Long Time Correlations in Lagrangian Dynamics: A Key to Intermittency in Turbulence

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Using a new experimental technique, based on the scattering of ultrasounds, we perform a direct measurement of particle velocities, in a fully turbulent flow. This allows us to approach intermittency in turbulence from a dynamical point of view and to analyze the Lagrangian velocity fluctuations in the framework of random walks. We find experimentally that the elementary steps in the walk have random uncorrelated directions but a magnitude that is extremely long range correlated in time. Theoretically, a Langevin equation is proposed and shown to account for the observed one- and two-point statistics. This approach connects intermittency to the dynamics of the flow.

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Traditional experimental studies of velocity fluctuations in turbulence rely on velocimetry measurement at a fixed point in space. A local velocity probe yields time traces of the velocity fluctuations which are then related to spatial velocity profiles using the Taylor hypothesis [1]. In this case, the flow is analyzed in terms of the Eulerian velocity field $u(x, t)$. One of the most peculiar features of homogeneous three-dimensional turbulence is its intermittency, well established in the Eulerian framework [2]. The statistical properties of the flow depend on the length scale at which it is analyzed. For instance, the functional form of the probability of measuring an Eulerian velocity increment $\Delta_s u(x) = u(x+s) - u(x)$ varies with the magnitude of the length scale s. Many studies devoted to the understanding of this feature have been developed along the lines of Kolmogorov and Obukhov pioneering ideas [3]. In this case, intermittency is analyzed in terms of the anomalous scaling of the moments of velocity increments in space. It is attributed to the inhomogeneity in space of the turbulent activity and often analyzed in terms of *ad hoc* multiplicative cascade models [2]. Although very successful at describing the data, these models have failed to connect intermittency with the specific dynamics of turbulence. Here, we adopt a Lagrangian point of view. It is a natural framework for mixing and transport problems in turbulence [4]. In addition it has been shown in the passive scalar problem that intermittency is strongly connected to the particular properties of Lagrangian trajectories [5,6]. In the Lagrangian approach, the flow is parametrized by $v(x_0, t)$, the velocity of a fluid particle initially at position x_0. Experimentally, we follow the motion of a single tracer particle and we consider the increments in time of its velocity fluctuations: $\Delta_\tau v(t) = v(t+\tau) - v(t)$. Our first observations [7] have established and described intermittency in this Lagrangian framework. Since our measurements give access to the individual motion of fluid particles, we study intermittency from a dynamical point of view. We show, within a Langevin approach, that the anomalous scaling in the Lagrangian velocity increments traces back to the existence of long time correlations in the particle accelerations, i.e., the hydrodynamic forces that drive the particle motion.

In order to study the motion of Lagrangian tracers, we need to resolve their velocity fluctuations across a wide range of scales. To this end, we use a confined flow with no mean advection, so that fluid particles remain for long times inside a given measurement volume. The tracking of small tracer particles is achieved using a new acoustic technique based on the principle of a "continuous Doppler sonar." The flow volume is continuously insonified with a monochromatic ultrasound which is then scattered by the tracer particle [7]. This scattered sound is detected by two transducer arrays which yield a measurement of both the particle position, by direct triangulation, and of its velocity, from the Doppler shift. Indeed, for an incoming sound with frequency f_0, the scattered sound at the receiver has frequency $f(t) = f_0 + \mathbf{k} \cdot \mathbf{v}(t)$, where $\mathbf{v}(t)$ is the velocity of the tracer particle and $\mathbf{k}$ is the scattering wave vector. This frequency modulation in the acoustic signal is extracted numerically, using a high-resolution parametric method [8]. Figure 1(a) shows the experimental setup and an example of a particle trajectory; Fig. 1(b) gives an example of the time variation of one component of its velocity. A water flow of the von Kármán swirling type [7,9] is generated inside a cylinder by counterrotation at $\Omega = 7$ Hz of two disks with radius $R = 9.5$ cm, fitted with eight blades of height 0.5 cm and set 18 cm apart. The large scale flow is axisymmetric and the fluctuations in its center approximate well the conditions of local homogeneous and isotropic turbulence. The flow power consumption is $\epsilon = 25$ W/kg, with velocity fluctuations $u_{\rm rms} = 0.98$ m/s. The characteristic size of the velocity gradients is $\ell = (15u_{\rm rms}^2/\epsilon)^{1/2} = 880$ μm, larger than the diameter (250 μm) of the neutrally buoyant tracer particle (density of 1.06). The turbulent Reynolds number of the flow is $R_\ell = u_{\rm rms}\ell/\nu = 740$. The integral length and time scales are $L = 4$ cm and $T_L = 22.4$ ms, while the dissipative scales are $\eta = 15$ μm and $\tau_\eta = 0.2$ ms, respectively.

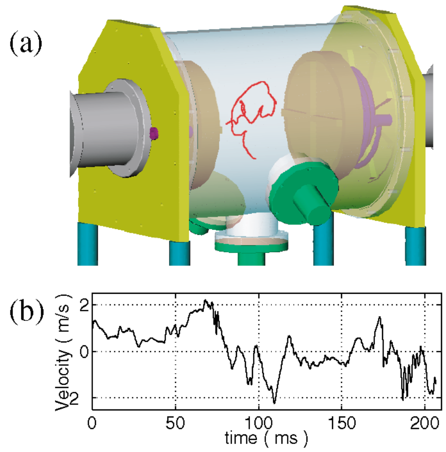

FIG. 1 (color online). (a) Experimental setup of the von Kármán flow, with an example of a particle 3D trajectory; (b) corresponding velocity variation (one component shown).

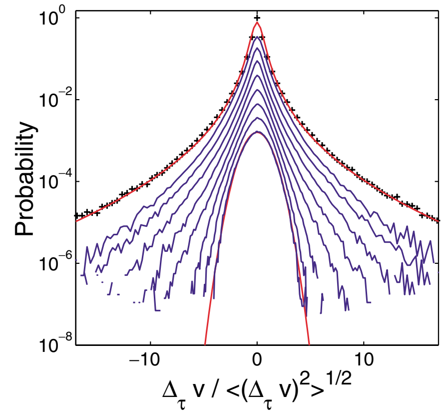

FIG. 2 (color online). PDF $\Pi_\tau(\Delta v)$ calculated for time lags $100\tau/T_L = 1.3, 2.7, 5.4, 11.2, 22.4, 44, 89.3, 174$. The curves are displayed with a vertical shift for clarity. Crosses correspond to the MRW model with $\lambda^2 = 0.115$.

The flow is insonified at 2.5 MHz, with the transducers located at the flow wall. The receiver arrays are placed at 45° on each side of the emission direction. The measurement region is the intersection of the emission and detection cones. The particles act as Lagrangian tracers for times longer than 1 ms (below which inertia cuts off their response), up to times as long as they will stay confined inside the measurement volume, i.e., between one and ten T_L, the Lagrangian integral time scale ($T_L = 22.4$ ms, computed from the Lagrangian velocity autocorrelation function). Four thousand such events are analyzed, for a total of 1.9×10^6 data points sampled at 6.5 kHz.

The probability density functions (PDFs) of the Lagrangian time increments $\Pi_\tau(\Delta v)$ are shown in Fig. 2. They are Gaussian at integral scale ($\tau > T_L$) and vary continuously towards the development of stretched exponential tails as the time increments decrease towards the dissipative time scale [7,10]. The outer curve in Fig. 2 is the PDFs of Lagrangian acceleration measured by La Porta *et al.* [9] in the same flow geometry and at a comparable turbulent Reynolds number. This evolution of the PDFs $\Pi_\tau(\Delta v)$ leads to an anomalous scaling of the velocity structure functions $\langle|\Delta_\tau v|^q\rangle \sim \tau^{\zeta(q)}$, with $\zeta(q)$ a nonlinear function of q. In practice, exponents are estimated via the relative scaling $\langle|\Delta_\tau v|^q\rangle \sim \langle|\Delta_\tau v|^2\rangle^{\xi(q)}$, where $\xi(q) = \zeta(q)/\zeta(2)$ [11]. In Kolmogorov K41 phenomenology [2], the Lagrangian second order structure function is assumed to scale linearly with the time increment τ so that $\zeta(2)$ is considered as being equal to one [7]. Given the available sample size, moments are investigated up to order 6. We observe that $\xi(q)$ as a function of q is well approximated by a quadratic law in the range $0 \le q \le 6$ [7]:

$$\xi(q) = (1/2 + \lambda_L^2)q - \lambda_L^2 q^2/2, \quad \lambda_L = 0.115 \pm 0.01. \tag{1}$$

The Lagrangian value of the intermittency parameter ($\lambda_L^2 = 0.115$) is larger than the Eulerian value ($\lambda_E^2 = 0.025$), measured from hot-wire anemometry or in direct numerical simulations [12]. This mainly comes from the fact that time increments are measured here and that the reference structure function is the second order one [13].

Intermittency is thus observed and quantified in both Lagrangian and Eulerian frameworks. In contrast to traditional Eulerian studies where intermittency is described in terms of multiplicative processes, we look here for a dynamical origin. We consider the statistics of the fluid particles fluctuating velocity in analogy with a random walk. We write a velocity increment over a time lag τ as the sum of contributions over small times τ_1:

$$\Delta_\tau v(t) = v(t+\tau) - v(t) = \sum_{n=1}^{\tau/\tau_1} \Delta_{\tau_1} v(t + n\tau_1). \tag{2}$$

If the incremental "steps" of duration τ_1 were independent (and identically distributed), the PDF $\Pi_\tau(\Delta v)$ would readily be obtained as a convolution of the elementary distribution at scale τ_1, $\Pi_{\tau_1}(\Delta v)$—plus an eventual convolution kernel to account for stationarity at large scales. Such a regular convolution process corresponds to the Kolmogorov K41 picture of turbulence [2]; the particle velocity fluctuations are Brownian and the scaling is monofractal. A first important result of our analysis is to show that the elementary steps are not independent. The autocorrelation of the signed increments, $\Delta_{\tau_1} v(t)$, decays very rapidly (Fig. 3): the correlation coefficient drops under 0.05 for time separations larger than $2\tau_1$. However, if one considers the amplitude of the steps $|\Delta_{\tau_1} v(t)|$, one finds that the autocorrelation decays very slowly and only vanishes at the largest time scales of the turbulent motion. Recast in terms of the random walk, our results show that the amplitudes of the steps are long range correlated in time although their directions are not. As this point is fundamental, we have verified it using a Lagrangian tracking algorithm in a direct numerical simulation of the Navier-Stokes equations, using a pseudospectral solver, at $R_\ell = 75$, and for the same ratio τ_1/T_L (Fig. 3). The results are in remarkable agreement with our measurements. All increments are correlated for $\Delta t < \tau_1$, the time over which they are computed. Above τ_1, the correlation of the signed increments rapidly drops while the correlation coefficient of their absolute values decays very slowly, to vanish only for $\Delta t > 3\,T_L$. This behavior is observed for τ_1 chosen from the smallest resolved time scales to inertial range values. These observations persist in the limit of very small time increments, and thus presumably for the acceleration of the fluid particle and thus for the forces acting on it.

Theoretically, one would like to understand this behavior from the hydrodynamic forces in the Navier-Stokes equations. Such a direct analytical treatment is out of reach at present. We propose, as a natural first step, to study a surrogate dynamical equation of the Langevin type. In this procedure, one considers a one-dimensional variable, $W(t)$, representing the particle velocity, driven by a stochastic force:

$$dW = -\frac{1}{T_L} W dt + dF(t). \tag{3}$$

If this force is chosen as a white noise then $W(t)$ has the dynamics of Brownian motion (discarding the effect of

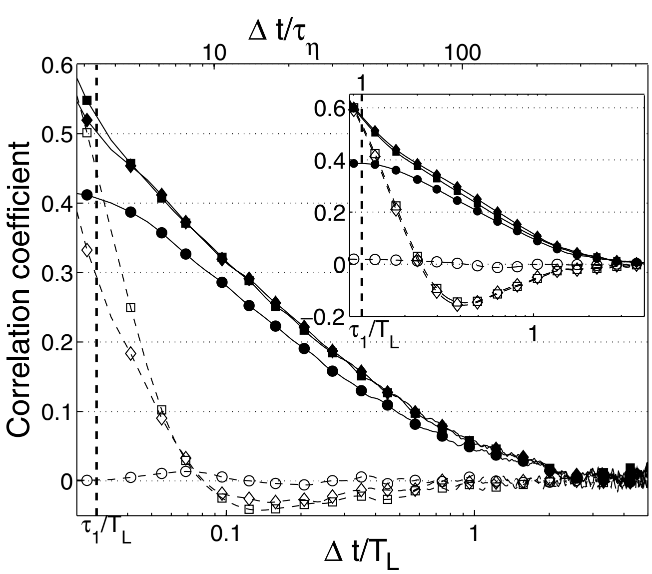

FIG. 3. Variation of the normalized correlation coefficient $\chi(f, g)(\Delta t) = \langle [f(t + \Delta t) - \langle f \rangle][g(t) - \langle g \rangle] \rangle / \sigma_f \sigma_g$. Two velocity components are considered: the squares and diamonds mark the $\chi(\Delta_{\tau_1} v_x, \Delta_{\tau_1} v_x)$ and $\chi(\Delta_{\tau_1} v_y, \Delta_{\tau_1} v_y)$ autocorrelation functions while the circles mark the cross correlation $\chi(\Delta_{\tau_1} v_x, \Delta_{\tau_1} v_y)$; curves with filled symbols are computed using the absolute value of the increments while the curves with open symbols are computed using the full signed increments. The main curve corresponds to the experiment at $R_\ell = 740$, with $\tau_1 = 0.03 T_L$. The inset shows similar results for a direct numerical simulation at $R_\ell = 75$.

the drift term that accounts for stationarity), its statistics is monofractal with a similarity exponent equal to 1/2—the increments scale as $\langle |W(t + \tau) - W(t)|^p \rangle_t \sim \tau^{p/2}$, corresponding to the nonintermittent Kolmogorov picture. In order to account for intermittency, one needs to ascribe other properties to the stochastic force. Guided by our experimental results, we build a stochastic force $F(t) = A(t)G(t)$, having a random direction and a long-range correlated magnitude. Specifically, its direction is modeled by a Gaussian variable $G(t)$, chosen white in time, with zero mean and unit variance. Its amplitude, $A(t)$, being a positive variable, is written $A(t) = \exp[\omega(t)]$ where the magnitude $\omega(t)$ is a stochastic process that satisfies

$$\langle \omega(t)\omega(t + \Delta t) \rangle_t = -\lambda^2 \ln(\Delta t / T_L) \quad \text{for } \Delta t < T_L \tag{4}$$

and 0 otherwise, λ^2 being an adjustable parameter. When discretized, this dynamics corresponds to a one-dimensional multifractal random walk (MRW) [14]. Analytical calculations show that the resulting dynamical variable $W(t)$ has multiscaling properties. The moments have scaling laws, $\langle |\Delta_\tau W|^q \rangle \sim \tau^{\zeta(q)}$, with $\zeta(q) = (1/2 + \lambda^2)q - \lambda^2 q^2/2$, so that λ^2 in Eq. (4) is the intermittency parameter of the model [14]. It is a fundamental point that the same parameter λ^2 governs both the evolution of the PDFs of the increments (one-time statistics) and the time correlation of the process (two-time statistics).

We find that this model captures the essential features of the Lagrangian data. First, in order to test the relevance of Eq. (4), we have computed, from the experimental and numerical data, the autocorrelation function of the logarithm of the amplitude of infinitesimal Lagrangian velocity increments. Figure 4(a) confirms that the logarithmic decrease build in the MRW model [Eq. (4)] is observed both in the experimental and the numerical data; it yields the estimate $\lambda^2 = 0.115 \pm 0.01$. Second, we check the relevance of the model for the description of the one-time statistics of the Lagrangian increments $\Delta_\tau v$. We note (Fig. 2, upper curve) that the choice $\lambda^2 = 0.115$ yields a PDF for the stochastic force that is in remarkable agreement with experimental measurements of fluid particle accelerations [9]. The agreement at larger time scales is evidenced on the behavior of the first two cumulants. Cumulants are computed with more reliability that the moments and are related to them through $\langle |\Delta_\tau v|^q \rangle = \langle \exp(q \ln|\Delta_\tau v|) \rangle = \exp[\sum_n C_n(\tau) q^n / q!]$. In the MRW model, one can analytically derive [14]

$$C_1(\tau) = (1 + \lambda^2)\ln(\tau), \qquad C_2(\tau) = -\lambda^2 \ln(\tau), \tag{5}$$

all higher order cumulants being null. $C_1(\tau)$ and $C_2(\tau)$ computed from the experimental and numerical data are shown in Figs. 4(b) and 4(c) and compared to MRW model predictions when the intermittency parameter is set to the value $\lambda^2 = 0.115$ derived from the correlations in the dynamics. In each case the agreement is excellent;

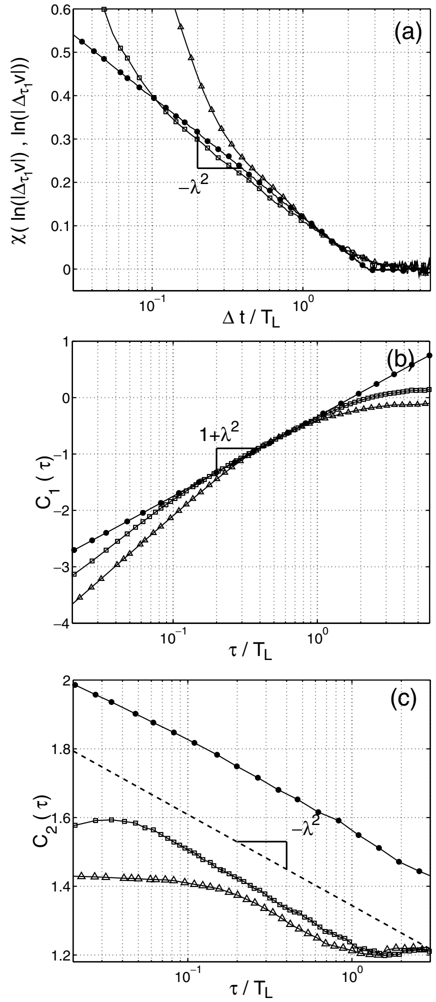

FIG. 4. Experimental (open squares) and numerical data (open triangles) compared to the predictions of the MRW model (filled circles). (a) Correlation in time of the magnitude of one component of the velocity increments, $\chi(\ln|\Delta_{\tau_1} v_x|, \ln|\Delta_{\tau_1} v_x|)$, computed for a time lag $\tau_1 = 0.03T_L$. (b),(c): first and second order cumulants versus time scale τ.

the slope of the variation $\partial C_{1,2}(\tau)/\partial \ln\tau$ in the inertial range is correctly given by Eq. (5). The same intermittency parameter thus governs the anomalous scaling of the Lagrangian velocity increments and their long time dynamical correlations. We note however, that while the scale dependence ($\partial C_2/\partial \ln\tau = -\lambda^2$) is correctly given by the MRW model, the predicted value of C_2 is about 20% higher than the experimental value. This prevents a complete fitting of the experimental PDFs of velocity increments; only their rate of change across the scales is correctly given and directly related to the anomalous intermittency exponents.

We therefore believe that long time correlations in the Lagrangian dynamics are a key feature for the understanding of intermittency, which leads to a new dynamical picture of turbulence. Long time correlations and the occurrence of very large fluctuations at small scales dominate the motion of a fluid particle. It can be understood if, along its trajectory, the particle encounters very intense small-scale structures over a more quiet background. Intermittency is then due to the nature and distribution of these small-scale structures. Indeed, the analogy with a random walk suggests that the statistics at all scales can be recovered if one ascribes two properties to the small scales: (1) the probability density function of fluid particle accelerations and (2) the functional form of their time correlations. In the Lagrangian framework, these features are directly linked to the Navier-Stokes equations that govern the elementary changes in the velocity (momentum) of the fluid particles. It thus gives a possibility to derive intermittency from the constitutive physical equations. Although this may be quite a theoretical challenge, direct numerical simulations look promising as they allow the study of the flow dynamical fields (pressure, velocity gradient tensor, etc.) along the trajectory of individual fluid particles.

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[1] A. M. Monin and A. S. Yaglom, *Statistical Fluid Mechanics* (MIT Press, Cambridge, MA, (1987).
[2] U. Frisch, *Turbulence* (Cambridge University Press, Cambridge, (1995).
[3] A. N. Kolmogorov, J. Fluid Mech. **13**, 82 (1962); A. M. Obhukov, J. Fluid Mech. **13**, 77 (1962).
[4] For example, see J. M. Ottino, *The Kinematics of Mixing* (Cambridge University Press, Cambridge, (1989).
[5] G. Falkovich, K. Gawedzki, and M. Vergassola, Rev. Mod. Phys. **73**, 913 (2001).
[6] A Pumir, B. Shraiman, and M. Chertkov, Phys. Rev. Lett. **85**, 5324 (2001).
[7] N. Mordant, P. Metz, O. Michel, and J.-F. Pinton, Phys. Rev. Lett. **87**, 214501 (2001).
[8] N. Mordant, O. Michel, and J.-F. Pinton, J. Acoust. Soc. Am. **112**, 108–119 (2002).
[9] A. L. La Porta *et al.*, Nature (London) **409**, 1017 (2001).
[10] P. K. Yeung and S. B. Pope, J. Fluid Mech. **207**, 531 (1989).
[11] R. Benzi, S. Ciliberto, C. Baudet, G. Ruiz-Chavarria, and C. Tripiccione, Europhys. Lett. **24**, 275 (1993).
[12] J. Delour, J.-F. Muzy, and A. Arnéodo, Eur. Phys. J. B **23**, 243 (2001).
[13] M. S. Borgas, Philos. Trans. R. Soc. London A **342**, 379 (1993).
[14] E. Bacry, J. Delour, and J.-F. Muzy, Phys. Rev. E **64**, 026103 (2001).