Scattering of Cosmic Rays by Magnetohydrodynamic Interstellar Turbulence

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Recent advances in understanding of magnetohydrodynamic (MHD) turbulence call for substantial revisions in our understanding of cosmic ray transport. We use recently obtained scalings of MHD modes to calculate the scattering frequency for cosmic rays. We consider gyroresonance with MHD modes (Alfvénic, slow, and fast) and transit-time damping by fast modes. We conclude that the gyroresonance with fast modes is the dominant contribution to cosmic ray scattering for the typical interstellar conditions. In contrast to earlier studies, we find that Alfvénic and slow modes are inefficient because they are far from the isotropy usually assumed.

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Introduction.—The propagation of cosmic rays (CRs) is affected by their interaction with magnetic field. This field is turbulent and therefore, the resonant interaction of cosmic rays with MHD turbulence has been discussed by many authors as the principal mechanism to scatter and isotropize cosmic rays [1]. Although cosmic ray diffusion can happen while cosmic rays follow wandering magnetic fields [2], the acceleration of cosmic rays requires efficient scattering.

While most investigations are restricted to Alfvén waves propagating along an external magnetic field (the so-called slab model of Alfvénic turbulence), obliquely propagating MHD waves have been included in [3] and later studies [4,5]. The problem, however, is that the Alfvénic turbulence considered in their studies is isotropic turbulence, and this is contrary to the modern understanding of MHD turbulence ([6], see [7] for a review and references therein).

A recent study [8] found a strong dependence of scattering on turbulence anisotropy. Therefore the calculations of CR scattering must be done using a realistic MHD turbulence model. An important attempt in this direction was carried out in [9]. There Alfvén modes were treated in the spirit of Goldreich-Shridhar [6] (1995, henceforth GS95) model of incompressible turbulence and marginal scattering was obtained. However, a more accurate description is now available [10] and thus there is a need to revisit the problem. Moreover, [9] did not consider compressible modes, while we show below that these modes provide the dominant contribution to the scattering.

MHD Statistic.—MHD perturbations can be decomposed into Alfvénic, slow, and fast modes (see [11]). Alfvénic turbulence is considered by many authors as the default model of interstellar magnetic turbulence. This is partially motivated by the fact that unlike compressible modes, the Alfvén ones are essentially free of damping in fully ionized medium (see [11,12]).

Unlike hydrodynamic turbulence, Alfvénic turbulence is anisotropic, with eddies elongated along the magnetic field. This happens because it is easier to mix the magnetic field lines perpendicular to the direction of the magnetic field rather than to bend them. The GS95 model describes *incompressible* Alfvénic turbulence, which formally means that plasma $\beta = P_{\rm gas}/P_{\rm mag} = 2C_s^2/V_A^2$ is infinity. It was first conjectured in [13] that GS95 scaling should be approximately true for moderately compressible plasma. For low β plasma Cho and Lazarian [14] (henceforth CL02) showed that the coupling of Alfvénic and compressible modes is weak and that the Alfvénic modes follow the GS95 spectrum [15]. This is consistent with the analysis of observational data [16,17]. In what follows, we consider both Alfvén modes and compressible modes and use the description of those modes obtained in CL02 to study CR scattering by MHD turbulence in a medium with energy injection scale $L = 100$ pc, density $n = 10^{-4}$ cm^{-3} temperature $T = 2 \times 10^6$ K. Recent observations [18] suggest that matter in the galactic halos is magnetic-dominant, corresponding to low β medium, here we choose $\beta \simeq 0.1$. The injection length scale is important as Alfvénic turbulence exhibits scale-dependent anisotropy that increases with the decrease of the scale.

We describe MHD turbulence statistics by correlation functions. Using the notations from [9], we get the expressions for the correlation tensors in Fourier space

$$\begin{aligned}\langle B_i(\mathbf{k})B_j^*(\mathbf{k}')\rangle/B_0^2 &= \delta(\mathbf{k}-\mathbf{k}')M_{ij}(\mathbf{k}),\\ \langle v_i(\mathbf{k})B_j^*(\mathbf{k}')\rangle/V_AB_0 &= \delta(\mathbf{k}-\mathbf{k}')C_{ij}(\mathbf{k}),\\ \langle v_i(\mathbf{k})v_j^*(\mathbf{k}')\rangle/V_A^2 &= \delta(\mathbf{k}-\mathbf{k}')K_{ij}(\mathbf{k}),\end{aligned} \tag{1}$$

where $B_{\alpha,\beta}$ is the magnetic field fluctuations.

The isotropic tensor usually used in the literature is

$$K_{ij}(\mathbf{k}) = C_0\{\delta_{ij} - k_ik_j/k^2\}k^{-11/3}. \tag{2}$$

The normalization constant C_0 can be obtained if the energy input at the scale L is defined. Assuming equipartition, the kinetic energy density $\epsilon_k = \int dk^3 \sum_{i=1}^3 K_{ii}\rho V_A^2/2 \sim B_0^2/8\pi$, we get $C_0 = L^{-2/3}/12\pi$. The analytical fit to the anisotropic tensor for Alfvén modes, obtained in [10] is

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$$K_{ij}(\mathbf{k}) = C_a I_{ij} k_\perp^{-10/3} \exp(-L^{1/3} k_\| / k_\perp^{2/3}), \tag{3}$$

where $I_{ij} = \{\delta_{ij} - k_i k_j / k_\perp^2\}$ is a 2D matrix in x-y plane, $k_\|$ is the wave vector along the local mean magnetic field (see [7]), $k_\perp$ is the wave vector perpendicular to the magnetic field and the normalization constant $C_a = L^{-1/3}/6\pi$. The tensors in [9] used step function instead of the exponent. We assume that for the Alfvén modes $M_{ij} = K_{ij}$, $C_{ij} = \sigma M_{ij}$ where the fractional helicity $-1 < \sigma < 1$ is independent of $\mathbf{k}$ [9].

Numerical calculations in CL02 demonstrated that slow modes follow GS95 scalings. The correlation tensors for slow modes in low β plasma are [19]

$$\begin{bmatrix} M_{ij}(\mathbf{k}) \\ C_{ij}(\mathbf{k}) \\ K_{ij}(\mathbf{k}) \end{bmatrix} = \frac{C_a \beta^2}{16} \sin^2(2\theta) J_{ij} k_\perp^{-10/3} \times \exp\left(-\frac{L^{1/3} k_\|}{k_\perp^{2/3}}\right) \begin{bmatrix} \cos^2\theta \\ \sigma \cos\theta \\ 1 \end{bmatrix},$$

where $\cos\theta = k_\|/k$, $J_{ij} = k_i k_j / k_\perp^2$ is also a 2D tensor in the x-y plane.

According to CL02, fast modes are isotropic and have one dimensional spectrum $E(k) \propto k^{-3/2}$. In low β medium, the velocity fluctuations are always perpendicular to $\mathbf{B}_0$ for all $\mathbf{k}$, while the magnetic fluctuations are perpendicular to $\mathbf{k}$. Thus K_{ij}, M_{ij} of fast modes are not equal, their x-y components are [20]

$$\begin{bmatrix} M_{ij}(\mathbf{k}) \\ C_{ij}(\mathbf{k}) \\ K_{ij}(\mathbf{k}) \end{bmatrix} = \frac{L^{-1/2}}{8\pi} J_{ij} k^{-7/2} \begin{bmatrix} \cos^2\theta \\ \sigma \cos\theta \\ 1 \end{bmatrix}, \tag{4}$$

In high β medium, the velocity fluctuations are radial, i.e., along the direction of $\mathbf{k}$. Fast modes in this regime are essentially sound waves compressing magnetic field ([6,13], Cho and Lazarin [22]). The compression of magnetic field depends on plasma β. The corresponding x-y components of the tensors are

$$\begin{bmatrix} M_{ij}(\mathbf{k}) \\ C_{ij}(\mathbf{k}) \\ K_{ij}(\mathbf{k}) \end{bmatrix} = \frac{L^{-1/2}}{8\pi} \sin^2\theta J_{ij} k^{-7/2} \begin{bmatrix} \cos^2\theta/\beta \\ \sigma \cos\theta/\beta^{1/2} \\ 1 \end{bmatrix}. \tag{5}$$

Scattering by Alfvénic turbulence.—Particles get into resonance with MHD perturbations propagating along the magnetic field if the resonant condition is fulfilled, namely, $\omega = k_\| v \mu + n\Omega$ ($n = \pm 1, 2, \ldots$), where ω is the wave frequency, $\Omega = \Omega_0/\gamma$ is the gyrofrequency of relativistic particle, $\mu = \cos\alpha$, where α is the pitch angle of particles. In other words, resonant interaction between a particle and the transverse electric field of a wave occurs when the Doppler shifted frequency of the wave in the particle's guiding center rest frame $\omega_{gc} = \omega - k_\| v \mu$ is a multiple of the particle gyrofrequency. For high energy particles, the resonance happens for both positive and negative n.

We employ quasilinear theory (QLT) to obtain our estimates. QLT has been proved to be a useful tool in spite of its intrinsic limitations [9,21,23]. For moderate energy cosmic rays, the corresponding resonant scales are much smaller than the injection scale. Therefore the fluctuation on the resonant scale $\delta B \ll B_0$ even if they are comparable at the injection scale. QLT disregards diffusion of cosmic rays that follow wandering magnetic field lines [2] and this diffusion should be accounted separately. Obtained by applying the QLT to the collisionless Boltzmann-Vlasov equation, the Fokker-Planck equation is generally used to describe the involvement of the gyrophase-average distribution function f,

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \mu}\left(D_{\mu\mu} \frac{\partial f}{\partial \mu} + D_{\mu p} \frac{\partial f}{\partial p}\right) + \frac{1}{p^2} \frac{\partial}{\partial p}\left[p^2 \left(D_{\mu p} \frac{\partial f}{\partial \mu} + D_{pp} \frac{\partial f}{\partial p}\right)\right],$$

where p is the particle momentum. The Fokker-Planck coefficients $D_{\mu\mu}$, $D_{\mu p}$, D_{pp} are the fundamental physical

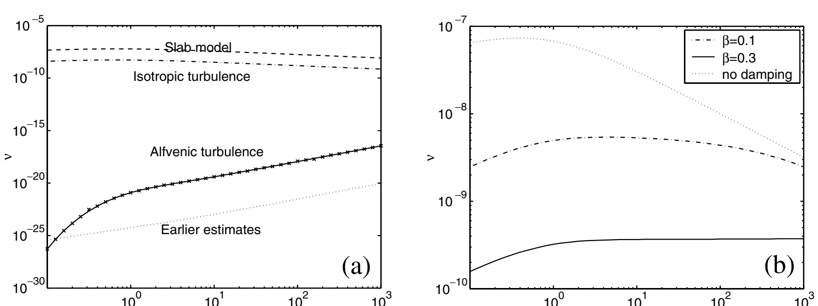

FIG. 1. The scattering frequency ν vs the kinetic energy E_k of cosmic rays (a) by Alfvénic turbulence, (b) by fast modes. In (a), the dash-dotted line refers to the scattering frequency for isotropic turbulence. The "×" represents our numerical result for anisotropic turbulence, the solid line is our analytical result from Eq. (8). Also plotted (dashed line) is the previous result for anisotropic turbulence in [9]. In (b), the dashed line represents the scattering by fast modes immune from damping, the solid and dashdot line are the results taking into account collisionless damping.

parameter for measuring the stochastic interactions, which are determined by the electromagnetic fluctuations [24]:

$$\begin{aligned} \langle B_i(\mathbf{k})B_j^*(\mathbf{k}')\rangle &= \delta(\mathbf{k}-\mathbf{k}')P_{ij}(\mathbf{k}), \qquad \langle B_i(\mathbf{k})E_j^*(\mathbf{k}')\rangle = \delta(\mathbf{k}-\mathbf{k}')T_{ij}(\mathbf{k}), \\ \langle E_i(\mathbf{k})B_j^*(\mathbf{k}')\rangle &= \delta(\mathbf{k}-\mathbf{k}')Q_{ij}(\mathbf{k}), \qquad \langle E_i(\mathbf{k})E_j^*(\mathbf{k}')\rangle = \delta(\mathbf{k}-\mathbf{k}')R_{ij}(\mathbf{k}). \end{aligned} \tag{6}$$

From Ohm's law $\mathbf{E}(\mathbf{k}) = -(1/c)\mathbf{v}(\mathbf{k}) \times \mathbf{B}_0$, we can express the electromagnetic fluctuations T_{ij}, R_{ij} in terms of correlation tensors C_{ij}, K_{ij}. Adopting the approach in [24], we can get the Fokker-Planck coefficients in the lowest order approximation of V_A/c,

$$\begin{aligned} \begin{bmatrix} D_{\mu\mu} \\ D_{\mu p} \\ D_{pp} \end{bmatrix} &= \frac{\Omega^2(1-\mu^2)}{2B_0^2} \begin{bmatrix} 1 \\ mc \\ m^2c^2 \end{bmatrix} \mathcal{R}e \sum_{n=-\infty}^{n=\infty} \int_{k_{\min}}^{k_{\max}} dk^3 \int_0^\infty dt e^{-i(k_\parallel v_\parallel - \omega + n\Omega)t} \\ &\times \left\{ J_{n+1}^2\left(\frac{k_\perp v_\perp}{\Omega}\right) \begin{bmatrix} P_{\mathcal{RR}}(\mathbf{k}) \\ T_{\mathcal{RR}}(\mathbf{k}) \\ R_{\mathcal{RR}}(\mathbf{k}) \end{bmatrix} + J_{n-1}^2\left(\frac{k_\perp v_\perp}{\Omega}\right) \begin{bmatrix} P_{\mathcal{LL}}(\mathbf{k}) \\ -T_{\mathcal{LL}}(\mathbf{k}) \\ R_{\mathcal{LL}}(\mathbf{k}) \end{bmatrix} \right. \\ &\left. + J_{n+1}\left(\frac{k_\perp v_\perp}{\Omega}\right) J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega}\right) \left(e^{i2\phi} \begin{bmatrix} -P_{\mathcal{RL}}(\mathbf{k}) \\ T_{\mathcal{RL}}(\mathbf{k}) \\ R_{\mathcal{RL}}(\mathbf{k}) \end{bmatrix} + e^{-i2\phi} \begin{bmatrix} -P_{\mathcal{LR}}(\mathbf{k}) \\ -T_{\mathcal{LR}}(\mathbf{k}) \\ R_{\mathcal{LR}}(\mathbf{k}) \end{bmatrix} \right) \right\}, \end{aligned} \tag{7}$$

where $k_{\min} = L^{-1}$, $k_{\max} = \Omega_0/v_{\rm th}$ corresponds to the dissipation scale, $m = \gamma m_H$ is the relativistic mass of the proton, $v_\perp$ is the particle's velocity component perpendicular to $\mathbf{B}_0$, $\phi = \arctan(k_y/k_x)$, $\mathcal{L}, \mathcal{R} = (x \pm iy)/\sqrt{2}$ represent left and right-hand polarization.

The integration over time gives us a delta function $\delta(k_\parallel v_\parallel - \omega + n\Omega)$, corresponding to static magnetic perturbations [24,25]. For cosmic rays, $k_\parallel v_\parallel \gg \omega = k_\parallel V_A$ so that the resonant condition is just $k_\parallel v\mu + n\Omega = 0$. From this resonance condition, we know that the most important interaction occurs at $k_\parallel = k_{\rm res} = \Omega/v_\parallel$.

Noticing that the integrand for small $k_\perp$ is substantially suppressed by the exponent in the anisotropic tensor [see Eq. (3)] so that the large scale contribution is not important, we can simply use the asymptotic form of Bessel function for large argument. Then if the pitch angle α not close to 0, we can derive the analytical result for anisotropic turbulence,

$$\begin{bmatrix} D_{\mu\mu} \\ D_{\mu p} \\ D_{pp} \end{bmatrix} = \frac{v^{2.5}\cos\alpha^{5.5}}{2\Omega^{1.5}L^{2.5}\sin\alpha} \Gamma[6.5, k_{\max}^{-\frac{2}{3}} k_{\rm res} L^{\frac{1}{3}}] \begin{bmatrix} 1 \\ \sigma m V_A \\ m^2 V_A^2 \end{bmatrix}, \tag{8}$$

where $\Gamma[a, z]$ is the incomplete gamma function. The presence of this gamma function in our solution makes our results orders of magnitude larger than those in [9,26] for the most of energies considered [see Fig. 1(a)]. However, the scattering frequency $\nu = 2D_{\mu\mu}/(1-\mu^2)$ are much smaller than the estimates for isotropic model. Unless we consider very high energy CRs ($\geq 10^8$ GeV) with the corresponding Larmor radius comparable to the turbulence injection scale, we can neglect scattering by the Alfvénic turbulence. What is the alternative way to scatter cosmic rays?

Scattering by fast modes.—Our result that anisotropic turbulence is inefficient in CR scattering agrees well with the conclusions reached in [8,9]. The contribution from slow modes is not larger than that by Alfvén modes since the slow modes have the similar anisotropies and scalings. More promising are fast modes, which are isotropic [14]. For fast modes we discuss two types of resonant interaction: gyroresonance and transit-time damping; the latter requires longitudinal motions. However, fast modes are subject to collisionless damping which suppresses scattering [27]. The damping rate $\gamma_d = \tau_d^{-1}$ for the low β case [11] is

$$\begin{aligned} \gamma_d &= \frac{\sqrt{\pi\beta}}{4} V_A k \frac{\sin^2\theta}{\cos\theta} \\ &\times \left[\sqrt{\frac{m_e}{m_H}} \left(-\frac{m_e}{m_H \beta \cos^2\theta} \right) + 5\exp\left(-\frac{1}{\beta\cos^2\theta} \right) \right], \end{aligned} \tag{9}$$

where m_e is the electron mass. We see that the damping increases with β. According to CL02, fast modes cascade over time scales $\tau_{fk} = \tau_k \times V_A/v_k = (k \times k_{\min})^{-1/2} \times V_A/V^2$, where $\tau_k = (kv_k)^{-1}$, V is the turbulence velocity at the injection scale.

Consider gyroresonance scattering in the presence of collisionless damping. The cutoff of fast modes corresponds to the scale where $\tau_{fk}\gamma_d \simeq 1$ and this defines the cutoff scale k_c^{-1}. As we see from Eq. (9), the damping increases with θ unless θ is close to $\pi/2$.

Using the tensors given in Eq. (4) we obtain the corresponding $D_{\mu\mu}$ for the CRs interacting with fast modes by integrating Eq. (7) from $k_{\min}$ to k_c [see Fig. 1(b)]. When k_c^{-1} is less than r_L, the results of integration for damped and undamped turbulence coincides.

Since the k_c decreases with β, the scattering frequency decreases with β.

Adopting the tensors given in Eq. (5), it is possible to calculate the scattering frequency of CRs in high β medium. For instance, for density $n = 0.5\ \mathrm{cm}^{-3}$, temperature $T = 8000$ K, magnetic field $B_0 = 1\ \mu$G, the mean free path is smaller than the resonant wavelength for the particles with energy larger than 0.1 GeV, therefore collisional damping rather than Landau damping should be taken into account. Nevertheless, our results show that the fast modes still dominate the CRs's scattering in spite of the viscous damping.

Apart from the gyroresonance, fast modes potentially can scatter CRs by transit-time damping (TTD) [22]. TTD happens due to the resonant interaction with parallel magnetic mirror force $-(mv_\perp^2/2B)\nabla_\parallel \mathbf{B}$. For small amplitude waves, particles should be in phase with the wave so as to have a secular interaction with wave. This gives the Cherenkov resonant condition $\omega - k_\parallel v_\parallel \sim 0$, corresponding to the $n = 0$ term in Eq. (7). From the condition, we see that the contribution is mostly from nearly perpendicular propagating waves ($\cos\theta \sim 0$). According to Eq. (4),we see that the corresponding correlation tensor for the magnetic fluctuations M_{ij} are very small, so the contribution from TTD to scattering is not important.

Self-confinement due to the streaming instability has been discussed by different authors [9,28,29] as an effective alternative to scatter CRs and essential for CR acceleration by shocks. However, we will discuss in our next paper that in the presence of the turbulence the streaming instability will be partially suppressed owing to the nonlinear interaction with the background turbulence.

Thus the gyroresonance with the fast modes is the principle mechanism for scattering cosmic rays. This demands a substantial revision of cosmic ray acceleration/propagation theories, and many related problems may need to be revisit. For instance, our results may be relevant to the problems of the Boron to Carbon abundances ratio. We shall discuss the implications of the new emerging picture elsewhere.

Summary.—In the paper above we have shown the following: (i) Scattering by fast modes is the dominant scattering process provided that turbulent energy is injected at large scales. (ii) Gyroresonance is the most important for pitch angle scattering. Transit-time damping (TTD) of the resonant waves is subdominant because the corresponding magnetic fluctuations are nearly perpendicular to the mean magnetic field. (iii) The scattering frequency by fast modes depends on collisionless damping for viscous damping, therefore it varies with plasma β.

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