Chirped Femtosecond Solitonlike Laser Pulse Form with Self-Frequency Shift

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Ultrashort laser pulse propagation in a generalized nonconservative system is considered. Slopes appearing in the form of the third-order time derivative for narrow pulse widths, nonlinear dispersion, and self-frequency shift arising from stimulated Raman scattering are taken into account. An exact analytical solitonlike solution is presented for a femtosecond solitary laser pulse. The stability of the latter has been shown numerically by applying perturbations in amplitude and chirp, as well as adding white noise. The results indicate stability in a broad parameter range. In addition, we have also found that the solution acts as an attractor when starting with a quite arbitrary Gaussian pulse as an initial condition.

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During the past decade, the capacity of light-wave systems was dramatically improved by the impressive developments in laser, amplifier, and fiber technologies as well as multiplexing techniques. The better understanding of the underlying physics [1,2] will lead to the Tbyte/sec operation regime in the next generation of ultrahigh speed optical telecommunication systems. High bit rates correspond to narrow pulse widths per channel. Thereby, new physical effects will become important in the Tbyte/sec regime. The new phenomena are similar to those observed in the generation of ultrashort [femtosecond (fs) up to the attosecond (as)] laser pulses [3].

In the past, several authors, e.g., [1,2,4], have discussed the new processes which are present in the ultrashort pulse regime. In view of the many applications, the question of whether stable ultrashort pulse solutions exist becomes of utmost interest. That question may also be posed in the form of whether fs pulses possessing a solitary (solitonlike) structure exist. The soliton concept has turned out to be very powerful in many fields of modern science. At present, among the various soliton realizations, the optical soliton in the picosecond (ps) regime has the highest potential for practical applications.

In its original version, a soliton is a stable, localized, nonlinear solution of an integrable, dispersive (Hamiltonian) system, such as the cubic nonlinear Schrödinger (NLS) equation. However, during the past few years stable, localized, solitary solutions have been also found in driven and dissipative-dispersive systems, such as the driven and damped NLS equation and the (quintic) complex Ginzburg-Landau (CGL) equation. These pulse solutions may be called solitary or solitonlike since they share some properties with solitons, such as preserving shape and size during propagation [5–10].

For intensive and short light pulses whose widths are shorter than 100 fs, several new processes, such as third-order dispersion (TOD), nonlinear dispersion, and self-frequency shift (SFS) arising from stimulated Raman scattering, become important. For such cases, the NLS equation has been extended to higher order nonlinear Schrödinger (HNLS) equations [11]. Many authors [12–21] have analyzed the HNLS equations from different points of view (e.g., Painlevé analysis, Hirota direct method, inverse scattering transform, Darboux-Bäcklund transform, etc.). They obtained solitonlike solutions under the balance between group velocity dispersion, self-phase modulation (SPM), TOD, and self-steepening effects, respectively. Also, the effect of fourth-order dispersion has been investigated [22]. It should be noted that, in all of these investigations, the important SFS has been ignored. Blow *et al.* [23] have employed the bandwidth-limited gain to suppress the SFS and used an adiabatic perturbation theory as well as numerical simulations to determine the laser pulse form. Besides for the HNLS, also for the CGL equations several generalizations have been discussed. A quintic nonlinearity term has been added to investigate the corrections caused by nonlinear amplification (absorption) and nonlinear refraction, resulting in new interesting solutions [24]. Deissler and Brand [25] have investigated numerically the effect of a nonlinear gradient term and found significant modifications.

In this Letter, we present an exact analytical solitonlike solution for a fs laser pulse, including TOD, nonlinear dispersion, and SFS. By employing numerical methods, we prove the stability of the solitonlike solution under perturbations of amplitude, white noise, and chirp. We investigate the evolution of a Gaussian pulse as an initial condition and report how it gradually approaches the analytically predicted solution.

For an optical system, including bandwidth-limited gain and higher order effects, a so-called distributed model can be used when the period of the perturbation is small compared to the amplifiers spacing. The resulting equation for the propagation of femtosecond (e.g., 100 fs) pulses can be written in the form [4,9]

$$E_z - i\frac{D}{2}E_{tt} - i|E|^2E - \delta E - i\sigma E_t - \varepsilon E_{tt} - \chi|E|^2E - \lambda E_{ttt} - \mu(|E|^2E)_t - \nu E(|E|^2)_t = 0, \quad (1)$$

where $E(z, t)$ is the complex envelope of the electric field, z is the normalized propagation distance, and t is the retarded time. The model parameters δ, σ, ε, and χ are real constants; λ, μ, and ν can be complex. The coefficient $D = +1\ (-1)$ corresponds to anomalous (normal) dispersion. Furthermore, $\delta > 0\ (< 0)$ is the linear excess gain (loss) at the carrier frequency ω_0, σ and the imaginary part λ_i of λ result from the difference between the pulse carrier frequency ω_0 and the gain-center frequency ω_a (and are proportional to $\delta\omega = \omega_a - \omega_0$), ε describes the effect of spectral limitation due to gain bandwidth-limited amplification and/or spectral filtering (which are inversely proportional to gain and/or spectral filtering bandwidth, respectively); χ accounts for nonlinear gain and/or other absorption processes (in the following, we shall call it effective nonlinear gain). The real part λ_r of λ represents the net TOD from material. The real part μ_r of μ is the nonlinear dispersion term; it is responsible for self-steepening at the pulse edge. The imaginary part μ_i of μ describes the combined effect of nonlinear gain and/or absorption processes; ν is the nonlinear gradient term which results from the time-retarded induced Raman process. In fact, the imaginary part ν_i of ν is usually responsible for the soliton SFS, and the real part ν_r of ν could be zero [1].

In this Letter, we present the analytic form of a stable solitary pulse solution for the dissipative ($\delta < 0$ and $\varepsilon > 0$) and driven ($\chi > 0$) situation ($\mu_i = \nu_r = 0$). The values for the various parameters will be discussed later. Proceeding with the analysis of Eq. (1) similar to Ref. [9], we separate $E(z, t)$ into the amplitude $A(\tau)$ and *nonlinear* phase according to $E(z, t) = A(\tau)\exp\{i[\Omega t - Kz + \beta\ln A(\tau)]\}$, where τ is the retarded time defined by $\tau = t - \rho z$, and β describes the possible chirp effects. Substituting that ansatz into Eq. (1), one can separate the complex envelope equation into two complex *ordinary* differential equations:

$$0 = n(\gamma_1 - \rho)A_\tau A^2 + n\gamma_3\{(n-1)(n-2)A_\tau^3 + 3(n-1)AA_\tau A_{\tau\tau} + A^2A_{\tau\tau\tau}\} + [n\gamma_4 + (2-n)\gamma_5]A_\tau A^4, \quad (2)$$

$$\gamma_2 n[(n-1)AA_\tau^2 + A^2A_{\tau\tau}] + \gamma_n A^5 + \gamma_0 A^3 = 0, \quad (3)$$

where $n = 1 + i\beta$, and the coefficients γ_j follow straightforwardly from the previous definitions.

Under the compatibility requirements $\gamma_2[n\gamma_4 + (2-n)\gamma_5] = (n+2)\gamma_3\gamma_n$ and $\gamma_2(\gamma_1 - \rho) = \gamma_3\gamma_0$ (for zero boundary conditions), we can solve either Eq. (2) or Eq. (3), and find the solution in the form $A(\tau) = A_0\,\mathrm{sech}(\eta\tau)$, with $n(n+1)\gamma_2\eta^2 = \gamma_n A_0^2$, $\gamma_2 n^2\eta^2 + \gamma_0 = 0$.

Returning to the electric field envelope, we get

$$E(z, t) = A_0\{\mathrm{sech}[\eta(t - \rho z)]\}^{1+i\beta}\exp[i(\Omega t - Kz)]. \quad (4)$$

By the present procedure we generate eight real algebraic equations for the free parameters. We do not present them in detail here, but discuss their solutions. In general, these eight equations will determine eight parameters of the problem. Six of them (A_0, η, ρ, β, Ω, and K) describe the characteristics of the solitonlike pulses. The other two will give compatibility conditions for Eq. (1), i.e., conditions under which damping and driving balance for stationary solutions.

Before proceeding, we remind the reader on a special case which is known in literature. In the absence of higher order terms, Eq. (1) simplifies to [1] $E_z - \frac{i}{2}E_{tt} - i|E|^2E = \delta E + \varepsilon E_{tt}$. In that special case, our eight algebraic equations reduce to six equations. By directly solving them, one can easily obtain the solitonlike solution which has been presented in Ref. [26], $E(z, t) = A_0\,\mathrm{sech}(\eta t)\exp\{-i[Kz + i\beta\ln\mathrm{sech}(\eta t)]\}$, where $\beta = (-3 \pm \sqrt{9 + 32\varepsilon^2})/4\varepsilon$, $\eta^2 = 3\delta/\varepsilon(1+\beta^2)$, $|A_0|^2 = -[3\beta\varepsilon + (2-\beta^2)/2]\eta^2$, $K = -[(1-\beta^2)/2 + 2\varepsilon\beta]\eta^2$. However, as pointed out in Ref. [26], that chirped solitonlike solution exists only for $\delta > 0$ because of $\varepsilon > 0$. It means that the background is unstable, and the noise may increase linearly with the distance.

For the general case, the relationship among the parameters is much more complicated. First, two equations can be used to determine ρ and K directly. Second, from two other equations one can rewrite the amplitude A_0 and the inverse pulse width η as

$$|A_0| = \eta\left[\frac{6(1-\beta^2)\lambda_r - \beta(11-\beta^2)\lambda_i}{3\mu_r - \beta\mu_i + 2\nu_r}\right]^{1/2}, \quad (5)$$

$$\eta = \left[\frac{-\sigma - 2\varepsilon\Omega + 3\lambda_i\Omega^2}{\lambda_i(1-\beta^2) + 2\lambda_r\beta}\right]^{1/2}. \quad (6)$$

Third, the frequency shift Ω can be obtained by a combination of two other equations, and, fourth, the chirp parameter β can be determined in a similar way. Instead of presenting the general expression for β, we only mention that the chirp parameter β depends on the higher order terms. It implies that a chirp will generally occur in the presence of higher order terms, except when the parameters satisfy the condition $3(\lambda_i\mu_r - \lambda_r\mu_i) = 2(\lambda_r\nu_i - \lambda_i\nu_r)$. Thus, in principle, also a solitary wave solution without chirp can exist in dispersive systems such as the HNLS equation.

The remaining two equations will provide constraints on the model parameters.

In summary, narrow, chirped, solitonlike pulse solutions of Eq. (1) can be calculated systematically in the form (4). The parameters follow from algebraic equations.

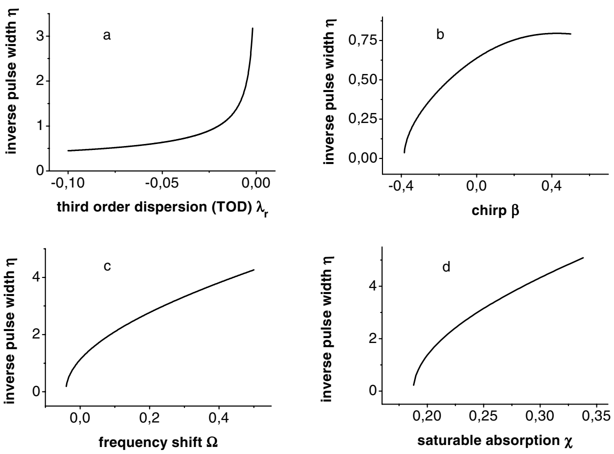

FIG. 1. The inverse pulse width η in dependence on typical parameters

We have investigated the dependence of the pulse widths on the parameters TOD λ_r, effective nonlinear gain χ, chirp β, and frequency shift Ω. As shown in Fig. 1(a), the smaller absolute TOD will lead to the narrower pulse widths. This is consistent with the well-known experimental results from ultrashort pulse lasers. Figure 1(b) predicts that for the positive chirp narrower pulses will occur than for the negative chirp. From Figs. 1(c) and 1(d), we can recognize that larger frequency shifts and stronger effective nonlinear gains will produce narrower pulses. These results are also consistent with the experimental observations in some laser and fiber amplifier systems (see, e.g., [27–29]).

A complete stability analysis of the solution (4) is complex, both analytically and numerically, since the parameter space is at least six dimensional. By employing a numerical split-step Fourier code to investigate the evolution of different initial pulses, we have found stable solitary pulses. For a typical example, we choose the following values: pulse duration 141 fs, pulse peak power 138.6 W, linear loss $1.3152\,\mathrm{m}^{-1}$, saturable absorption $0.63153\,\mathrm{W}^{-1}\mathrm{km}^{-1}$, nonlinear index $1.3\times10^{-22}\,(\mathrm{m/V})^2$, group velocity dispersion $-6.3761\,\mathrm{ps}^2\mathrm{km}$, third-order dispersion $0.79535\,\mathrm{ps}^3\mathrm{km}$ at wavelength $1.55\,\mu\mathrm{m}$, small signal gain $1.3147\,\mathrm{m}^{-1}$, gain bandwidth 20.335 THz, and saturable gain coefficient $0.19411\,\mathrm{W}^{-1}\mathrm{km}^{-1}$. These values give the following dimensionless parameters: $\delta=-0.00151$, $\sigma=0.06293$, $\varepsilon=0.50967$, $\chi=0.19158$, $\lambda_r=-0.03204$, $\lambda_i=0.00772$, $\mu_r=-0.04137$, and $\nu_i=-0.0256$. Then the solution (4) has the parameters $A_0=0.81585$, $\eta=0.65333$, $\Omega=-0.053115$, $\beta=0.4182$, $\rho=-0.03935$, and $K=-0.35887$ (without loss of generality, the parameters μ_i and ν_r have been set zero).

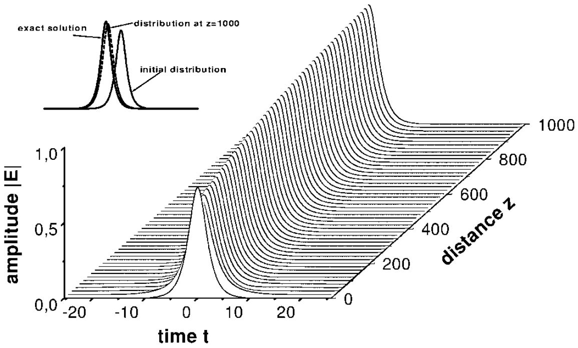

FIG. 2. Evolution of an initial pulse whose amplitude is 10% smaller than that of the exact solution, i.e., for $A_0=0.73472$. The other parameters are the same as specified in the text.

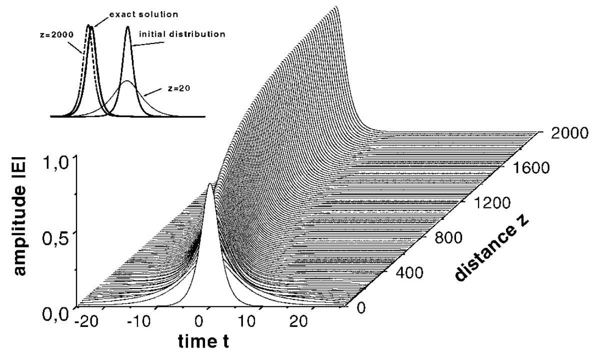

FIG. 3. Evolution of an initial pulse $E(0,t)=A_0[\mathrm{sech}(\eta t)]\exp[i(\Omega t+0.46t^2)]$. The parameters A_0, η, and Ω are specified in the text.

To demonstrate stability with respect to finite perturbations, we performed three types of numerical experiments. First, we perturbed the amplitude in the initial distribution. Second, we looked for the evolution of pulse forms with different initial chirps. Finally, we added white noise. Figure 2 shows the evolution of an initial pulse whose amplitude A_0 is 10% smaller than the theoretical prediction, i.e., 0.734 27. We clearly see that, after a short period of adjustment, the theoretically predicted pulse form evolves. Figure 3 depicts the evolution of a solution under the perturbation of chirp. The initial pulse is $E(0,t)=A_0[\mathrm{sech}(\eta t)]\exp[i(\Omega t+0.46t^2)]$, where A_0, η, and Ω have the values of the exact solution presented above. Note that the form for the initial chirp is different

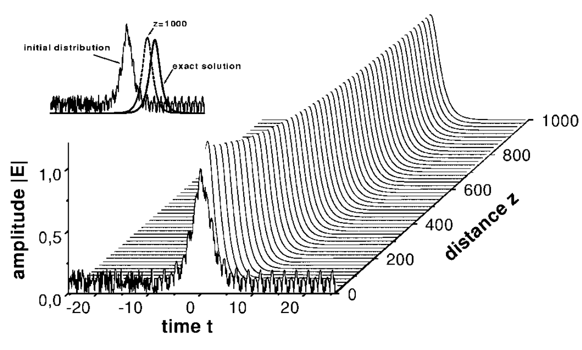

FIG. 4. Evolution of the exact solution under the perturbation of white noise whose maximal value is 0.2.

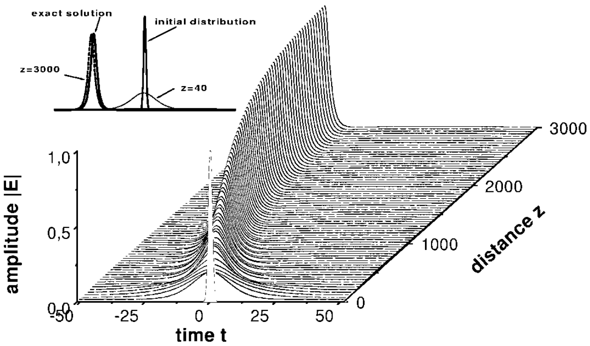

FIG. 5. Evolution of an initial Gaussian pulse $E(0, t) = \exp(-t^2)$.

from that in the exact solution. This difference explains the abrupt changes in the beginning. Nevertheless, finally the solution approaches the analytical one. Figure 4 shows the evolution under the perturbation of white noise whose maximal value is 0.2. From the evolution behaviors of the pulses, we can conclude that both, the solitonlike solution and the background, are stable under finite initial perturbations (see also the inlays in Figs. 2–5, which show comparisons of pulses at typical distances z with the initial as well as exact distributions).

Finally, we investigated the question of whether the predicted pulse form is an attractor under quite general conditions. Figure 5 shows the evolution of an initial Gaussian pulse $E(0, t) = \exp(-t^2)$. For a comparison with the analytical solution, we have fitted the parameter values at $z = 3000$. The results are $A_0 = 0.81051 \pm 0.01617$, $\eta = 0.64991 \pm 0.01857$, $\beta = 0.40776 \pm 0.01303$, and $\Omega = -0.05683 \pm 0.00417$. From these evaluations we conclude that the solitonlike solution (4) is an attractor in the parameter region under consideration. A theoretical model, making use of collective coordinates, is in progress.

In conclusion, we have obtained analytically the solitonlike solution for a generalized nonconservative system which models ultrashort laser pulse propagation. The stability of the solution has been investigated numerically. The results show that parameter regions exist in which both, the solitonlike solution and the background, are stable. In addition, we have also considered the evolution of an initial Gaussian pulse and found that it gradually approaches the analytically predicted solution, which acts as an attractor.

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