Decisive Role of Fragmentation Functions in Hard Hadron Production

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(Received 2 May 2002; published 16 December 2002)

It is demonstrated that the fragmentation functions at large momentum fraction play a key role in hard hadron production from relativistic proton-proton collisions. We find that this region of the fragmentation functions is not strongly constrained by the electron-positron data. This freedom can be used (together with the transverse-momentum distribution of partons) to reproduce hard pion-to-proton ratio data in relativistic proton-proton collisions.

DOI: 10.1103/PhysRevLett.89.272301 PACS numbers: 25.75.–q, 13.85.–t, 13.87.Fh

Hadron spectra at large transverse-momentum (p_T), where perturbative quantum chromodynamics (pQCD) has good predictive power, are very important for our understanding of the physics at the Relativistic Heavy Ion Collider (RHIC) and at the planned Large Hadron Collider (LHC). Various experiments at these colliders focus on hard (high p_T) hadron spectra. Jet "tomography" (the study of the strongly interacting medium via energy loss of hard partons) has been proposed to detect the quark-gluon plasma, using high-p_T hadron production [1,2].

Suppression of total charged hadron production in Au + Au collisions relative to a nucleon-nucleon reference is reported at RHIC [3,4]. However, for the proton-to-pion (p/π) ratio, the PHENIX experiment reports an anomalous enhancement in Au + Au collisions at $\sqrt{s} = 130$ GeV [5]. An explanation for the p/π enhancement was proposed recently, combining pQCD with soft physics and jet quenching [6]. It should be kept in mind in this context that, while pQCD is quite successful for total charged hadron ($h^+ + h^-$) and pion production at large p_T in pp collisions, proton production in pp is not well understood using the language of pQCD. In fact, pQCD underestimates the p/π^+ ratio by a factor of 3–10 in pp collisions [see Fig. 3(a) of this work for $\sqrt{s} = 27.4$ GeV and, e.g., Ref. [6] for Tevatron energy].

In this Letter, we look into how pQCD is used to calculate p_T spectra in pp collisions. We focus on the role of a nonperturbative ingredient, the *fragmentation function* (obtained by fitting data) in the production of hadronic final states. Most of the information included in the fits comes from $h^+ + h^-$ data. The fragmentation function (FF) of pions is studied in some detail. Less direct information is available on kaons, and the FF of protons is even less well known. In the following, we concentrate on the proton FF as an example of the role of the FF in the hadroproduction of hard particles. We find (for all types of hadrons) that the value of the FF in a region of phase space where it is least constrained plays a decisive role for hadron production at RHIC and LHC.

To predict the p_T spectra of final state hadrons in the framework of pQCD, perturbative partonic cross sections need to be convoluted with parton distribution functions (PDFs) and FFs according to the factorization theorem [9]. Perturbative QCD has nothing to say about the details of FFs, apart from describing their scale evolution. The FFs are assumed to be *universal,* meaning that once extracted from a limited set of data via a fitting procedure, they can be used with predictive power in other reactions [10–13]. The situation is conceptually identical to that of the PDFs. We work with this formalism in the present Letter, applying the Kniehl-Kramer-Potter (KKP) fragmentation functions [11] throughout. As an alternative, phenomenological models (e.g., string or cluster models) can, of course, be implemented in Monte Carlo programs [14,15].

The available fragmentation functions are obtained by fitting mostly e^+e^- data. [If information from pp ($p\bar{p}$) data is included, complications related to k_T smearing [16,17] may result at small p_T.] Note, however, that the phase space for hadronic collisions is different from that of e^+e^- reactions. This manifests itself in different values for z, the momentum fraction of the fragmenting parton carried by the final hadron. In order to study the role of different regions of z in the FF, we define the ratio R_z as

$$R_z(p_T) = \int_0^z dz' \frac{d\sigma}{p_T dp_T dz'} \Bigg/ \int_0^1 dz' \frac{d\sigma}{p_T dp_T dz'}, \quad (1)$$

where the cross section $d\sigma/(p_T dp_T dz)$ is differential in p_T and in z, and the ratio depends on the upper limit of the integration in the numerator.

Figure 1 displays $R_z(p_T)$ as a function of z for three different values of p_T in the case of proton production in pp collisions at center-of-mass (c.m.) energies $\sqrt{s} = 27.4$ GeV [1(a)] and $\sqrt{s} = 130$ GeV [1(b)]. In Fig. 1(a) we show $p_T = 2$ GeV (solid line), 4 GeV (dashed line), and 6 GeV (dotted line). In Fig. 1(b), where the higher

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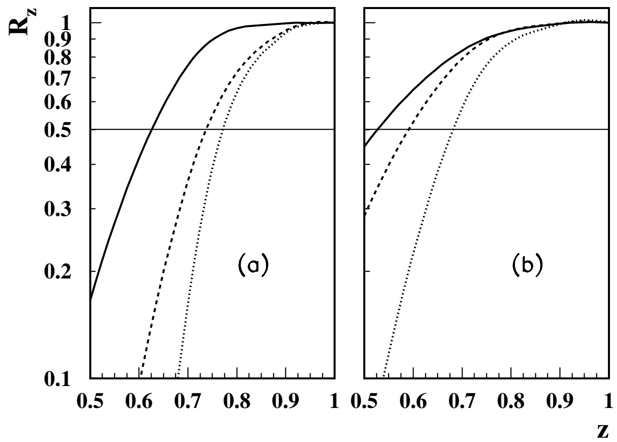

FIG. 1. The ratio $R_z(p_T)$ defined in Eq. (1) as a function of z for different p_T values (a) at $\sqrt{s} = 27.4$ GeV with $p_T = 2$ GeV (solid line), $p_T = 4$ GeV (dashed line), and $p_T = 6$ GeV (dotted line), and (b) at $\sqrt{s} = 130$ GeV with $p_T = 4$ GeV (solid line), $p_T = 8$ GeV (dashed line), and $p_T = 16$ GeV (dotted line).

c.m. energy allows higher p_T values, the solid line corresponds to $p_T = 4$ GeV, the dashed line means $p_T = 8$ GeV, and the dotted line represents $p_T = 16$ GeV. It can be read off from Fig. 1(a) that when $p_T = 4$ GeV, about 90% of the cross section comes from the contribution of $z > 0.6$ at $\sqrt{s} = 27.4$ GeV. When $p_T = 6$ GeV, 90% of the cross section arises from $z > 0.7$. Figure 1(b) shows that at $\sqrt{s} = 130$ GeV, when $p_T = 4$ GeV, approximately 60% of the cross section comes from the contribution of $z > 0.5$; when $p_T = 8$ (16) GeV, 70% (95%) of the cross section is from $z > 0.5$.

In summary, the large z part of the fragmentation function dominates the proton production cross section in hadronic collisions. When p_T increases, the effect becomes even more pronounced. This conclusion is consistent with results for Drell-Yan processes at large p_T, where virtual photon fragmentation functions are introduced to resum large logarithms [18].

The above results depend on the shape of the FF. If the corresponding FFs fall less steeply (are "harder") than the KKP proton FFs, the dominance of the large z region is even more significant. Thus, the above effect is also very important for the hadroproduction of pions and kaons, which have harder FFs. Let us examine, therefore, what kind of constraints are placed on the FFs in the large z region by e^+e^- data.

In e^+e^- collisions, the hadron production cross section (differential in momentum fraction x) is given by [10]

$$\frac{d\sigma^h}{dx} = \sum_a \int_x^1 \frac{dz}{z} \frac{d\hat{\sigma}_a}{dy}(y, \mu_F, \mu_R) D_a^h(z, \mu_F), \qquad (2)$$

where $d\hat{\sigma}_a/dy$ is the differential cross section of a partonic subprocess producing parton a (as a function of $y = x/z$, of the factorization scale μ_F, and of the renormalization scale μ_R), and $D_a^h(z, \mu_F)$ is the FF for parton a to fragment into hadron h with momentum fraction z. The contributions from different partons (quarks of flavor i and gluons g) are summed over.

Through next-to-leading order (NLO), in the $\overline{\rm MS}$ scheme, the partonic subprocess cross section for quark flavor i can be written as [10]

$$\frac{d\hat{\sigma}_{q_i}}{dy} = \sigma_0 N_c e_{q_i}^2 \left\{ \delta(1-y) + \frac{\alpha_s(\mu_R^2)}{2\pi} \times \left[P_{qq}^{0,T}(y) \ln\frac{s}{\mu_F^2} + K_q^T(y) + K_q^L(y) \right] \right\}, \qquad (3)$$

where σ_0 is the corresponding total cross section for $e^+e^- \to \gamma^* \to \mu^+\mu^-$, N_c is the number of colors, e_{q_i} is the charge of the quark, α_s is the strong coupling constant, and the interested reader is referred to [10] for the functions P and K. The corresponding expression including the Z^0 contribution is similar, but more complicated due to the weak coupling of the Z^0 [19].

For gluons,

$$\frac{d\hat{\sigma}_g}{dy} = \sigma_0 N_c \sum_{i=1}^{2} N_f e_{q_i}^2 \frac{\alpha_s(\mu_R^2)}{2\pi} \times \left[P_{qg}^{0,T}(y) \ln\frac{s}{\mu_F^2} + K_g^T(y) + K_g^L(y) \right], \qquad (4)$$

in a similar notation (N_f is the number of active flavors).

The usual parametrization of the fragmentation functions at an input scale μ_0 is [10–13]

$$D(z, \mu_0^2) = N z^\alpha (1-z)^\beta, \qquad (5)$$

where α and β are fixed by fitting to a given set of data. The parameter α describes the behavior of the FF in the small z region, while β determines the behavior in the large z region. The FFs are not as well studied as the PDFs. The KKP fragmentation functions [11] provide one of the few sets that contain FFs for protons. Since we are particularly interested in proton production, we use KKP FFs [11] in the following calculations. Figure 2 presents a comparison of e^+e^- data at $\sqrt{s} = 91$ GeV [20] to the KKP analytical approximation (solid line) and to results obtained when β is arbitrarily modified. A complete solution of the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equations is, of course, important for a more quantitative study of FFs. Here, as users, we apply the KKP analytical approximation to illustrate that the e^+e^- data do not strongly constrain the large z region.

In the leading order approximation, one has $x = z$. Therefore, at sufficiently high c.m. energies, e^+e^- data points at large x should constrain the fragmentation function at large z. It can be seen from Fig. 2 that the number of available data points at large x is quite limited and that the uncertainty associated with the highest-x point, in

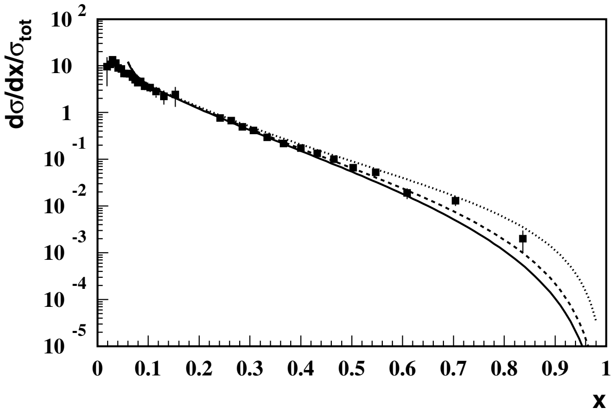

FIG. 2. Perturbative QCD calculations using FFs with different large z behavior compared to e^+e^- data at $\sqrt{s} = 91$ GeV [20]. The solid line is the result with $\beta = \beta_{\rm KKP}$ [11]. The dashed (dotted) line represents $\beta = 0.8\beta_{\rm KKP}$ ($\beta = 0.5\beta_{\rm KKP}$) for modified u, d ($\bar{u}, \bar{d}$) quark and gluon FFs.

particular, is rather large. This is not surprising, since this point is close to the phase space limit of the experiment. Since there is no reason to expect that a NLO calculation would change the state of affairs at the high energies considered here, we use the leading order pQCD results in the following discussion. We use $\mu_F = \sqrt{s}$ [this sets the log terms in (3) and (4) to zero].

Figure 2 shows that there is significant freedom in the choice of the large z behavior of FFs based on e^+e^- experiments. To illustrate the large uncertainty in the proton FFs in the large z region [21], we show the cross section calculated with the analytical approximation given by KKP and the original KKP value [11] of β in Eq. (5) (solid line), together with results with $\beta = 0.8\beta_{\rm KKP}$ (dashed line) and $\beta = 0.5\beta_{\rm KKP}$. We conclude from Fig. 2 that taking, e.g., $\beta = 0.8\beta_{\rm KKP}$ does not change the quality of the fit to the e^+e^- data at this energy. (The value of χ^2/degree of freedom for the interval $0.1 \le x \le 0.8$ is, in fact, 0.99 for the solid line and 0.43 for the dashed curve. However, we expect that the full 46-parameter KKP fit is only slightly influenced by this type of modification.) It is also clear from the above discussion how different parametrizations of FFs [10–13] can be rather different in the large z region even for $h^+ + h^-$ fragmentation (the FFs studied in most detail).

The situation concerning gluon FFs at large z is even less certain than it is for quarks. The contribution of gluon fragmentation to the e^+e^- hadron production cross section appears only as a NLO correction. In hadronic collision, the contribution from gluon processes is much more important than in e^+e^- [18]. Since the probability of finding a gluon in the proton (the gluon PDF) increases rapidly as $x \sim 2p_T/\sqrt{s}$ decreases, gluon fragmentation plays an amplified role at RHIC and LHC energies, compared to fixed target energies. It is thus very important to note that the large z gluon FF for $h^+ + h^-$ obtained including some pp data can be an order of magnitude larger than the one not including pp information [13].

As we have seen, the e^+e^- data do not strongly constrain the FFs in the large z region. It is therefore felt that one has some freedom to fit the proton p_T spectra in pp collisions by modifying the large z behavior of the proton fragmentation functions. To illustrate this idea, we compare the results for the p/π^+ ratio calculated using the KKP form of the proton FFs based on the standard pQCD formalism [16], but varying the value of the parameter β. In addition, the transverse-momentum distribution of the partons in the proton needs to be taken into account in a more complete calculation. As long as no first-principles treatment of this effect is available, the width of the transverse-momentum distribution $\langle k_T^2 \rangle$ provides another phenomenologically adjustable parameter.

Figure 3 demonstrates that the remaining pQCD freedom represented by the above parameters may be sufficient to achieve a better description of the available data on p/π^+ ratios, without invoking any other mechanism. Our goal here is not to fit the data; Fig. 3 serves only as an illustration. The dotted line in Fig. 3(a) shows that the calculated $R = p/\pi^+$ ratio using the original KKP proton FFs underestimates the data (dots) [7] at $\sqrt{s} = 27.4$ GeV by up to a factor of 10. If we set $\beta = 0.8\beta_{\rm KKP}$

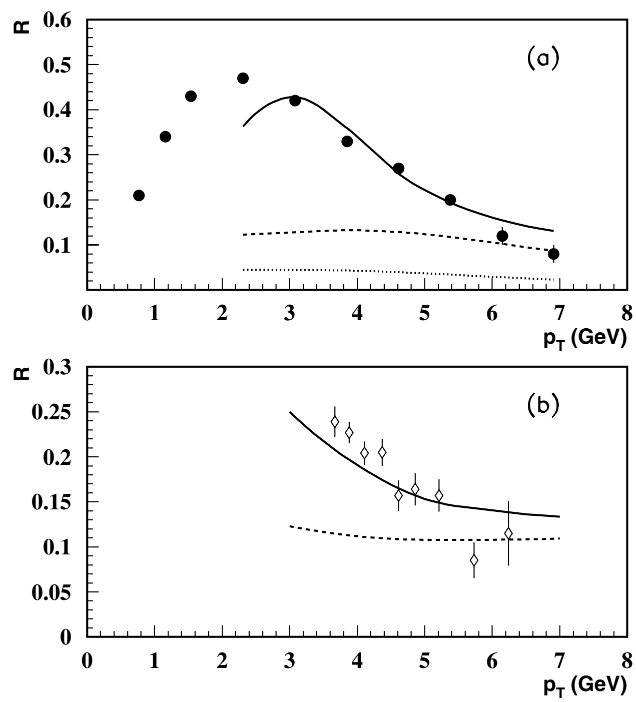

FIG. 3. Comparison of p/π ratio data and pQCD results in pp collisions: (a) at $\sqrt{s} = 27.4$ GeV (dotted line is $\beta = \beta_{\rm KKP}$, dashed line is $\beta = 0.8\beta_{\rm KKP}$, solid line is calculated with different $\langle k_T^2 \rangle$ values for the pion and the proton as described in the text; data are from Ref. [7]); (b) at $\sqrt{s} = 61$ GeV (ISR data [8]) and prediction for RHIC at $\sqrt{s} = 130$ GeV (dashed line).

in the proton FF (dashed line), the pQCD result for the p/π^+ ratio comes close to the experimental data for $p_T > 6$ GeV. In the above calculation, we used $\langle k_T^2 \rangle = 0.9$ GeV^2 for both proton and pion production. However, it is not clear that the input $\langle k_T^2 \rangle$ for proton and pion production needs to be identical. This nonuniversality of the k_T smearing can be motivated in part by the large difference between pion and proton masses, which we physically expect to enter a more complete formalism. Because of the large mass of the proton, the effective $\hat{s}$ (the energy involved in the partonic cross section) is larger for proton production than for pion production at the same p_T. Large $\hat{s}$ leads to more room for a dynamical intrinsic k_T, just like in the Drell-Yan case, where the larger Q^2 of the lepton pair leads to a larger k_T^2. Therefore, it is conceivable that a larger input $\langle k_T^2 \rangle$ is needed for proton production than for pion production [22].

Using $\langle k_T^2 \rangle_p = 2.6$ GeV^2 for the proton and keeping the value $\langle k_T^2 \rangle_\pi = 0.9$ GeV^2 for the pion in the calculation, we obtain the solid line in Fig. 3(a) for the p/π^+ ratio in pp collisions. It is fair to say that the result of this calculation is in satisfactory agreement with the data for $p_T \gtrsim 3$ GeV. (Below $p_T \approx 2$–3 GeV, pQCD is not considered reliable, and nonperturbative effects may become important [23].) We obtain similar results comparing to data at $\sqrt{s} = 38.8$ GeV.

In Fig. 3(b) we explore the energy dependence of the above proposition. Here, the solid line is the result of a pQCD calculation for the p/π^+ ratio in pp collisions at $\sqrt{s} = 61$ GeV, with $\beta = 0.8\beta_{\mathrm{KKP}}$ and $\langle k_T^2 \rangle_p = 3$ GeV^2 (while keeping $\langle k_T^2 \rangle_\pi$ constant). The calculated results are compared to intersecting storage ring (ISR) data (open symbols) [8]. We also give the prediction for RHIC energy, $\sqrt{s} = 130$ GeV, with $\langle k_T^2 \rangle_p = 3$ GeV^2 (dashed line). In this example, phenomenological $\langle k_T^2 \rangle$ effects are dominant over enlarging the FF at large z for $p_T \lesssim 5$ GeV. However, the treatment of the transverse-momentum degree of freedom is not universally agreed upon in the community and is used here only to illustrate such possibilities to augment the modification of the FFs that we found to be allowed by the e^+e^- data.

The main purpose of this Letter is to call attention to the importance of the behavior of fragmentation functions in the large z region for hard hadron production. Recently, similar ideas were considered for bottom hadroproduction at the Tevatron in terms of the moments of the fragmentation function (higher moments emphasizing the importance of large z) [24].

In conclusion, since the large z details of the fragmentation functions are very important for pQCD predictions of high p_T hadron production in both pp and AA collisions at RHIC and LHC, a comprehensive study of these fragmentation functions with particular attention to the large z region is strongly warranted. Additional complications not addressed in the present note include, e.g., the effect of changing β on the scale dependence, a first-principles treatment of the transverse-momentum degree of freedom, and various nuclear effects. These are left for future work. It appears that available p/π ratios from pp collisions at high p_T can be reproduced by adjusting the large z behavior of the fragmentation functions and the width of transverse-momentum distributions.

We are grateful to M. Gyulassy, S. Kretzer, and J.W. Qiu for stimulating discussions. This work was supported in part by the U.S. DOE under DE-FG02-86ER-40251, by the NSF under INT-0000211, and by Hungarian OTKA under T025579 and T032796.

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