QED Radiative Corrections to Parity Nonconservation in Heavy Atoms

M. Yu. Kuchiev* and V. V. Flambaum†
School of Physics, University of New South Wales, Sydney 2052, Australia
(Received 14 June 2002; published 31 December 2002)

The self-energy and vertex QED radiative corrections [$\sim Z\alpha^2 f(Z\alpha)$] are shown to give a large negative contribution to the parity nonconserving (PNC) amplitude in heavy atoms. The correction $-0.73(20)\%$ found for the $6s$-$7s$ PNC amplitude in ^{133}Cs brings the experimental result for this transition into agreement with the standard model. The calculations are based on a new relation that expresses the radiative corrections to the PNC matrix element via corrections to the energy shifts induced by the finite nuclear size.

DOI: 10.1103/PhysRevLett.89.283002 PACS numbers: 32.80.Ys, 11.30.Er, 31.30.Jv

It has been discovered recently that there exists a consistent deviation of experimental data on parity nonconservation (PNC) in atoms from predictions of the standard model. This paper demonstrates that this contradiction is removed by the self-energy and vertex QED radiative corrections, which prove to be much larger than anticipated. The corrections are evaluated with the help of a new relation that expresses them via similar radiative corrections to the energy shifts induced by the finite nuclear size (FNS).

Experimental investigation of the $6s$-$7s$ PNC amplitude in ^{133}Cs initiated by Bouchiat and Bouchiat [1], was carried further by Gilbert and Wieman [2], and by Wood *et al.* [3], who reduced the error to 0.3%, sparking an interest in the atomic PNC calculations that are crucial for the analysis of the experimental data. Accurate previous calculations of Refs. [4,5] have recently been revisited by Kozlov *et al.* [6] and Dzuba *et al.* [7,8]. Bennett and Wieman [9] analyzed the theoretical data [4,5], comparing it with available experimental data on dipole amplitudes, polarizabilities, and hyperfine constants for Cs, and suggested that the theoretical error for the PNC amplitude should be reduced from 1% to 0.4%. It has been recognized recently that several, previously neglected phenomena contribute at the required level of accuracy. Derevianko [10] found that the Breit corrections give -0.6%, the value confirmed in [6,11]. Sushkov [12] pointed out that the radiative corrections may be comparable with the Breit corrections. Johnson *et al.* [13] demonstrated that, indeed, the QED vacuum polarization gives 0.4%, the value confirmed in [7,14,15].

Reference [9] indicates that there is a 2.3σ deviation of the weak charge Q_W extracted from the atomic PNC amplitude [3] from predictions of the standard model [16]. More recent works [8,13], in which the Breit corrections (-0.6%) and the QED vacuum polarization (0.4%) were included, give similar deviations 2.2σ and 2.0σ, respectively. We show that this contradiction is removed by the self-energy and vertex radiative corrections. The corrections of this type were considered previously by Marciano and Sirlin [17] and Lynn and Sandars [18] using the plane wave approximation that resulted in a small value $\sim 0.1\%$. The expectation of Ref. [13] was that the Coulomb field of the atomic nucleus should not produce any drastic effect on these corrections, which should remain small. This assessment was supported by Ref. [14], which presented results of calculations indicating that the self-energy corrections are small. Note, however, that a rough estimate of the self-energy contribution in Ref. [7] ($\sim -0.6\%$) demonstrated that it may be significant.

Let us show that there is an approximate relation that expresses the QED radiative corrections to the PNC matrix element via similar radiative corrections to the energy shifts of the atomic electron induced by the FNS. This relation can be presented as

$$\delta_{\mathrm{PNC},\,sp} = \tfrac{1}{2}(\delta_{\mathrm{FNS},\,s} + \delta_{\mathrm{FNS},\,p}), \qquad (1)$$

where $\delta_{\mathrm{PNC},\,sp}$ is the relative radiative correction to the PNC matrix element between $s_{1/2}$ and $p_{1/2}$ orbitals

$$\delta_{\mathrm{PNC},\,sp} = \frac{\langle \psi_{s,1/2} | H_{\mathrm{PNC}} | \psi_{p,1/2} \rangle^{\mathrm{rad}}}{\langle \psi_{s,1/2} | H_{\mathrm{PNC}} | \psi_{p,1/2} \rangle}. \qquad (2)$$

The energy difference between the considered opposite parity orbitals $\sim 1\,\mathrm{eV}$ is much lower than a typical excitation energy $\sim m = 0.5\,\mathrm{MeV}$ (relativistic units $\hbar = c = 1$ are used, if not stated otherwise) that governs the radiative corrections. We can therefore neglect this difference assuming that $E_{s,1/2} = E_{p,1/2}$. This assumption makes the correction $\langle \psi_{s,1/2} | H_{\mathrm{PNC}} | \psi_{p,1/2} \rangle^{\mathrm{rad}}$ gauge invariant. (To make this argument even stronger, one can use the Coulomb approximation for the atomic field and consider the degenerate $ns_{1/2}$ and $np_{1/2}$ levels.) Similarly, $\delta_{\mathrm{FNS},\,s}$ and $\delta_{\mathrm{FNS},\,p}$ are the relative radiative corrections to the FNS energy shifts $E_{\mathrm{FNS},\,s}$, $E_{\mathrm{FNS},\,p}$, for the chosen $s_{1/2}$ and $p_{1/2}$ electron states,

$$\delta_{\mathrm{FNS},\,l} = E^{\mathrm{rad}}_{\mathrm{FNS},\,l} / E_{\mathrm{FNS},\,l}, \quad l = s, p. \qquad (3)$$

The operator H_{PNC} in (2) describes the PNC part of the electron Hamiltonian induced by the Z-boson exchange,

 0031-9007/02/89(28)/283002(4)$20.00

$$H_{\mathrm{PNC}} = (2\sqrt{2})^{-1} G_{\mathrm{F}} Q_{\mathrm{W}} \rho(r) \gamma_5. \tag{4}$$

Here G_{F} and Q_{W} are the Fermi constant and the nuclear weak charge, and $\rho(r)$ is the nuclear density. The FNS energy shifts can be presented as matrix elements of the potential $V_{\mathrm{FNS}}(r)$, which describes the deviation of the nuclear potential from the pure Coulomb one, $E_{\mathrm{FNS},l} = \langle \psi_{l,1/2} | V_{\mathrm{FNS}} | \psi_{l,1/2} \rangle$, $l = s, p$. Equality (1) may be established for the sum of all QED radiative corrections [$\sim Z\alpha^2 f(Z\alpha)$], or specified for any gauge invariant class of them. We concentrate our attention on the self-energy and vertex corrections in the lowest order of the perturbation theory described by the Feynman diagrams in Fig. 1, calling them the e-line corrections, though the vacuum polarization is also briefly discussed below.

The intermediate electron states in diagrams of Fig. 1 are described by the propagator $\hat{G} = (\gamma_\mu p^\mu + \gamma_0 U - m)^{-1}$, with $p^\mu = (\epsilon, -i\mathbf{\nabla})$, where ϵ is the virtual electron energy, and $U = U(r)$ is the atomic potential. The external legs describe the wave functions $\psi_{s,1/2}(\mathbf{r})$ and $\psi_{p,1/2}(\mathbf{r})$ for the considered $s_{1/2}$ and $p_{1/2}$ levels. We will need below the following relations:

$$\gamma_5 \hat{G} \cong -\hat{G} \gamma_5, \tag{5}$$

$$\psi_{s,1/2}(\mathbf{r}) \cong C \gamma_5 \psi_{p,1/2}(\mathbf{r}), \tag{6}$$

that are valid for short distances $r \ll m^{-1}$. In this region, one can neglect the mass term in the electron propagator $\hat{G}$. Then, using the identity $\gamma_5 \gamma_\mu = -\gamma_\mu \gamma_5$, one derives (5). Equation (6) can be verified using the Coulomb approximation, which is valid at small distances, for the atomic wave functions. The constant C in (6), which depends on normalization conditions, is irrelevant for us because we are interested in relative quantities in Eq. (1).

Consider the diagrams of Fig. 1 for the PNC amplitude, assuming that their left and right legs describe the $s_{1/2}$ and $p_{1/2}$ states, respectively. The PNC interaction (4) is located at small distances $r_{\mathrm{n}} \ll m^{-1}$, where r_{n} is the nuclear radius. We can use this fact taking the γ_5 matrix from Eq. (4) and, relying on Eq. (6), transforming the $\psi_{p,1/2}(\mathbf{r})$ external wave function into the $\psi_{s,1/2}(\mathbf{r})$ wave function. This transformation shows that we can look at the diagram 1(b) as the one that describes the diagonal $s_{1/2} - s_{1/2}$ transition induced by an effective scalar potential $V_{\mathrm{PNC,\,eff}}(r) = (2\sqrt{2})^{-1} G_{\mathrm{F}} Q_{\mathrm{W}} \rho(r)$ that arises when we drop the γ_5 matrix in the pseudoscalar Hamiltonian in Eq. (4). Considering the same diagram 1(b) for the FNS energy shift for the $s_{1/2}$ level, one observes its close similarity with the PNC problem. In both cases the diagram 1(b) describes the $s_{1/2} - s_{1/2}$ transitions that are induced by the short-range scalar potentials. The only distinction comes from the fact that the potentials $V_{\mathrm{PNC,\,eff}}(r)$ and $V_{\mathrm{FNS}}(r)$ exhibit different shapes. However, this difference manifests itself only via a normalization factor that is canceled out when we consider relative quantities in Eq. (1). We see that the relative contribution of the diagram 1(b) for the PNC amplitude equals half the contribution of the total self-energy correction to $\delta_{\mathrm{FNS},\,s}$. The mentioned factor one-half appears because the total self-energy correction to the FNS energy shift is given by two diagrams 1(b) and 1(c), which are identical for the FNS problem. Similarly, we verify that the contribution of the diagram 1(c) to $\delta_{\mathrm{PNC},\,sp}$ equals half of the self-energy correction to $\delta_{\mathrm{FNS},\,p}$. This discussion shows that the diagrams 1(b) and 1(c) comply with Eq. (1).

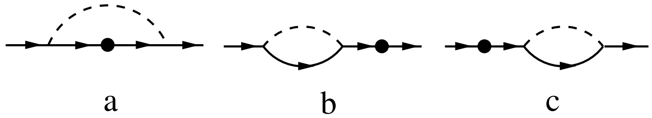

FIG. 1. The QED vertex (a) and self-energy (b),(c) corrections to the PNC matrix element and the FNS energy shifts.

Consider the vertex correction in Fig. 1(a). A range of distances r_{rad}, where the radiation processes take place, is proportional to the Compton radius, $r_{\mathrm{rad}} \sim m^{-1}$, and also depends on the chosen gauge for the electromagnetic field. (For example, the conventional length gauge favors larger distances, while velocity and acceleration gauges enhance contribution of smaller distances.) Therefore choosing the gauge appropriately we can force the radiative matrix element to originate from small distances $r_{\mathrm{rad}} < m^{-1}$. Using in this region Eq. (5), one commutes the γ_5 matrix from the weak interaction vertex (4) with the electron propagator and the electromagnetic vertex and brings it to the side, where Eq. (6) for the external wave function is used. This procedure shows that the relative contribution of the vertex diagram 1(a) to the PNC amplitude equals the relative contribution of the same diagram to the FNS energy shift. Deriving this conclusion, we again use the fact that the shape of the short-range potential is irrelevant for relative quantities. One can bring the γ_5 matrix in contact with either the right or the left external leg. Using both opportunities, one proves that the vertex diagram 1(a) gives the same contribution to the three different quantities: to the PNC amplitude, FNS energy shift for the $s_{1/2}$ state, and FNS energy shift for the $p_{1/2}$ state, all three measured in relative units. This shows that the vertex contributions satisfy Eq. (1) as well [19].

The discussion above shows that all three diagrams in Fig. 1 comply with Eq. (1), thus verifying that it holds for the e-line corrections. The derivation above used the gauge in which the radiation processes for the diagram 1(a) take place mostly at small separations $r_{\mathrm{rad}} < m^{-1}$. They should also take place outside the nucleus $r_{\mathrm{n}} < r_{\mathrm{rad}}$, as is necessary to justify our presumption that the shape of the potential inside the nucleus is irrelevant. From the last two inequalities, we find that our derivation relies on a parameter $mr_{\mathrm{n}} \sim 0.01$. This determines the magnitude, few percent, of the error of Eq. (1)

Similarly, we consider the contribution of the QED vacuum polarization. Reference [15] presents explicit variations of $s_{1/2}$ and $p_{1/2}$ wave functions at the origin induced by the vacuum polarization [see Eq. (43) of

Ref. [15]]. Using these wave functions to calculate corrections to the PNC matrix element and FNS energy shifts, we find that for the vacuum polarization Eq. (1) holds as well.

Note that we do not consider here radiative corrections of the order $\sim\alpha/\pi$ which appear in the plane wave approximation. These contributions have been included into the radiative corrections to the weak charge Q_W (and the renormalization of the charge and electron mass in the case of FNS energy shifts). Correspondingly, we subtract the contribution of the plane waves from Eq. (1), considering only the part of the corrections that depends on the atomic potential $\sim Z\alpha^2 f(Z\alpha)$. For heavy atoms, this subtlety is insignificant numerically because the considered Z-dependent part of the correction is bigger than the omitted Z-independent one, as we will see below.

Equation (1) presents the e-line corrections to the PNC matrix element, which are difficult to calculate, in terms of the corrections to the FNS energy shifts that have been well studied both numerically, by Johnson and Soff [20], Blundell [21], Cheng *et al.* [22], and Lindgren *et al.* [23], and analytically, by Pachucki [24] and Eides and Grotch [25]. Reference [22] presents the e-line radiative corrections to the FNS energy shifts for $1s_{1/2}$, $2s_{1/2}$, and $2p_{1/2}$ levels in hydrogenlike ions with atomic charges $Z = 60$, 70, 80, and 90. Equation (1) contains relative corrections; therefore we need to calculate the FNS energy shifts E_{FNS}. We did this by solving the Dirac equation with the conventional Fermi-type nuclear distribution $\rho(r) = \rho_0/\{1 + \exp[(r - a)/c]\}$. Parameters a, c were taken the same as in [22], namely $a = 0.523$ fm and c chosen to satisfy $R_{\mathrm{rms}} = 0.836A^{1/3} + 0.570$ fm. Using the results of [22] and this calculation, we obtained the relative radiative corrections shown in Fig. 2. In order to include the interesting case $Z = 55$ and to account for all values of $55 \le Z \le 90$, we used interpolation formulas presented in [22]. The relative corrections for the $1s$ and $2s$ levels are approximately the same size. This indicates that the radiative processes responsible for the correction take place at separations much smaller than the K-shell radius, $r \ll (Z\alpha m)^{-1}$, which is consistent with the assumption $r < m^{-1}$ above. For these separations, we can assume that, first, the screening of the nuclear Coulomb field in many-electron atoms does not produce any significant effect, and, second, the relative corrections does not depend on the atomic energy level because for small separations all atomic $ns_{1/2}$-wave functions exhibit similar behavior. These arguments remain valid for the $p_{1/2}$ states as well, permitting us to presume that the results shown in Fig. 2 for the $2s_{1/2}$ levels and $2p_{1/2}$ levels of hydrogenlike ions remain valid for $s_{1/2}$ and $p_{1/2}$ states of the valence electron in a many-electron atom. We obtain the e-line radiative corrections for the PNC matrix element using Eq. (1) that expresses them via the found corrections to the FNS energy shifts. The found PNC corrections, presented in Fig. 2 by the dotted line, are negative and large (much larger than the neglected Z-independent part of the corrections).

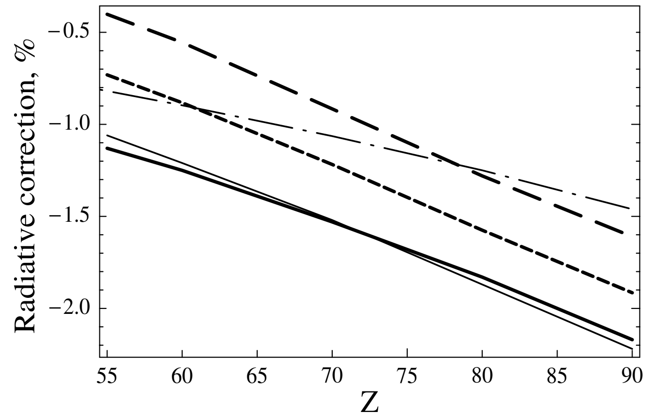

FIG. 2. The relative radiative corrections (in %) induced by the diagrams of Fig. 1. Thick, thin, and long-dashed lines—corrections for FNS energy shifts for $1s_{1/2}$, $2s_{1/2}$, and $2p_{1/2}$ levels extracted from [22]; short-dashed line—prediction of Eq. (1) for the PNC matrix element; dash-dotted line—correction to the HFI [27].

Let us discuss the implications for the $6s$-$7s$ PNC amplitude in ^{133}Cs. The standard model value for the nuclear weak charge for Cs [16] is

$$Q_W(^{133}\mathrm{Cs}) = -73.09 \pm (0.03). \tag{7}$$

Reference [8] refined previous calculations of Ref. [4] extracting from the experimental PNC amplitude of Ref. [3] the weak charge

$$Q_W(^{133}\mathrm{Cs}) = -72.18 \pm (0.29)_{\mathrm{expt}} \pm (0.36)_{\mathrm{theor}}, \tag{8}$$

with the theoretical error 0.5%. It is consistent with $Q_W(^{133}\mathrm{Cs}) = -72.21 \pm (0.28)_{\mathrm{expt}} \pm (0.34)_{\mathrm{theor}}$ that was adopted in [13] by taking the average of the results of Refs. [4–6], and accepting the theoretical error 0.4% proposed in [9]. The weak charge in Eq. (8) deviates from the standard model (7) by 2.0σ.

The e-line radiative correction derived from results presented in Fig. 2 is $-0.73 \pm (0.20)$%; the error reflects the uncertainty of the radiative corrections to the FNS energy shift for the $2p_{1/2}$ level in Cs $E^{\mathrm{rad}}_{\mathrm{FNS},2p,1/2} = -0.0001(1)$ eV [26]. Equation (8) combined with the e-line correction gives

$$Q_W(^{133}\mathrm{Cs}) = -72.71 \pm (0.29)_{\mathrm{expt}} \pm (0.39)_{\mathrm{theor}}, \tag{9}$$

which brings the PNC experimental amplitude of [3] within the limits of the standard model (7). For heavier atoms, the e-line corrections become larger, while the error diminishes. For the Tl atom, which presents another interesting case for applications, we find the e-line correction -1.61%.

Relations similar to (1) can be derived for any operator which is localized at distances smaller than the Compton radius. One can even try to apply it to the case of the hyperfine interaction (HFI), which has been thoroughly investigated previously, see, e.g., [27,28] and references therein, though the HFI has a long-range tail $\sim 1/r^3$ that presents an obstacle for our method. However, if convergence of the HFI matrix elements is fast, the relation $\delta_{\mathrm{FNS},\,s} \approx \delta'_{\mathrm{HFI},\,s}$ should hold. Here, $\delta'_{\mathrm{HFI},\,s}$ is the radiative correction to the HFI for s levels, the primed notation indicates that the Z-independent Schwinger term $\alpha/(2\pi)$ should be excluded (for heavy atoms this subtlety is not important). Figure 2 shows the e-line contribution to $\delta'_{\mathrm{HFI},\,s}$ that was extracted from [27] using interpolation for all considered values of Z. It agrees semiquantitatively with $\delta_{\mathrm{FNS},\,s}$, deviation is less than 33%. Thus, $\delta_{\mathrm{FNS},\,1s}$, $\delta_{\mathrm{FNS},\,2s}$, $\delta_{\mathrm{FNS},\,2p}$, and $\delta'_{\mathrm{HFI},\,s}$ all exhibit similar behavior; they all are negative and large regardless of the perturbative operator considered and quantum numbers of the wave functions involved, which is in line with the main argument of this paper (see also follow-ups mentioned in [29]).

In conclusion, large QED self-energy and vertex corrections to the parity nonconservation amplitude in heavy atoms reconcile the experimental results of Wood *et al.* [3] in Cs with the standard model.

This work was supported by the Australian Research Council. The authors are thankful to K.T. Cheng for his calculations of the self-energy corrections to the FNS energy shifts in Cs.

*Electronic address: kuchiev@newt.phys.unsw.edu.au
†Electronic address: flambaum@newt.phys.unsw.edu.au

[1] M. A. Bouchiat and C. Bouchiat, J. Phys. (Paris) I **35**, 899 (1974); **36**, 493 (1974).
[2] S. L. Gilbert and C. E. Wieman, Phys. Rev. A **34**, 792 (1986).
[3] C. S. Wood *et al.*, Science **275**, 1759 (1997).
[4] V. A. Dzuba, V. V. Flambaum, and O. P. Sushkov, Phys. Lett. A **141**, 147 (1989).
[5] S. A. Blundell, W. R. Johnson, and J. Sapirstein, Phys. Rev. Lett. **65**, 1411 (1990); S. A. Blundell, J. Sapirstein, and W. R. Johnson, Phys. Rev. D **45**, 1602 (1992).
[6] M. G. Kozlov, S. G. Porsev, and I. I. Tupitsyn, Phys. Rev. Lett. **86**, 3260 (2001).
[7] V. A. Dzuba, V. V. Flambaum, and J. S. M. Ginges, hep-ph/0111019.
[8] V. A. Dzuba, V. V. Flambaum, and J. S. M. Ginges, hep-ph/0204134.
[9] S. C. Bennett and C. E. Wieman, Phys. Rev. Lett. **82**, 2484 (1999);**82**, 4153 (1999); **83**, 889 (1999).
[10] A. Derevianko, Phys. Rev. Lett. **85**, 1618 (2000).
[11] V. A. Dzuba, C. Harabati, W. R. Johnson, and M. S. Safronova, Phys. Rev. A **63**, 044103 (2001).
[12] O. P. Sushkov, Phys. Rev. A **63**, 042504 (2001).
[13] W. R. Johnson, I. Bednyakov, and G. Soff, Phys. Rev. Lett. **87**, 233001 (2001).
[14] A. I. Milstein and O. P. Sushkov, Phys. Rev. A **66**, 022108 (2002); hep-ph/0109257.
[15] M. Yu. Kuchiev and V. V. Flambaum, J. Phys. B **35**, 4101 (2002); hep-ph/0205012.
[16] D. E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[17] W. J. Marciano and A. Sirlin, Phys. Rev. D **27**, 552 (1983).
[18] B. W. Lynn and P. G. H. Sandars, J. Phys. B **27**, 1469 (1994).
[19] Another argument in favor of validity of Eq. (1) for the vertex contribution can be derived from the fact that the self-energy contribution satisfies this equation. To preserve the gauge invariance, the vertex contribution should satisfy this equation also. Moreover, the vertex contribution to Eq. (1) can be made small by using gauge transformation. For example, according to [14], the diagrams Figs. 1(b) and 1(c) are enhanced in the Landau gauge by the large logarithm $\ln mr_n$, while the vertex diagram Fig. 1(a) is not.
[20] W. R. Johnson and G. Soff, At. Data Nucl. Data Tables **33**, 405 (1985).
[21] S. A. Blundell, Phys. Rev. A **46**, 3762 (1992).
[22] K. T. Cheng, W. R. Johnson, and J. Sapirstein, Phys. Rev. A **47**, 1817 (1993).
[23] I. Lindgren, H. Persson, S. Salomonson, and A. Ynnerman, Phys. Rev. A **47**, R4555 (1993).
[24] K. Pachucki, Phys. Rev. A **48**, 120 (1993).
[25] M. I. Eides and H. Grotch, Phys. Rev. A **56**, R2507 (1997).
[26] K. T. Cheng (private communication).
[27] S. A. Blundell, K. T. Cheng, and J. Sapirstein, Phys. Rev. A **55**, 1857 (1997).
[28] P. Sunnergren *et al.*, Phys. Rev. A **58**, 1055 (1998).
[29] A preprint of the present paper [30] was followed by Refs. [31,32]. For the Cs atom, these works predict the e-line correction -0.9% and -0.85%, respectively, that agrees with $-0.7(2)\%$ of the present paper. The agreement persists for higher Z [33]. The small deviation $\sim 14\%$ of [32] from results of the present work and [31] for large Z may be attributed to the higher-order terms of the $Z\alpha$ expansion (omitted in [32]). Using them as fitting parameters, we verified that all data reported comply with Eq. (1).
[30] M. Yu. Kuchiev and V. V. Flambaum, hep-ph/0206124.
[31] M. Yu. Kuchiev, J. Phys. B **35**, L503 (2002).
[32] A. I. Milstein, O. P. Sushkov, and I. S. Terekhov, Phys. Rev. Lett. **89**, 283003 (2002).
[33] M. Yu. Kuchiev and V. V. Flambaum, hep-ph/0209052.