Anomalous Capacitive Sheath with Deep Radio-Frequency Electric-Field Penetration

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A novel nonlinear effect of anomalously deep penetration of an external radio-frequency electric field into a plasma is described. A self-consistent kinetic treatment reveals a transition region between the sheath and the plasma. Because of the electron velocity modulation in the sheath, bunches in the energetic electron density are formed in the transition region adjacent to the sheath. The width of the region is of order V_T/ω, where V_T is the electron thermal velocity, and ω is the frequency of the electric field. The presence of the electric field in the transition region results in a collisionless cooling of the energetic electrons and an additional heating of the cold electrons.

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The penetration of the electric field perpendicular to the plasma boundary was studied by Landau in the linear approximation [1]. He showed that an external electric field with amplitude E_0 is screened by the plasma electrons in the sheath region in a distance of order the debye length, and reaches a value E_0/ε in the plasma, where ε is the plasma dielectric constant. In many practical applications, the value of the external electric field is large: the potential drop in the sheath region $V_{\rm sh}$ is typically of order hundreds of volts and is much larger than electron temperature T_e, which is of order a few volts; and the field penetration has to be treated nonlinearly. The asymptotic solution of sheath structure has been studied in the limit $V_{\rm sh} \gg T_e$ [2,3]. In these treatments, the plasma-sheath boundary is considered to be infinitely thin and the position of the boundary is determined by the condition that the external electric field is screened in the sheath regions where electrons are absent. Electron interactions with the sheath electric field can be treated as collisions with a moving potential barrier (wall). It is well known that multiple electron collisions with an oscillating wall result in electron heating, provided there is sufficient phase-space randomization in the plasma bulk. It is common to describe the sheath heating by considering the electrons as test particles and neglecting the plasma electric fields [4]. As was proved in Refs. [3,5] accounting for the electric field in the plasma reduces the electron sheath heating, and the electron sheath heating vanishes completely in the limit of a uniform plasma. Therefore, an accurate description of the rf fields in the bulk of the plasma is necessary for calculating the sheath heating. The electron mean flow velocity is oscillatory in the sheath, and, as a result of this velocity modulation, the electron density bunches appear in the region adjacent to the sheath. The electron density perturbations decay due to phase mixing over a length of order V_T/ω, where V_T is the electron thermal velocity and ω is the frequency of the electric field. These electron density perturbations polarize the plasma and produce an electric field in the plasma bulk. This electric field, in turn, changes the velocity modulation and correspondingly influences the electron density perturbations. Therefore, electron sheath heating has to be studied in a self-consistent nonlocal manner assuming a finite temperature plasma.

Notwithstanding the fact that particle-in-cell simulations results have been widely available for the past decade [6–8] a basic understanding of the electron sheath heating is incomplete, because no one has studied the electric field in the plasma bulk using a nonlocal approach, similar to the anomalous skin effect for the inductive electric field [9]. As is shown below, the attempts to apply a fluid description in Refs. [7,8] yield inaccurate results. In this Letter, an analytical model is developed to explore the effects associated with the self-consistent nonlocal kinetic nature of the phenomenon.

The following assumptions about the discharge parameters have been adopted. The discharge frequency (ω) is assumed to be small compared with the electron plasma frequency (ω_p). Therefore, most of the external electric field is screened in the sheath region. The ion response time is typically longer than the time scale of discharge period, and a quasistationary ion density profile is assumed. The electron density profile is time dependent in response to the time-varying sheath electric field. Because a free-streaming electron flux is much larger than an ion flux, the electrode surface charges negatively and repels electrons. The negative surface charge is screened by the positive ion space charge. In the sheath region, the electrons are reflected by the large sheath electric field; therefore, electrons are covering and uncovering ions all the time. In the limit $V_{\rm sh} \gg T_e$, the debye length (λ_D) is much smaller than the sheath width ($L_{\rm sh}$) and an infinitely thin plasma-sheath boundary can be assumed [2,3]. Because $\omega_p \gg \omega$, electrons respond quasiadiabatically to the large sheath electric field, and, where and when the sheath electric field is screened, the quasineutrality condition holds [5]. As a result, $n_e = n_{\rm sh}$ for $x > x_{\rm sh}(t)$, where $n_{\rm sh}$ is the ion density in the sheath, and $n_e = 0$ for $0 < x < x_{sh}(t)$, where $x_{sh}(t)$ is the position of the plasma-sheath boundary and the electrode position is at $x = 0$ [2,3,5].

Because electrons penetrate into the region of the large electric field in the sheath only a very small distance of order $\lambda_D \ll L_{sh}$, and then are quickly reflected back with the time scale $1/\omega_p \ll 1/\omega$, the electron interaction with the sheath electric field can be modeled as collisions with a moving oscillating rigid barrier with velocity $V_{sh}(t) = dx_{sh}(t)/dt$ [3–5]. An electron with initial velocity $-u$ after a collision with the plasma-sheath boundary acquires a velocity $u_r = u + 2V_{sh}$. Therefore, the density of power deposition transfer from the oscillating plasma-sheath boundary to the plasma electrons is given by [3]

$$P_{sh} = \frac{m}{2}\left\langle \int_0^\infty [u + V_{sh}][u_r^2 - u^2] f_{sh} du \right\rangle, \qquad (1)$$

where m is the electron mass, $f_{sh}(-u, t)$ is the electron velocity distribution function in the sheath, and $\langle \cdots \rangle$ denotes a time average over the discharge period. Introducing a new velocity distribution function $g(-u', t) = f_{sh}[-u - V_{sh}(t), t]$, Eq. (1) yields

$$P_{sh} = 2m\left\langle V_{sh}(t) \int_0^\infty u'^2 g(-u', t) du' \right\rangle, \qquad (2)$$

where $-u' = -u - V_{sh}$ is the electron velocity relative to the oscillating rigid barrier. From Eq. (2) it follows that, if the function $g(u')$ is stationary, then $P_{sh} = 0$, and there is no collisionless power deposition due to electron interaction with the sheath [8,10]. For example, in the limit of a uniform ion density profile, $g(u')$ is stationary (*in an oscillating reference frame of the plasma-sheath boundary*), and the electron heating vanishes [3,5]. Indeed, in the plasma bulk the displacement current is small compared with the electron current ($\omega \ll \omega_p$), and from the conservation of the total current taken in the form $j_0 \sin(\omega t)$, it follows that the electron mean flow velocity in the plasma bulk, $V_b(t) = -j_0 \sin(\omega t)/|e| n_{sh}$, is equal to the plasma-sheath velocity $V_{sh}(t)$ [5]. Therefore, the electron motion in the plasma is strongly correlated with the plasma-sheath boundary motion. From the electron momentum equation it follows that there is an electric field, $E_b = m/e \, dV_b(t)/dt$, in the plasma bulk. In a frame of reference moving with the electron mean flow velocity, the sheath barrier is stationary, and there is no force acting on the electrons, because the electric field is compensated by the inertial force $[eE_b - mdV_b(t)/dt = 0]$ [3,5]. Therefore, an electron interaction with the sheath electric field is totally compensated by the influence of the bulk electric field, and the collisionless heating vanishes [5].

The example of a uniform density profile shows the importance of a self-consistent treatment of the collisionless heating in a plasma. If the function $g(u', t)$ is nonstationary, there is a net power deposition. In this Letter, a kinetic calculation is performed to yield the correct electron velocity distribution function $g(u', t)$ and, correspondingly, the net power deposition.

The authors of Ref. [8] determined the function $g(u', t)$ using the fluid approximation. The effective electron temperature was determined from the conservation of the electron heat flux, and the flow velocity was obtained from the current conservation. To make a closure they assumed that the plasma adjacent to the sheath is not perturbed. As a result they overestimated the temperature variations and, correspondingly, overestimated the reduction of the power dissipation in the sheath due to self-consistency. In fact, the electron density and the electric field is perturbed near the sheath. It was observed in particle-in-cell simulations [7] and was explained on the basis of the fluid approximation as electron acoustic waves. Analogous to the ion acoustic waves, the electron acoustic waves are possible if there are two populations of electrons with very different temperatures. This explanation is, in fact, misleading. A kinetic analysis [11,12] shows that, for the conditions typical for rf discharges, the collisionless damping of the electron acoustic waves (not accounted in the fluid theory) is large: it is comparable with the wave frequency. Moreover, in the fluid theory of Ref. [7], the temperature perturbations were neglected. It means that if this theory is applied to the calculation of the power dissipation of the sheath, $g(u')$ is stationary at the plasma-sheath boundary, and the power dissipation in the sheath vanishes completely, as discussed before. For all these reasons, for accurate calculations of the sheath power dissipation only the kinetic approach can be used, which requires solving the electron kinetic equation.

The full kinetic description can be done only numerically for a nonuniform ion density profile. Examples of similar simulations can be found in Refs. [13,14]. But to model the sheath-plasma interaction analytically, the ion density profile is assumed in a two-step approximation: the ion density n_b is uniform in the plasma bulk, and a lower ion density in the sheath region than in the plasma bulk, $n_{sh} < n_b$.

Throughout this Letter, a linear theory is used because the plasma-sheath boundary velocity and the mean electron flow velocity are small compared with the electron thermal velocity [5,6]. The important spatial scale is the length scale for phase mixing, $l_{mix} = V_T/\omega$. The sheath width satisfies $2V_{sh0}/\omega \ll l_{mix}$ because $V_{sh} \ll V_T$. Therefore, electron interactions with the sheath electric field are treated as a boundary condition for the plasma bulk, neglecting the sheath width. The collision frequency (ν) is assumed to be less than the discharge frequency ($\nu \ll \omega$), and, correspondingly, the mean free path is larger than the length scale for phase mixing l_{mix}. Therefore, the electron dynamics is assumed to be predominantly collisionless. The discharge gap is considered to be large compared with the electron mean free path, so that the influence of the opposite sheath is neglected. The effects of finite gap width can be incorporated, as discussed in Ref. [15].

Because of the assumed ion density discontinuity at the sheath-plasma boundary, in addition to the nonstationary sheath electric field, there is a stationary potential barrier for the electrons ($e\Phi_{sh}$). $e\Phi_{sh}$ is of order the electron temperature, so that only energetic electrons reach the sheath region. The electron motion is different for the low energy electrons with initial velocity in the plasma bulk $|u| < u_{sh}$, where $u_{sh}^2 = 2e\Phi_{sh}/m$, and for the energetic electrons with velocity $|u| > u_{sh}$. The low energy electrons with initial velocity in the plasma bulk $-u$ are reflected from the stationary potential barrier $e\Phi_{sh}$ and then return to the plasma bulk with velocity u. High energy electrons enter the sheath region with velocity $u' = -(u^2 - u_{sh}^2)^{1/2}$. They have velocity $u_2 = 2V_{sh} - u'$ colliding with the moving rigid barrier and then return to the plasma bulk with velocity $(u_2^2 + u_{sh}^2)^{1/2}$. As the electron velocity is modulated in time during reflections from the plasma-sheath boundary, so is the energetic electron density (by continuity of electron flux). The perturbations in the energetic electron density yield an electric field in the transition region adjusted to the sheath.

The electron velocity distribution function (EVDF) is taken to be a sum of a stationary isotropic part $f_0(u)$ and a nonstationary anisotropic part $f_1(x, u, t)$. All time-dependent variables are assumed to be harmonic functions of time, proportional to $\exp(-i\omega t)$, and, in the subsequent analysis, the multiplicative factor $\exp(-i\omega t)$ is omitted from the equations. The linearized Vlasov equation becomes

$$-i\omega f_1 + u\frac{\partial f_1}{\partial x} + \frac{eE(x)}{m}\frac{df_0}{du} = -\nu f_1, \qquad (3)$$

where the term on the right-hand side accounts for rare collisions. The EVDF must satisfy the boundary condition at the plasma-sheath boundary [12]

$$f_1(0, u) = f_1(0, -u), \qquad 0 < u < u_{sh}, \qquad (4)$$

$$f_1(0, u) = f_1(0, -u) + 2V_{sh}\frac{u'}{u}\frac{df_0}{du}, \qquad u > u_{sh}. \qquad (5)$$

The electric field is determined from the condition of conservation of the total current (ij_0), which gives

$$e\int_{-\infty}^{\infty} uf_1(x, u)du - \frac{i\omega}{4\pi}E(x) = ij_0, \qquad (6)$$

where the first term is the electron current and the second term corresponds to a small displacement current. Equations (3) and (6), together with the boundary conditions (4) and (5), comprise the full system of equations for the bulk plasma.

It is convenient to solve Eq. (3) by continuation into the region $x < 0$. First, we introduce the artificial force $F(x, u) = F_{sh}(u)\delta(x)$, where $F_{sh}(u) = 2mV_{sh}u'\Theta(|u| - u_{sh})$, $\delta(x)$ is the Dirac delta function, and $\Theta(u)$ is the Heaviside step function. This force accounts for the change of the energetic electron velocity in the sheath region. Adding the force $F(x, u)$ to the third term of Eq. (3) allows one to use boundary condition (4) for all electrons. It is convenient to divide the electric field in the plasma into two parts corresponding to $E(x) = E_1(x) + E_b\text{sgn}(x)$, where $E_1(x) \to 0$ for $x \to \infty$, and E_b is the value of the electric field far away from the sheath region. The Fourier transform of Eq. (6) with the utilization of Eq. (3) yields

$$\sigma(k)E_1(k) - \frac{2i}{k}[E_{sh}\sigma_{sh}(k) + \sigma(k)E_b] = \frac{2j_0}{k}, \qquad (7)$$

where $\sigma(k)$ is the electron conductivity, $\sigma_{sh}(k)$ is the effective conductivity due to electron interaction with the sheath, and $E_{sh} = (-i\omega + \nu)mV_{sh}/e$ is the effective electric field corresponding to V_{sh}; see [12] for details.

The profile for $E_1(x)$ given by Eq. (7) is shown at the top in Fig. 1. For $x < 6V_T/\omega$ the electric field profile is close to $E_1(x) \approx E_1(0)\exp(-\lambda x\omega/V_T)$, where $E_1(0) = -0.72$, and $\lambda = 0.19 + 0.77i$ for the conditions in Fig. 1. For $x > 6V_T/\omega$, the electric field profile is no longer a simple exponential function, similar to the case of the anomalous skin effect [1,16]. The three components of current corresponding to the first, second, and third terms in Eq. (7) are shown at the bottom in Fig. 1.

The power deposition is given by the sum of the power transferred to the electrons by the oscillating rigid barrier in the sheath region and by the electric field in the transition region, $P_{tot} = P_{sh} + P_{tr}$. P_{sh} is given by Eq. (1). And P_{tr} is the time averaged power deposition in the transition region, $P_{tr} = \int_0^\infty \langle jE\rangle dx$. Straightforward algebra yields [12] (similar to [16])

$$P_{tot} = -\int_0^\infty muD_u(u)\frac{df_0}{du}du, \qquad (8)$$

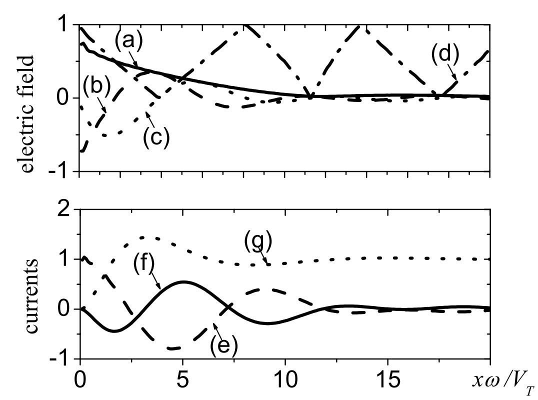

FIG. 1. Plots of the electric field and the current normalized to their respective values in the plasma bulk, E_b and $e^2nE_b/m\omega$, as functions of the normalized coordinate $x\omega/V_T$ for the following parameters: $n_{sh}/n_b = 1/3$, $\omega/\omega_p = 1/100$, and a Maxwellian EVDF. The upper graph shows profiles of $E_1(x)$: (*a*) amplitude—solid line, (*b*) real part—dashed line, (*c*) imaginary part—dotted line, and (*d*) phase with respect to the phase of E_b divided by 2π—dash-dotted line. The lower graph shows profiles of the imaginary part of currents: (*e*) j_{tr}—solid line, (*f*) j_{sh}—dashed line, and (*g*) j_b—dotted line.

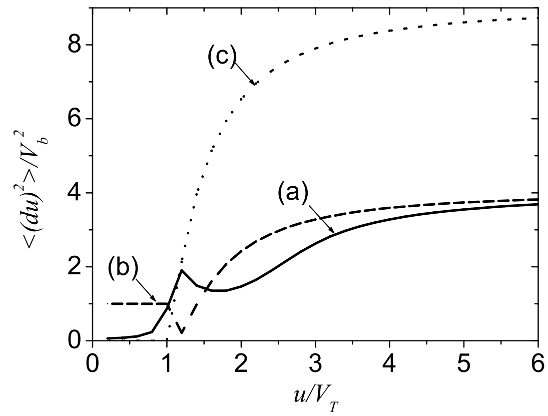

FIG. 2. Plot of the average square of the dimensionless velocity kick as a function of the dimensionless velocity for the conditions in Fig. 1, taking into account (*a*) both $E_1(x)$ and E_b—solid line, (*b*) only E_b—dashed line, and (*c*) no electric field—dotted line.

where $D_u(u) = u|du|^2/4$ is the diffusion coefficient in velocity space, and du is the sum of electron velocity "kicks" after passing through the transition region $(-2iV_b + du_{E_1})$ and after reflecting from the moving boundary in the sheath (idu_{sh}),

$$du(u) = i[du_{sh}(u) - 2V_b] + du_{E_1}(u), \quad (9)$$

$$du_{sh}(u) = 2V_b \frac{u'}{u}\frac{n_b}{n_{sh}} \Theta(|u| - u_{sh}), \quad (10)$$

$$du_{E_1} = \frac{eE_1(k = \omega/u)}{u}. \quad (11)$$

A plot of $|du|^2/2$ is shown in Fig. 2.

Taking into account the electric field in the plasma (both E_b and E_1) reduces $|du|$ for energetic electrons ($u > u_{sh}$) and increases $|du|$ for slow electrons ($u < u_{sh}$). Therefore, the electric field in the plasma cools the energetic electrons and heats the low energy electrons, respectively. Similar observations were made in the numerical simulations [7]. Taking into account the electric field in the plasma (both E_b and E_1) reduces the total power deposited in the sheath region. Interestingly, the numerical simulations shown in Fig. 3 demonstrate that taking into account only the uniform electric field E_b gives a result close within a few percent to the case when both E_b and E_1 are accounted for.

The reason is the electric field E_1 redistributes the power deposition from the energetic electrons to the low energy electrons but does not change the total power deposition. Therefore, the total power deposition due to sheath heating can be calculated approximately from Eq. (8), including only the electric field E_b. This gives

$$P_{tot} \approx -\frac{1}{4} m V_b^2 \int_0^\infty u^2 [du_{sh}(u) - 2V_b]^2 \frac{df_0}{du} du. \quad (12)$$

The result of the self-consistent calculation of the power dissipation in Eq. (12) differs from the test-particle estimate [3] by the last term in Eq. (12). It contributes correction of order $2n_{sh}/n_b$ to the main term, which is small, if $2n_{sh} \ll n_b$.

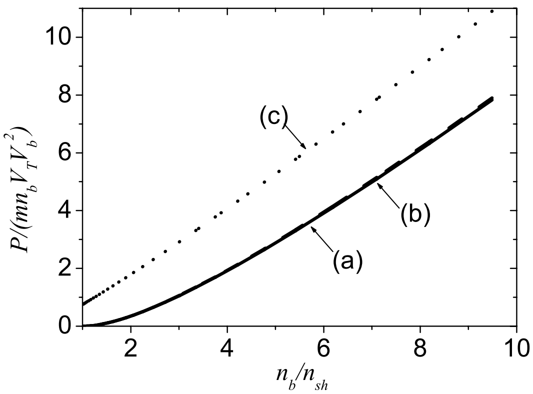

FIG. 3. Plot of the dimensionless power density as a function of the ratio of the bulk plasma density to the sheath density, taking into account (*a*) both $E_1(x)$ and E_b—solid line, (*b*) only E_b—dashed line, and (*c*) no electric field—dotted line.

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