Experimental Demonstration of Continuous Variable Polarization Entanglement

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We report the experimental transformation of quadrature entanglement between two optical beams into continuous variable polarization entanglement. We extend the inseparability criterion proposed by Duan *et al.* [Phys. Rev. Lett. **84**, 2722 (2000)] to polarization states and use it to quantify the entanglement. We propose an elaboration utilizing two quadrature entangled pairs for which all three Stokes operators between a pair of beams are entangled.

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The polarization state of light has been extensively studied in the quantum mechanical regime of single (or few) photons. The demonstration of entanglement of the polarization of pairs of photons has been of particular interest. This entanglement has facilitated the study of many interesting quantum phenomena such as Bell's inequality [1]. Comparatively, research on continuous variable quantum polarization states has been cursory. Recently, however, interest in the field has increased due to the demonstration of transfer of continuous variable quantum optical polarization states to the spin state of atomic ensembles [2]; and to its potential for local oscillator-free continuous variable quantum communication networks. A number of theoretical papers have now been published [3,4], of particular interest is the work of Korolkova *et al.* [5] which introduces the new concept of continuous variable polarization entanglement, and proposes methods for its generation and characterization. Previous to the work presented here, however, only the squeezing of polarization states had been experimentally demonstrated [2,6,7].

In this paper we report the experimental transformation of the commonly studied and well understood entanglement between the phase and amplitude quadratures of two beams (quadrature entanglement) [8] onto a polarization basis. Quadrature entanglement can be characterized using the inseparability criterion proposed by Duan *et al.* [9]. We generalize this criterion to an arbitrary pair of observables and apply it to the Stokes operators that define quantum polarization states. We experimentally generate entanglement of Stokes operators between a pair of beams, satisfying both the inseparability criterion, and the product of conditional variances which is a signature of the EPR paradox [10]. Interacting this entanglement with a pair of distant atomic ensembles could entangle the atomic spin states. We also analyze the polarization state generated by combining two quadrature entangled pairs. We show that if the quadrature entanglement is strong enough to beat a bound $\sqrt{3}$ times lower than that for the inseparability criterion, then all three Stokes operators can be simultaneously entangled.

The polarization state of a light beam can be described as a Stokes vector on a Poincaré sphere and is determined by the four Stokes operators [11]: $\hat{S}_0$ represents the beam intensity whereas $\hat{S}_1$, $\hat{S}_2$, and $\hat{S}_3$ characterize its polarization and form a Cartesian axis system. If the Stokes vector points in the direction of $\hat{S}_1$, $\hat{S}_2$, or $\hat{S}_3$, the polarized part of the beam is horizontally, linearly at 45°, or right-circularly polarized, respectively. Quasimonochromatic laser light is almost completely polarized, in this case $\hat{S}_0$ is a redundant parameter, determined by the other three operators. All four Stokes operators can be measured with simple experiments [5]. Following [11] we expand the Stokes operators in terms of the annihilation $\hat{a}$ and creation $\hat{a}^\dagger$ operators of the constituent horizontally (subscript H) and vertically (subscript V) polarized modes

$$\hat{S}_0 = \hat{a}_H^\dagger \hat{a}_H + \hat{a}_V^\dagger \hat{a}_V, \qquad \hat{S}_2 = \hat{a}_H^\dagger \hat{a}_V e^{i\theta} + \hat{a}_V^\dagger \hat{a}_H e^{-i\theta},$$
$$\hat{S}_1 = \hat{a}_H^\dagger \hat{a}_H - \hat{a}_V^\dagger \hat{a}_V, \qquad \hat{S}_3 = i\hat{a}_V^\dagger \hat{a}_H e^{-i\theta} - i\hat{a}_H^\dagger \hat{a}_V e^{i\theta}, \tag{1}$$

where θ is the phase difference between the H, V-polarization modes. Equations (1) are an example of the well-known bosonic representation of angular momentum type operators in terms of a pair of quantum harmonic oscillators introduced by Schwinger [12]. The commutation relations of the annihilation and creation operators $[\hat{a}_k, \hat{a}_l^\dagger] = \delta_{kl}$ with $k, l \in \{H, V\}$ directly result in Stokes operator commutation relations, $[\hat{S}_i, \hat{S}_j] = 2i\hat{S}_k$, where $i, j, k = 1, 2, 3$ are cyclically interchangeable. These commutation relations dictate uncertainty relations, which indicate that entanglement is possible between the Stokes operators of two beams, we term this continuous variables polarization entanglement. Three observables are involved, compared to two for quadrature entanglement, and the entanglement between two of them relies on the mean value of the third. To provide a proper definition of this entanglement, we have chosen to extend the inseparability criterion proposed by Duan *et al.* [9]. The inseparability criterion characterizes the separability of, and therefore the entanglement between, the amplitude $\hat{X}^+$ and phase $\hat{X}^-$ quadratures of a pair of optical beams (denoted throughout by the subscripts x and y) with Gaussian noise statistics. These quadrature operators are observables and can be obtained from the annihilation and creation operators, $\hat{X}^+ = \hat{a} + \hat{a}^\dagger$, $\hat{X}^- = i(\hat{a}^\dagger - \hat{a})$. In this paper we restrict ourselves to the symmetric situation where all experimental outcomes are independent of exchange of beams x and y, in this case the inseparability criterion can be written as

$$\Delta^2_{x\pm y}\hat{X}^+ + \Delta^2_{x\pm y}\hat{X}^- < 4. \quad (2)$$

Throughout this paper $\Delta^2\hat{O} = \langle \delta\hat{O}^2\rangle$ where $\hat{O} = \langle\hat{O}\rangle + \delta\hat{O}$. $\Delta^2_{x\pm y}\hat{O}$ is the smaller of the sum and difference variances of the operator $\hat{O}$ between beams x and y, $\Delta^2_{x\pm y}\hat{O} = \min\langle(\delta\hat{O}_x \pm \delta\hat{O}_y)^2\rangle$. Note that for physically realistic entanglement between two observables, one observable must be correlated, and the other anticorrelated between subsystems x and y. The minimization utilized in calculating $\Delta^2_{x\pm y}\hat{O}$ selects the relevant sign for each observable. The measure in Eq. (2) relies explicitly on the uncertainty relation between the amplitude and phase quadrature operators. Given the general Heisenberg uncertainty relation $\Delta^2\hat{A}\Delta^2\hat{B} \geq |\langle\delta\hat{A}\delta\hat{B}\rangle|^2 = |[\delta\hat{A}, \delta\hat{B}]|^2/4 + |\langle\delta\hat{A}\delta\hat{B} + \delta\hat{B}\delta\hat{A}\rangle|^2/4$ [13] it can be generalized to any pair of observables $\hat{A}$, $\hat{B}$. Unlike the commutation relation $|[\delta\hat{A}, \delta\hat{B}]|$, the correlation function $|\langle\delta\hat{A}\delta\hat{B} + \delta\hat{B}\delta\hat{A}\rangle|$ is state dependant. In this work we assume it to be zero and arrive at the sufficient condition for inseparability

$$\Delta^2_{x\pm y}\hat{A} + \Delta^2_{x\pm y}\hat{B} < 2|[\delta\hat{A}, \delta\hat{B}]|. \quad (3)$$

To allow direct analysis of our experimental results, we define the degree of inseparability $I(\hat{A}, \hat{B})$, normalized such that $I(\hat{A}, \hat{B}) < 1$ guarantees the state is inseparable

$$I(\hat{A}, \hat{B}) = \frac{\Delta^2_{x\pm y}\hat{A} + \Delta^2_{x\pm y}\hat{B}}{2|[\delta\hat{A}, \delta\hat{B}]|}. \quad (4)$$

An arbitrary pair of polarization modes may be constructed by combining horizontally and vertically polarized modes on a pair of polarizing beam splitters. In the symmetric situation, which this paper is restricted to, the horizontally (vertically) polarized input beams must be interchangeable; therefore, their expectation values and variances must be the same ($\alpha_H = \langle\hat{a}_{H,x}\rangle = \langle\hat{a}_{H,y}\rangle$, $\alpha_V = \langle\hat{a}_{V,x}\rangle = \langle\hat{a}_{V,y}\rangle$, $\Delta^2\hat{X}^\pm_H = \Delta^2\hat{X}^\pm_{H,x} = \Delta^2\hat{X}^\pm_{H,y}$, $\Delta^2\hat{X}^\pm_V = \Delta^2\hat{X}^\pm_{V,x} = \Delta^2\hat{X}^\pm_{V,y}$), and the relative phase between horizontally and vertically polarized modes for subsytems x and y must be related by $\theta = \theta_x = \pm\theta_y + m\pi$ where m is an integer. Given these assumptions it is possible to calculate $I(\hat{S}_i, \hat{S}_j)$ from Eqs. (1). We choose to simplify the situation further, providing results that may be directly related to our experiment. We assume that the horizontal and vertical inputs are not correlated, and that each input beam does not exhibit internal amplitude/phase quadrature correlations. Finally, we assume that the vertically polarized input modes are bright ($\alpha_V^2 \gg 1$) so that second order terms are negligible. The denominators of Eq. (4) for the three possible combinations of Stokes operators are then found to be

$$\begin{aligned} |[\delta\hat{S}_1\delta, \hat{S}_2]| &= 4\alpha_H\alpha_V\sin\theta, \\ |[\delta\hat{S}_1, \delta\hat{S}_3]| &= 4\alpha_H\alpha_V\cos\theta, \\ |[\delta\hat{S}_2, \delta\hat{S}_3]| &= 2|\alpha_H^2 - \alpha_V^2|. \end{aligned} \quad (5)$$

In our experiment the phase θ between the horizontally and vertically polarized input modes was controlled to be $\pi/2$, in this situation $|[\delta\hat{S}_1, \delta\hat{S}_3]| = 0$ which means that using the inseparability criterion of Eq. (4) it is impossible to verify entanglement between $\hat{S}_1$ and $\hat{S}_3$. On the other hand $|[\delta\hat{S}_1, \delta\hat{S}_2]|$ and $|[\delta\hat{S}_2, \delta\hat{S}_3]|$ both have finite values and therefore the potential for entanglement. We experimentally determined $I(\hat{S}_1, \hat{S}_2)$ and $I(\hat{S}_2, \hat{S}_3)$ from measurements of α_V, α_H, and $\Delta^2_{x\pm y}\hat{S}_i$.

The experimental transformation between quadrature and polarization entanglement demonstrated here becomes clearer if $\Delta^2_{x\pm y}\hat{S}_i$ are expressed in terms of quadrature operators. Assuming that $\alpha_H^2 \ll \alpha_V^2$ we find from Eqs. (1) that $\Delta^2_{x\pm y}\hat{S}_1 = \alpha_V^2\Delta^2_{x\pm y}\hat{X}^+_V$, $\Delta^2_{x\pm y}\hat{S}_2 = \alpha_V^2\Delta^2_{x\pm y}\hat{X}^-_H$, and $\Delta^2_{x\pm y}\hat{S}_3 = \alpha_V^2\Delta^2_{x\pm y}\hat{X}^+_H$. $I(\hat{S}_1, \hat{S}_2)$ and $I(\hat{S}_2, \hat{S}_3)$ can then be written

$$I(\hat{S}_1, \hat{S}_2) = \frac{\alpha_V}{\alpha_H}\left(\frac{\Delta^2_{x\pm y}\hat{X}^+_V + \Delta^2_{x\pm y}\hat{X}^-_H}{8}\right), \quad (6)$$

$$I(\hat{S}_2, \hat{S}_3) = \left(1 + \frac{\alpha_H^2}{\alpha_V^2}\right)\left(\frac{\Delta^2_{x\pm y}\hat{X}^+_H + \Delta^2_{x\pm y}\hat{X}^-_H}{4}\right). \quad (7)$$

Equation (6) shows that as α_V/α_H increases the level of correlation required for $I(\hat{S}_1, \hat{S}_2)$ to fall below unity and therefore to demonstrate inseparability quickly becomes experimentally unachievable. In particular, if the horizontal inputs are vacuum states $I(\hat{S}_1, \hat{S}_2)$ becomes infinite and verification of entanglement is not possible. In contrast, Eq. (7) shows that in this situation $I(\hat{S}_2, \hat{S}_3)$ becomes identical to the criterion for quadrature entanglement [Eq. (2)] between the two horizontally polarized inputs. Therefore, quadrature entanglement between the horizontally polarized inputs is transformed to polarization entanglement between $\hat{S}_2$ and $\hat{S}_3$. In the following section we experimentally demonstrate this transformation. The asymmetry of these results arises because the Stokes vector of the output mode of each polarizing beam splitter is aligned almost exactly along $\hat{S}_1$ (since $\alpha_H \gg \alpha_V$). This creates an asymmetry in the commutation relations and a corresponding bias in the uncertainty relations that define the inseparability criteria.

In our experiment two equal power 1064 nm amplitude squeezed beams were produced in a pair of spatially separated optical parametric amplifiers (OPAs). The

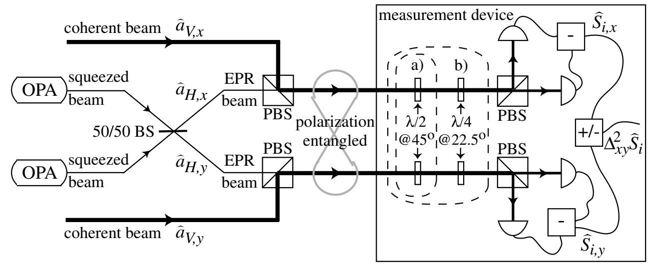

FIG. 1. Experimental production and characterization of continuous variable polarization entanglement. The optics within (a) are included to measure $\hat{S}_2$, and those within (b) to measure $\hat{S}_3$. (P)BS: (polarizing) beam splitter.

OPAs were optical resonators constructed from hemilithic MgO:LiNbO$_3$ crystals and output couplers and are described in detail in Ref. [7]. We combined the squeezed beams with $\pi/2$ phase shift on a 50/50 beam splitter with interference efficiency of 97.8%. The output beams exhibited the conventional quadrature entanglement [8]. We modematched each entangled beam into a homodyne detector that provided amplitude or phase quadrature measurements and characterized the entanglement with the inseparability measure given in Eq. (2). We obtained the result $I(\hat{X}^+, \hat{X}^-) = (\Delta^2_{x\pm y}\hat{X}^+ + \Delta^2_{x\pm y}\hat{X}^-)/4 = 0.44$, which is well below unity. We also determined the product of conditional variances between the beams $(\min_g[\langle(\delta\hat{X}^+_x + g\delta\hat{X}^+_y)^2\rangle\langle(\delta\hat{X}^-_x - g\delta\hat{X}^-_y)^2\rangle] < 1)$, which was propose by Reid and Drummond [10] as a signature of the EPR paradox. We observed a value of 0.58 which is also well below unity.

We transformed the entanglement onto a polarization basis by combining each entangled beam (horizontally polarized) with a much more intense vertically polarized coherent beam ($\alpha_V^2 = 30\alpha_H^2$) with measured modematching efficiency for both of 91% (see Fig. 1). The relative phase between the horizontal and vertical input modes θ was controlled to be $\pi/2$. The two resultant beams were polarization entangled. We verified this entanglement by measuring correlations of the Stokes operators between the beams.

Each beam was split on a polarizing beam splitter and the two outputs were detected on a pair of high quantum efficiency photodiodes. Dependent on the inclusion of wave plates before the polarizing beam splitter, the difference photocurrent between the two photodiodes yielded instantaneous values for $\hat{S}_1$, $\hat{S}_2$, or $\hat{S}_3$ (see Fig. 1). The variance of the unity gain electronic sum or subtraction of the Stokes operator measurements between the polarization entangled beams was obtained in a spectrum analyzer that had a resolution bandwidth of 300 kHz and video bandwidth of 300 Hz. This resulted in values for $\Delta^2_{x\pm y}\hat{S}_i$. All of the presented results were taken over the sideband frequency range from 2 to 10 MHz and are the average of ten consecutive traces. Every trace was more than 4.5 dB above the measurement dark noise which was taken into account. We determined α_V^2 directly by blocking the horizontal modes and measuring the power spectrum of the subtraction between the two homodynes, this also gave α_H^2 since the ratio α_V^2/α_H^2 was measured to equal 30.

Figure 2 shows our experimental measurements of $I(\hat{S}_1, \hat{S}_2)$ and $I(\hat{S}_2, \hat{S}_3)$. The dashed lines indicate the results a pair of coherent beams would produce. Both traces are below this line throughout almost the entire measurement range; this is an indication that the light is in a nonclassical state. At low frequencies both traces were degraded by noise introduced by the relaxation oscillation of our laser. $I(\hat{S}_2, \hat{S}_3)$ shows polarization entanglement, however as expected $I(\hat{S}_1, \hat{S}_2)$ is far above unity. The best entanglement was observed at 6.8 MHz with $I(\hat{S}_2, \hat{S}_3) = 0.49$ which is well below unity.

By electronically adding or subtracting the Stokes operator measurements with a gain g chosen to minimize the resulting variance we observed a signature of the EPR paradox for polarization states. In this case the product of the conditional variances of $\hat{S}_2$ and $\hat{S}_3$ from one beam after utilizing information gained through measurement of the other must be less than the Heisenberg uncertainty product $(\min_g[\langle(\delta\hat{S}_{2,x} \pm g\delta\hat{S}_{2,y})^2\rangle\langle(\delta\hat{S}_{3,x} \pm g\delta\hat{S}_{3,y})^2\rangle] < |[\delta\hat{S}_2, \delta\hat{S}_3]|^2/4)$. We observed a conditional variance product of $0.77|[\delta\hat{S}_2, \delta\hat{S}_3]|^2/4$.

Polarization entanglement has more degrees of freedom than quadrature entanglement because three observables, rather than two, are involved. In the following section we consider the continuous variable situation most analogous to single photon polarization entanglement where the correlation is independent of the basis of measurement, and demonstrate theoretically that all three Stokes operators can be simultaneously entangled. We extend the work of Ref. [5], and arrange the entanglement such that Eqs.. (5) are equal and the mean value of the

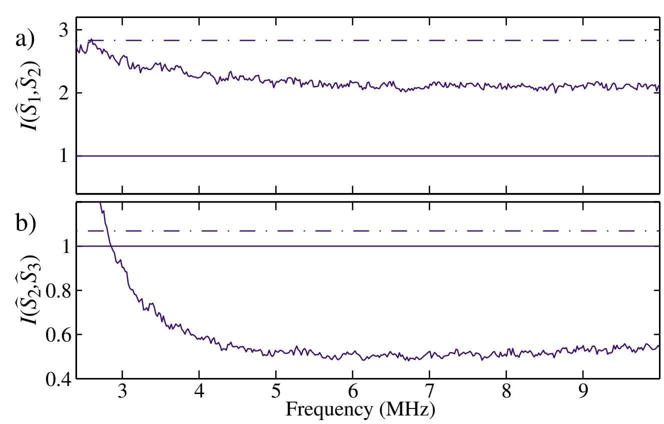

FIG. 2 (color online). Experimental measurement of (a) $I(\hat{S}_1, \hat{S}_2)$ and (b) $I(\hat{S}_2, \hat{S}_3)$, values below unity indicate entanglement. The dashed line is the corresponding measurement inferred between two coherent beams.

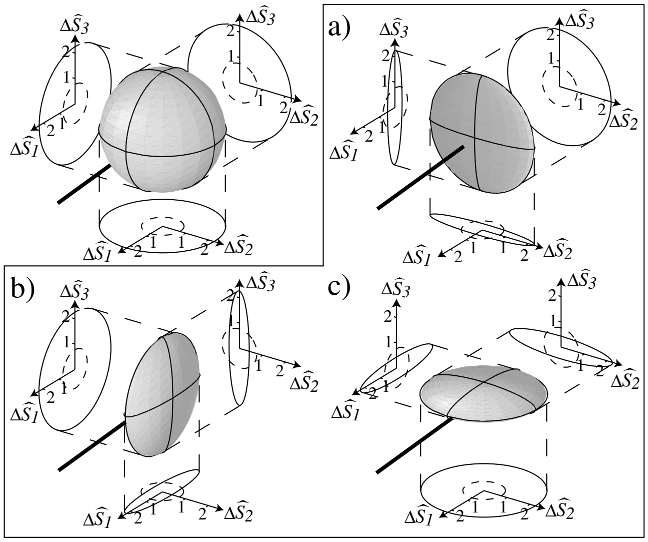

FIG. 3. Calculated polarization entanglement produced from four pure quadrature squeezed beams with squeezed quadrature variances of 0.1; axes normalized to 1 for a coherent state. The top left figure represents the knowledge of beam y before any measurement of beam x. (a), (b), and (c) represent the conditional knowledge of beam y given measurements of $\hat{S}_1$, $\hat{S}_2$, and $\hat{S}_3$, respectively, on beam x. If the conditional knowledge is better than the dashed circles the state is entangled.

three Stokes operators are the same ($|\langle\hat{S}_i\rangle| = \alpha^2$). This leads to $\alpha_V^2 = [(\sqrt{3}-1)/2]\alpha^2$, $\alpha_H^2 = [(\sqrt{3}+1)/2]\alpha^2$, $\theta_x = \pi/4 + n_x\pi/2$, and $\theta_y = \pi/4 + n_y\pi/2$, where n_x and n_y are integers. We assume that the two horizontally polarized inputs, and the two vertically polarized inputs, are quadrature entangled with the same degree of correlation such that $\Delta^2_{x\pm y}\hat{X}^\pm_H = \Delta^2_{x\pm y}\hat{X}^\pm_V = \Delta^2_{x\pm y}\hat{X}$. In this configuration, Eqs. (5) become $|\langle\delta\hat{S}_i\delta\hat{S}_j\rangle| = \alpha^2$, for all $i \neq j$. To simultaneously minimize all three degrees of Stokes operator inseparability $[I(\hat{S}_i, \hat{S}_j)]$ it is necessary that $\theta_x = -\theta_y + n\pi$. After making this assumption we find that $\Delta^2_{x\pm y}\hat{S}_i = \sqrt{3}\alpha^2\Delta^2_{x\pm y}\hat{X}$ for all i. Hence, in this situation $I(\hat{S}_i, \hat{S}_j)$ are all identical, and the entanglement is equivalent between any two Stokes operators. The condition for entanglement can then be expressed as a simple criterion on the quadrature entanglement between the input beams

$$I(\hat{S}_i, \hat{S}_j) < 1 \Longleftrightarrow I(\hat{X}^+, \hat{X}^-) < 1/\sqrt{3}, \qquad (8)$$

where $I(\hat{X}^+, \hat{X}^-) = I(\hat{X}^+_H, \hat{X}^-_H) = I(\hat{X}^+_V, \hat{X}^-_V)$. The factor of $1/\sqrt{3}$ arises from the projection of the quadrature properties onto a polarization basis in which the Stokes vector is pointing at equal angle $[\cos^{-1}(1/\sqrt{3})]$ from all three Stokes operator axes. In principle it is possible to have all three Stokes operators perfectly entangled. In other word, ideally the measurement of any Stokes operator of one of the beams could allow the exact prediction of that Stokes operator from the other beam (see Fig. 3). The experimental production of such a field is a straightforward extension of the experiment reported here, given the availability of four independent squeezed beams. Maximal single photon polarization entanglement enables tests of Bells inequality [1]. It has recently been shown that continuous variable polarization entanglement of the form discussed above can also exhibit Bell-like correlations [4]. This entanglement resource would also enable the demonstration of maximal continuous variable polarization teleportation.

To conclude, we have presented the first generation of continuous variable polarization entanglement. The scheme presented transforms the well-understood quadrature entanglement to a polarization basis. The two Stokes operators orthogonal to the Stokes vectors of the polarization entangled beams easily fulfill a generalized version of the inseparability criterion proposed by Duan *et al.*. We have also demonstrated that in the limiting case of our experimental configuration where $\alpha_V^2 \gg 1$ and $\alpha_H^2 = 0$ it is not possible to verify entanglement between any other pair of Stokes operators. Finally, we have shown that using four squeezed beams it is possible for all three Stokes operators to be perfectly entangled, although with a bound $\sqrt{3}$ times lower (stronger) than that for quadrature entanglement.

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