in our work do not agree with this relation. The "derivation" in Ref. [30] is based on the wrong assumption that there is a gauge in which the vertex contributions to δ_{ef}, Δ_s, and Δ_p vanish simultaneously. Although it is possible to set the vertex correction to each of these quantities to zero by choosing

an appropriate gauge, these are three different gauges. So the above relation is wrong.

Now we can perform a consistent analysis of the experimental data on atomic parity violation since all the contributions are known. In our analysis for Cs, we have included the theoretical value of the PNC amplitude from Refs. [7–10] as well as the -0.61% correction due to the Breit interaction [12], the -0.85% radiative correction calculated in the present work, the $+0.42\%$ vacuum polarization correction [15,16], the -0.2% neutron skin correction [31], the -0.08% correction due to the renormalization of Q_{W} from the atomic momentum transfer $q \sim 30$ MeV down to $q = 0$ [16], and the $+0.04\%$ contribution from the electron-electron weak interaction [16]. The theoretical uncertainty from the $Z^2\alpha^3/\pi$ term unaccounted in (8) is about 0.05%–0.1%. Using these results, we obtain from the data [4] the following value of the nuclear weak charge Q_W at zero momentum transfer

$$Cs: Q_{\mathrm{W}} = -72.90 \pm (0.28)_{\mathrm{exp}} \pm (0.35)_{\mathrm{theor}}. \qquad (10)$$

This value agrees with prediction of the standard model, $Q_{\mathrm{W}} = -73.09 \pm 0.03$ (see Ref. [5]). We have used the neutron skin correction in our analysis. However, in our opinion, the status of this correction is not quite clear because data on the neutron distribution used in Ref. [31] are not quite consistent with the data on neutron distributions obtained from proton scattering (see, e.g., Ref. [32]).

In the analysis for Tl, we have included the theoretical value of the PNC amplitude from Refs. [6], as well as the -0.88% correction due to the Breit interaction [33], the -1.41% radiative correction calculated in the present work, the $+0.90\%$ vacuum polarization correction [16], the -0.2% neutron skin correction, the -0.08% correction due to the renormalization of Q_{W} from the atomic momentum transfer $q \sim 30$ MeV down to $q = 0$ [16], and the $+0.01\%$ contribution from the electron-electron weak interaction [16]. Using these theoretical results, we obtain from the data [3] the following value of the nuclear weak charge Q_{W} at zero momentum transfer:

$$Tl: Q_{\mathrm{W}} = -116.7 \pm (1.2)_{\mathrm{exp}} \pm (3.4)_{\mathrm{theor}}. \qquad (11)$$

This agrees with prediction of the standard model, $Q_{\mathrm{W}} = -116.7 \pm 0.1$ (see Ref. [5]).

In conclusion, we have calculated analytically for the first time the $Z\alpha^2$ and $Z^2\alpha^3 \ln(\lambda_C/r_0)$ radiative corrections to the effect of atomic parity violation. This calculation has allowed us to perform a consistent analysis of the experimental data. Agreement with the standard model is within 0.5σ.

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