High-Intensity Laser-Field Amplification between Two Foils

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Interaction of two oppositely directed ultraintense laser pulses with two closely placed thin foils is modeled analytically and investigated by particle-in-cell simulation. It is shown that laser energy can be trapped and accumulated between the foils. The intensity could reach a 100-fold that of the pump lasers. The trapping is found to be bistable and the parameters for stable energy confinement and enhancement are given. The ultrahigh fields that can be produced have many potential applications, including that of verifying nonlinear quantum electrodynamics effects.

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Many modern applications of the laser depend on the intensity of its electromagnetic wave field. When the normalized amplitude a ($=eA/mc^2$, where e, m, A, and c are the electron charge, mass, vector potential, and light speed, respectively) is larger than unity, corresponding to laser intensities larger than 10^{18} W/cm^2 for a typical optical frequency, the electron motion in the laser field enters the relativistic regime [1]. Novel effects such as relativistic harmonic generation [2] can be attributed to the relativistic quiver motion of the electron. When the laser electric field is still higher, the Schwinger critical field ($\eta = eA\hbar\omega/m^2c^4 \geq 1$) may be reached and one enters the nonlinear quantum electrodynamics (QED) regime [3]. For this the laser intensity must be larger than 2×10^{29} W/cm^2, or $a = 10^4$ for a 1 μm laser and $a = 1$ for a 0.1 nm laser. Certain important QED effects, such as vacuum pair production [4] and the Hawking-Unruh radiation [5], would then become realizable. In practice, because of polarization or exponential tail effects [6], one can expect QED effects even at lower intensities. To attain the QED regime, one can use either future exa/zettawatt lasers [7] or x-ray free-electron lasers [8].

On the other hand, rapid progress in short pulse laser technology [9] has made possible the realization of laser-plasma interaction at ultrarelativistic intensities. The laser pulse and plasma are nonlinearly self-modulated such that an intensity higher than that in vacuum can occur. For example, when the laser power is larger than the critical power $P_c = 17n_c/n$ GW, relativistic self-focusing can appear. The plasma is expelled by the ponderomotive force of the pulse [10,11], and the latter propagates in the self-created channel at an intensity higher than that in vacuum. The relativistic soliton is another structure that can form. Solitons in both underdense and overdense plasmas have been investigated theoretically and experimentally [12–19]. In an overdense plasma, a laser pulse will reflect from the critical surface where the electron density is $n_c = 1.1\times10^{21}\lambda^{-2}$ cm^{-3} (where λ is the wavelength in μm) and a stationary soliton of relativistic intensity can appear there. In an underdense plasma, although the laser pulse is severely distorted and depleted by stimulated Raman scattering, a part of it can experience a frequency down-shift to below the local plasma frequency and become trapped [19]. However, the latter scenario is difficult to control. In view of these natural occurrences of laser energy enhancement in a plasma, one can ask the question if it is possible to stably trap and amplify laser fields in tailored plasma traps. We propose here a simple scheme for trapping and amplifying the laser field between two foils.

We consider the one-dimensional problem of circularly polarized electromagnetic (EM) wave trapping between two layers of enhanced electron density. Ion dynamics shall be neglected. We shall first discuss an analytical solution in the form of a stationary soliton in an infinite plasma. Based on this solution we propose a laser-foil interaction (LFI) model consisting of two closely spaced thin plasma layers, with trapped EM waves between them, and impinged upon by EM waves at their outer sides. This model also allows for stationary analytical solutions, from which we can obtain a relation between the EM field intensity at the center of the gap and the gap width. The solutions of the soliton as well as the LFI models are verified by particle-in-cell (PIC) simulations using relativistic LPIC++ code [20] and their stability investigated. In order to see how the trapped fields and the electron layers behave in the real situation, complete PIC simulations on the evolution of the LFI are performed. Our results show that laser energy trapping and accumulation are indeed possible, and field intensities 100 times larger than that of the pump lasers can be attained. Furthermore, the stationary states are bistable. The conditions for stable wave trapping and enhancement are obtained.

We shall start with the relativistic cold plasma equations in the form given by Shen *et al.* [21]. The normalized vector potential of the laser field of frequency ω_L in an electron layer can be written as $a = a_0(\xi)\times \exp^{i\omega_L t + i\theta(\xi)}$, where $\xi = \omega_L z/c$. There are two constants of motion [22]

$$M = (\partial\theta/\partial\xi)(\gamma^2 - 1), \quad (1)$$

$$W = [(\partial\gamma/\partial\xi)^2 + M^2]/2(\gamma^2 - 1) + \gamma^2/2 - N_i, \quad (2)$$

where $\gamma = (1 + a^2)^{1/2}$ is the relativistic factor, and $N_i = n_i/n_c$ is the constant normalized ion density.

If there is no net energy flow, we have in the electron layer $M = 0$, and Eq. (2) can be integrated to [23]

$$\gamma(\xi) = \frac{1 + \sqrt{q^2+1} + (\sqrt{q^2+1} - 1)\mathrm{cn}^2[\sqrt{\varepsilon}(\xi - \xi_0)]}{1 + \sqrt{q^2+1} - (\sqrt{q^2+1} - 1)\mathrm{cn}^2[\sqrt{\varepsilon}(\xi - \xi_0)]}, \quad (3)$$

for $W \geq 1/2 - N_i$, and

$$\gamma(\xi) = \frac{p^2 + \mathrm{sn}^2[\sqrt{(\varepsilon+1)^2 - N_i^2}(\xi - \xi_0)/2]}{p^2 - \mathrm{sn}^2[\sqrt{(\varepsilon+1)^2 - N_i^2}(\xi - \xi_0)/2]}, \quad (4)$$

for $W < 1/2 - N_i$, where $\varepsilon = (N_i^2 + 2W)^{1/2}$, $q = [(\varepsilon + N_i)^2 - 1]^{1/2}$, $p = [(\varepsilon + N_i + 1)/(\varepsilon + N_i - 1)]^{1/2}$, $k = \{[N_i^2 - (\varepsilon - 1)^2]/4\varepsilon\}^{1/2}$, and $\bar{k} = \{[(\varepsilon - 1)^2 - N_i^2]/[(\varepsilon + 1)^2 - N_i^2]\}^{1/2}$. In obtaining the soliton solutions, we have used the boundary condition $a = 0$ at $\xi = \pm\infty$.

For an infinite overdense plasma, we have $W = 1/2 - N_i$, and [24]

$$\gamma(\xi) = \frac{N_i\mathrm{ch}^2[\sqrt{N_i - 1}(\xi - \xi_0)] - 1 + N_i}{N_i\mathrm{ch}^2[\sqrt{N_i - 1}(\xi - \xi_0)] + 1 - N_i}, \quad (5)$$

where again $M = 0$. For $n = 1.5n_c$, Esirkepov *et al.* [24] obtained a soliton solution and showed by simulation that it is also stable. When a higher electron density such as $n = 4n_c$ is used, we find that the electrons at the center of the localized structure can be completely expelled by the ponderomotive force of the trapped electromagnetic waves. In this case two stationary solutions are found. Figure 1(a) shows a solution with the peak amplitude at the center of the soliton, and Fig. 1(b) shows one with zero field amplitude at the center. Both solutions have been confirmed by PIC simulations of the corresponding configuration. The simulations also show that both of these stationary analytical solutions survive for several hundred laser cycles without apparent change. In fact, it is found that even for not very high plasma densities such localized states involving high-intensity fields can exist for a fairly long time. The problem is how to realize this wave-trapping configuration and the corresponding solutions in the real world.

In view of the above result, we propose here a method for producing high-intensity electromagnetic fields by confining and accumulating electromagnetic waves in the vacuum gap between two closely spaced thin foils, impinged upon on their outer sides by two oppositely directed circularly polarized laser pulses. This easily realizable practical configuration differs somewhat from that of the soliton solution given above in that here the plasma background is absent. One can model this LFI configuration by two closely spaced plasma layers with impinging laser fields on their outer sides and trapped EM fields in the gap between them. Thus the only plasma in the system is that from the ionization of the two foils by the lasers. Self-consistent analytical solutions can also be obtained for the LFI model by applying the analytical solutions in the different regions and matching the solutions at the boundaries with the usual EM boundary conditions [20,21]. In Fig. 2, the field amplitude at the center ($z = 0$) of the gap versus the distance Δ (normalized by λ) between the two foils is given. In Fig. 2(a) the amplitude of the pump lasers is $a_0 = 3$. The line $ABCDE$ is for $W \geq 1/2 - N_i$ and line FG for $W < 1/2 - N_i$, corresponding to the analytical solutions given by Eqs. (3) and (4), respectively. One observes that for any given Δ, there can be up to three solutions. For example, for $\Delta = 0.284\lambda$, the points B, D, and E all represent possible solutions.

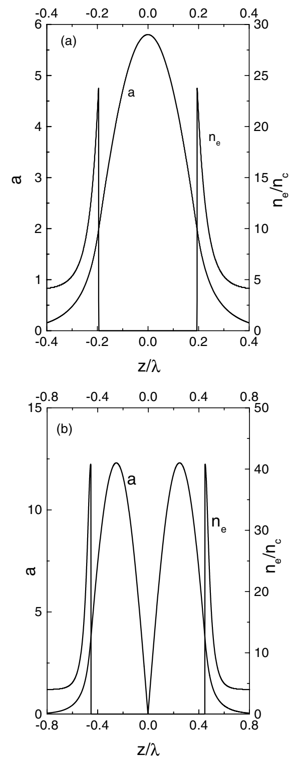

FIG. 1. Analytical solutions of the EM wave amplitude a and electron density n_e as a function of space z/λ, for the soliton model with background ion density $n_i/n_c = 4$: (a) single hump solution, (b) double hump solution.

In the simulation of the LFI model, two electron layers (together with the fixed ions) of thickness $\omega_L z/c = 0.1$ and density $n = 100n_c$ are used. In order to investigate the stability of the stationary states of the LFI model, we first perform the simulation with the trapped electromagnetic field already in place, using the analytical solutions as a guide in setting up the "initial" states. We found that

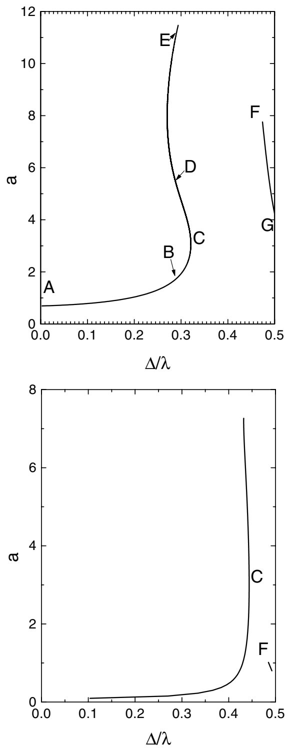

FIG. 2. Analytical results of the EM field amplitude a at the center of vacuum gap between the two foils as a function of the gap distance for the LFI model. The two electron layers are of thickness $\omega_L z/c = 0.1$ and density $n/n_c = 100$. The amplitudes of the two pump-laser pulses are (a) $a_0 = 3$, and (b) $a_0 = 0.5$.

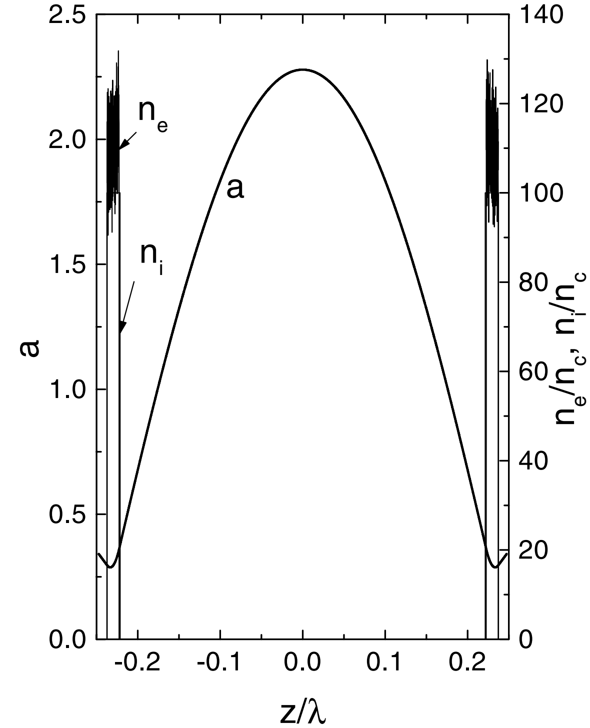

FIG. 3. PIC simulation of the evolution of the laser-plasma interaction. Snapshots of the laser field a, electron density n_e, and ion density n_i after 9 laser cycles. The pump laser amplitude rises from zero to $a = 0.5$ in three laser cycles and then remains constant. The gap width is $\Delta/\lambda = 0.443$. The electron layers are the same as in Fig. 2.

the solutions B and E can be obtained in the simulation but D not. In fact, the solution D is extremely short lived and evolves to solution B in only a few laser cycles. That is, self-consistent relativistic trapping of electromagnetic energy between two electron layers is bistable. As expected, simulation of the real situation (initially no field inside the gap) shows that only the solution B appears.

The curve $ABCDE$ in Fig. 2(a) can be used to determine the largest (at C) possible stationary field amplitude between the two foils. For the example shown, this amplitude is lower than the pump-laser amplitude $a = 3$. It would thus seem to be difficult to realize ultraintense EM fields since the point E is difficult to attain. However, if a lower pump amplitude, say, $a_0 = 0.5$, is used, one obtains the curve shown in Fig. 2(b). At the point C ($\Delta = 0.443\lambda$) the amplitude is about $a = 2$, or 4 times larger than the amplitude of the pump lasers. We have performed realistic PIC simulations for this case by letting the amplitude of the pump lasers rise from zero to $a_0 = 0.5$ in three laser cycles and then remain constant. One sees that while much of the laser energy is reflected from the foils, a part of the energy propagates through the foils and accumulates in the vacuum gap. Figure 3 shows a snapshot after nine laser cycles, when steady state is reached. The simulation also confirms the amplitude $a = 2.2$ at $z = 0$, as predicted by the analytical solution of the LFI model. Furthermore, this result shows that the maximum field amplitude depends more on the thickness of the foils than the amplitude of the pump lasers.

There is no stationary solution in the section between C and F in Fig. 2(b). Nevertheless, in many applications the intensity rather than the lifetime of the trapped electromagnetic field is of primary concern. It is therefore also of interest to investigate the evolution of nonsteady interactions. For example, for $\Delta = 0.46\lambda$, letting the pump lasers rise from zero to $a = 0.5$ in six laser cycles and then keeping it constant, we obtain by PIC simulation the time dependence (Fig. 4) of the amplitude of the EM field at $z = 0$. One observes that the amplitude reaches its maximum $a = 5$ after 26 laser cycles. This amplitude is 10 times larger than that of the pump laser. In terms of the intensity, it is 100 times larger. If one uses thicker foils, even larger electromagnetic fields can be realized, but an even longer interaction time is needed. The largest electric field occurs at the gap center where the magnetic field is null. It may be used to test the Unruh radiation as suggested by Chen and Tajima [6].

In conclusion, based on the analytical solitonlike solutions for relativistic EM wave-plasma interaction, we propose here a scheme to accumulate EM energy by

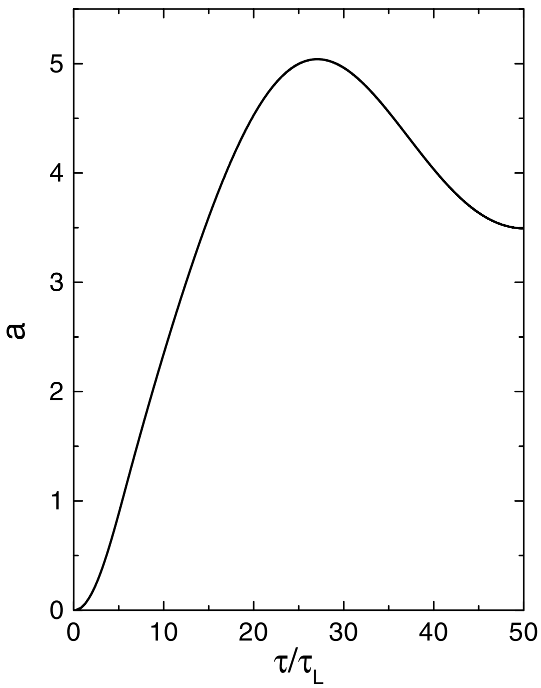

FIG. 4. The field amplitude a at the center of the vacuum gap between the two foils as a function of time (normalized by the period τ_L of a laser cycle). The pump laser amplitude rises from zero to $a = 0.5$ in six laser cycles and then remains constant. The gap width is $\Delta/\lambda = 0.46$. The electron layers are the same as in Fig. 2.

trapping the laser fields between two closely spaced thin foils impinged upon by two high-intensity short-wavelength circularly polarized laser pulses from opposite sides. It is shown that intensities 100 times greater than that of the pump lasers can be realized if the thickness of the two foils and the distance between them are appropriate. The trapping time in our example is about 26 laser cycles (about 70 fs for a laser pulse with 1 μm wavelength). For larger pump-laser intensities, thicker foils must be used to provide the electrons needed for good trapping. Similar laser field trapping in other configurations are also possible. For example, by inserting a thin foil in front of a solid target, one can use a single circularly polarized laser to produce in the gap between the foil and the solid target an electromagnetic field much stronger than that of the pumping laser pulse if the foil and gap are of suitable size. However the field structure in this case is no longer symmetric because of the obviously asymmetrical geometry. In the present paper we have not studied the lateral stability of the foils. Clearly, pulses of high lateral uniformity are required to maintain the planar interaction geometry. On the other hand, the phenomenon studied here can also occur in higher dimensions. One may, namely, trap laser energy in vacuum spherical microballoons of appropriate size and wall thickness. We also have not studied that long-time behavior of the wave-trapping phenomenon. The problem is somewhat more complicated since parametric instabilities, electron heating and expansion, ion dynamics, etc., can all play a role [1,2,6,7,25].

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[1] C. J. Joshi and P. B. Corkum, Phys. Today **48**, No. 1, 36 (1995).
[2] P. Norreys *et al.*, Phys. Rev. Lett. **76**, 1832 (1996).
[3] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[4] D. L. Burke *et al.*, Phys. Rev. Lett. **79**, 1626 (1997).
[5] W. Unruh, Phys. Rev. D **14**, 870 (1976).
[6] P. Chen and T. Tajima, Phys. Rev. Lett. **83**, 256 (1999).
[7] T. Tajima and G. Mourou, Phys. Rev. ST Accel. Beams **5**, 031301 (2002).
[8] N. Patel, Nature (London) **415**, 110 (2002).
[9] G. A. Mourou, C. P. J. Barty, and M. D. Perry, Phys. Today **51**, No. 1, 22 (1998).
[10] A. Pukhov and J. Meyer-ter-Vehn, Phys. Rev. Lett. **79**, 2686 (1997).
[11] H. Yang *et al.*, Phys. Rev. E **65**, 016406 (2002).
[12] V. A. Kozlov, A. G. Litvak, and E. V. Suvorov, Sov. Phys. JETP **49**, 75 (1979).
[13] M. Y. Yu, P. K. Shukla, and N. L. Tsintzadze, Phys. Fluids **25**, 1049 (1982); P. K. Shukla, N. N. Rao, M. Y. Yu, and N. L. Tsintsadze, Phys. Rep. **138**, 1 (1986).
[14] S. V. Bulanov *et al.*, Phys. Fluids B **4**, 1935 (1992).
[15] S. V. Bulanov *et al.*, Phys. Rev. Lett. **82**, 3440 (1999).
[16] N. M. Naumova *et al.*, Phys. Rev. Lett. **87**, 185004 (2001).
[17] Y. Sentoku *et al.*, Phys. Rev. Lett. **83**, 3434 (1999).
[18] K. Mima *et al.*, Phys. Plasmas **8**, 2349 (2001).
[19] M. Lontano *et al.*, Phys. Plasmas **9**, 2562 (2002).
[20] R. E. W. Pfund, R. Lichters, and J. Meyer-ter-Vehn, in *Super Strong Field in Plasmas*, edited by M. Lontano *et al.*, AIP Conf. Proc. No. 426 (American Institute of Physics, Melville, NY, 1998), p. 141. Modifications are made for this problem.
[21] B. Shen and J. Meyer-ter-Vehn, Phys. Plasmas **8**, 1003 (2001); Phys. Rev. E **65**, 016405 (2002); B. Shen and Z. Xu, Phys. Rev. E **64**, 056406 (2001).
[22] C. S. Lai, Phys. Rev. Lett. **36**, 966 (1976); P. Kaw and J. Dawson, Phys. Fluids **13**, 472 (1970); C. Max and F. Perkins, Phys. Rev. Lett. **27**, 1342 (1971).
[23] A. Kim *et al.*, JETP Lett. **72**, 241 (2000).
[24] T. Zh. Esirkepov *et al.*, JETP Lett. **68**, 36 (1998).
[25] S. Guérin *et al.*, Phys. Plasmas **3**, 2693 (1996).