Collective Cooling and Self-Organization of Atoms in a Cavity

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We theoretically investigate the correlated dynamics of N coherently driven atoms coupled to a standing-wave cavity mode. For red detuning between the driving field and the cavity as well as the atomic resonance frequencies, we predict a light force induced self-organization of the atoms into one of two possible regular patterns, which maximize the cooperative scattering of light into the cavity field. Kinetic energy is extracted from the atoms by superradiant light scattering to reach a final kinetic energy related to the cavity linewidth. The self-organization starts only above a threshold of the pump strength and atom number. We find a quadratic dependence of the cavity mode intensity on the atom number, which demonstrates the cooperative effect.

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The modification of light forces within an optical resonator has been predicted to provide for new possibilities to trap and cool neutral atoms [1,2]. This has been demonstrated in some recent experiments [3,4]. As an essential ingredient of these schemes, the energy dissipation takes place dominantly via cavity decay and not via spontaneous emission. Hence, several problems of other cooling schemes, like photon reabsorption, pumping into unwanted states, or recoil heating, can be reduced. Moreover, no particular atomic level structure is needed. The final temperature $kT \approx \hbar\kappa$ is basically limited by the cavity linewidth κ and can be well below the Doppler limit $kT \approx \hbar\gamma$ set by the atomic linewidth γ.

While most conventional optical cooling schemes rely on single atom processes, all atoms in the cavity are simultaneously coupled to the same field modes as dissipation channels. This induces atomic cross talk via the cavity field and influences the cooling process of one atom by the motion of the others. As in other ensemble based cooling schemes, such as stochastic cooling [5], the price to pay is a limited scalability of the system. Using system size scaling arguments [5,6], one finds a linear increase of the cooling time with atom number, which seems to render such systems impractical for very large ensembles.

Here we present an extra phenomenon which leads to a dramatically different scaling behavior exploiting the cavity-mediated atom-atom coupling as an essential ingredient to enhance cooling. For this we introduce a rather innocent looking change in the setup and illuminate the atoms in the cavity by a red detuned field from the side instead of sending the pump light directly into the cavity field. For a single atom such a change yields some quantitative changes in the cooling rate and temperature [7], but no drastic modifications of the system properties occur. However, putting more atoms in the mode, one finds strong cooperative action which leads to a fast and efficient trapping and cooling of the whole ensemble.

Let us first qualitatively identify the origin of this cooperative effect. The atoms being pumped perpendicular to the cavity axis coherently scatter photons into the cavity. The scattered field amplitude for each atom depends on its position in the cavity and pump fields. Assuming, e.g., a standing-wave mode function $\cos(kx)$, atoms in the nodes do not contribute, while a maximum scattering occurs from atoms close to antinodes. The key point is that the phase of the coherent part of the scattered field is position dependent. Thus, atoms separated by half a wavelength radiate equal fields with opposite phase and their contributions cancel. Hence, destructive interference prevents a coherent field buildup for a uniform distribution along the cavity axis.

However, any distribution of a finite number of freely moving atoms has density fluctuations, which eventually leads to a small field buildup with random phase. For red detuning this creates an attractive optical potential towards the antinodes of the cavity field redistributing the atoms. The induced periodic localization of the atoms, in turn, strongly enhances the scattering into certain directions in analogy to the coherent Bragg scattering by a regular atomic pattern. If the atomic pattern formed induces strong scattering into the cavity mode direction, the periodic potential is further deepened and the corresponding atomic pattern is stabilized in a runaway process. For suitable operating conditions the cavity loss dissipates the kinetic energy that the atoms gain by falling into the potential wells, which leads ultimately to a stable trapping. This localization process is, of course, counteracted by heating processes, such as spontaneous emission and cavity intensity fluctuations, and will stop at some finite atomic position spread.

There is still one subtle open question. In an optical standing wave the distance between neighboring antinodes is $\lambda/2$ with a field phase difference of π. Hence the radiated field from atoms in adjacent potential wells cancels on average for equal population and no such runaway process should occur. However, we show in the

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following that the gas of N atoms starting from a uniform distribution breaks the translational invariance and finds a very particular state by self-organization: all the atoms accumulate in either the "even" or all in the "odd" wells, i.e., in positions x_j $(j = 1, \ldots, N)$ such that $\cos(kx_j)$ is close to 1 or close to -1 for all. The constructive interference of the scattered light in this final state then gives rise to a quadratic dependence of the stationary mode intensity on the atom number N, which can serve as a signature of reaching the self-organized state and as an unambiguous evidence for the atomic cooperative dynamics.

Our first minimal model consists of a single 1D cavity mode $\cos(kx)$ with resonance frequency ω_C and linewidth κ, interacting with N identical, pointlike two-level atoms with transition frequency ω_A and linewidth γ. They are transversely driven by a plane-wave laser field of frequency ω. The atom-field coupling constant g is given in terms of the single-photon Rabi frequency at the field maximum, and η denotes the effective pumping strength. We restrict ourselves to the low-saturation regime, where the excited atomic state can be adiabatically eliminated, assuming large atom-pump detuning $\Delta_A \equiv \omega - \omega_A$.

We investigate the coupled dynamics of the field amplitude α and the center-of-mass motion of N dipoles at positions x_j along the cavity axis and with momenta p_j $(j = 1, \ldots, N)$ by means of a systematic semiclassical approximation including spontaneous emission and cavity decay [8]. This yields the coupled, stochastic Ito equations

$$\dot{\alpha} = i\left[\Delta_C - U_0 \sum_j \cos^2(kx_j)\right]\alpha - \left[\kappa + \Gamma_0 \sum_j \cos^2(kx_j)\right]\alpha - \eta_{\rm eff} \sum_j \cos(kx_j) + \xi_\alpha, \tag{1a}$$

$$\dot{p}_j = -\hbar U_0(|\alpha|^2 - 1/2)\frac{\partial}{\partial x_j}\cos^2(kx_j) - i\hbar(\eta^*_{\rm eff}\alpha - \eta_{\rm eff}\alpha^*)\frac{\partial}{\partial x_j}\cos(kx_j) + \xi_j, \tag{1b}$$

where $\Delta_C = \omega - \omega_C$, the parameters

$$U_0 = \frac{g^2\Delta_A}{\Delta_A^2 + \gamma^2}, \qquad \Gamma_0 = \frac{g^2\gamma}{\Delta_A^2 + \gamma^2}, \tag{2}$$

describe the dispersive and absorptive effects of the atoms, respectively, as they shift and broaden the resonance line of the cavity. There is an effective position dependent pumping term proportional to

$$\eta_{\rm eff} = \frac{\eta g}{-i\Delta_A + \gamma}, \tag{3}$$

responsible for the photon creation in the mode.

Damping processes are always accompanied by fluctuations, which imply Langevin-type noise terms for the momentum, ξ_j, and the complex field amplitude, ξ_α. These are defined by the second-order correlation coefficients, which include nontrivial cross correlations between the momentum noise terms ξ_j and the phase component of the field noise, defined as $\xi_\perp = (i/2)(-\alpha^*/|\alpha|\,\xi_\alpha + \alpha/|\alpha|\,\xi^*_\alpha)$. In the simulation we generate the proper random increment by decomposing the noise terms into uncorrelated and correlated components, $\xi_\alpha = \xi^{(0)}_\alpha + \xi^{(1)}_\alpha$ and $\xi_j = \xi^{(0)}_j + \xi^{(1)}_j$, which obey

$$\langle \xi^{(0)*}_\alpha \xi^{(0)}_\alpha \rangle = \kappa + \Gamma_0/2 \sum_j \cos^2(kx_j), \tag{4a}$$

$$\langle \xi^{(0)}_j \xi^{(0)}_j \rangle = 2(\hbar k)^2 \overline{u^2}\Gamma_0[(|\alpha|^2 - 1/2)\cos^2(kx_j) + \eta/g(\alpha + \alpha^*)\cos(kx_j) + (\eta/g)^2]. \tag{4b}$$

Here $k\sqrt{\overline{u^2}}$ is the mean projection of the momentum recoil along the cavity axis due to spontaneous emission (in the numerical examples we set $\overline{u^2} = 2/5$). The correlated components can be generated from N independent random numbers ζ_j $(\langle \zeta_j \zeta_k \rangle = \delta_{jk})$,

$$\xi^{(1)}_\alpha = i\frac{\alpha}{|\alpha|}\sqrt{\Gamma_0/2}\sum_j \cos(kx_j)\zeta_j, \tag{5a}$$

$$\xi^{(1)}_j = -\hbar k\sqrt{2\Gamma_0}|\alpha|\sin(kx_j)\zeta_j. \tag{5b}$$

In the strong-coupling regime ($g \gg \kappa, \gamma$) and for suitable detunings Δ_A and Δ_C, the cavity field exerts an efficient friction force and the atoms are driven into a stationary distribution characterized by a temperature depending primarily on κ [1,8]. In the present model, the nonconservative part of the dynamics is hidden in the coupled, nonlinear equations. However, the self-organization can be partly interpreted on the basis of the conservative forces. These are quite directly exposed in Eq. (1b). The first term corresponds to the dipole force in the $\cos^2(kx)$ optical lattice potential, having potential wells at $kx = n\pi$ with a periodicity of $\lambda/2$. The additional force term proportional to $\cos(kx)$ has opposite sign for positions around $kx = 2n\pi$ and $kx = (2n+1)\pi$. It originates from the recoil accompanying the atom-mediated pumping of the cavity mode. Let us assume a detuning such that $\Delta_C - U_0\sum\cos^2(kx_j) < 0$. If more atoms happen to be momentarily in the even wells so that $\sum\cos(kx_j) > 0$, then the cosine potential has wells at $kx = 2n\pi$ deepening the $\cos^2(kx)$ optical lattice, while it has hills at $kx = (2n+1)\pi$ that reduce or can even suppress the attractive wells in these points. An instantaneous unbalance of the populations in the different type of wells stimulates the increase of the asymmetry between the two wells. In favorable cases the process self-amplifies until all the atoms are in the same class of wells. Once such a phase has established it is further stabilized by destructive interference of the two forces (first and second terms) at the empty antinode sites.

The self-organization process and the effect of the remaining momentum fluctuations are illustrated in a typical example of trajectories in Fig. 1. As the number of 10 is rather low, the atoms are only weakly bound and show some hopping between different trapping positions even after a long time. The superradiant property of the system is exhibited in Fig. 2(a) where we show the quadratic dependence of the stationary photon number in the cavity as a function of the number of driven atoms N. For each number of N the detuning Δ_C is rescaled such that the effective mode detuning, including the "index" effect of the atoms, is kept constant provided the atoms are exactly in antinodes, i.e., $\Delta_C - NU_0 = \kappa$. In fact, the atoms oscillate around the antinodes with average light-shift proportional to the mean of $\cos^2(kx)$ less than 1. Hence the more atoms are in the cavity, the further the pumping field is detuned from the actual atom-shifted mode resonance. Yet, the photon number increases quadratically with the atom number. The maximum atomic saturation at the largest photon numbers is 0.06. The result of the numerical simulation is well described by the simple expression,

$$\overline{|\alpha|^2} = N^2 \frac{|\eta_{\rm eff}|^2}{\kappa^2} \frac{(1 - k^2\overline{x^2})}{1 + [1 + (U_0 N/\kappa) k^2 \overline{x^2}]^2}, \qquad (6)$$

which is obtained as the steady state of Eq. (1) without the noise term in the limit of large atomic detuning $\Delta_A \gg \gamma$ ($|U_0| \ll |\Delta_C|, \Gamma_0 \ll \kappa$) and for atoms closely localized at antinodes. The position uncertainty of a single atom $\overline{x^2}$ which provides for the best fit in Fig. 2 agrees well with the results of the simulation.

For a small number of atoms the self-organization is unlikely to occur, because the maximum number of photons generated in an optimally ordered state of occupying exactly every second well is too low to trap the atoms for long times. The same happens for weak pumping. In Fig. 2(b) the steady-state photon number is plotted as a function of the pumping strength η for the fixed number of atoms $N = 30$. For the chosen parameters, no cavity field builds up below $\eta = 400$, which is a critical pump strength for getting self-organization into potential wells deep enough to keep the atoms trapped at the steady-state temperature of the atomic cloud.

So far the atomic motion was confined to 1D along the cavity axis. One might suspect that the whole effect is washed out due to phase randomization, if the atoms are allowed to move transversely in the fields as well. In this case motion along the direction z of the plane-wave pump, induced by absorbing transverse photon momenta, should average out the contribution of various atoms and kill superradiance. However, this can easily be cured by employing a standing-wave transverse pump. In this case one gets a 2D lattice of cigar shaped potential wells generated by the pump and cavity field, where in the third remaining direction y the atoms are confined by the transverse Gaussian envelope of the cavity and pump fields. In each microtrap the relative phase of cavity and pump field is approximately constant.

Again the simulation shows that the atoms will self-organize and self-trap in a regular subset of these potentials in a way to maximize the atom-field coupling and generate the deepest possible wells. This is shown in Fig. 3 for the example of 20 atoms in a configuration where a TEM_{00} cavity mode is transversely crossed by a Gaussian standing-wave dipole trap field. The xz motion of the particles, after some transition phase, arranges in a rectangular pattern. At the same time we find an increase of the cavity field intensity with a decrease of the kinetic energy along the cavity, as shown in Fig. 4.

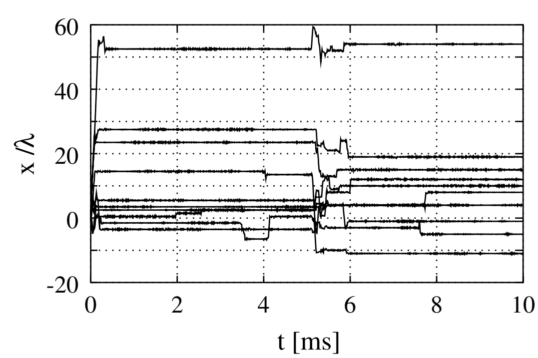

FIG. 1. Trajectories of 10 atoms exhibiting the self-organization for a weakly bound case. The system parameters are $(g, \kappa, \gamma) = (50, 10, 20)\ \mu s^{-1}$. The atomic detuning is very large, $\Delta_A = -500\gamma$, and the pumping constant is $\eta = 1000\ \mu s^{-1}$, resulting in a mean photon number about 7. The atomic mass number was chosen as 85 corresponding to Rb.

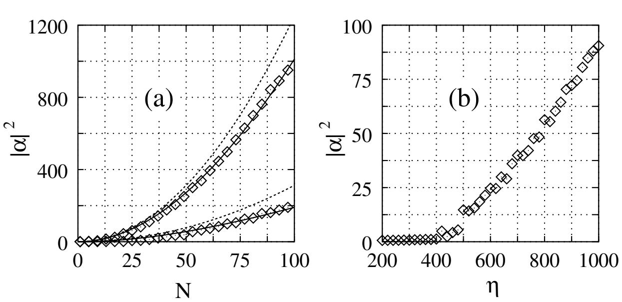

FIG. 2. The steady-state mean number of cavity photons $|\alpha|^2$ as a function of (a) the number of atoms N and (b) the pumping strength η for fixed number of atoms $N = 30$. In (a) the pumping strength η is 500 μs^{-1} for the lower curve, and 1000 μs^{-1} for the upper curve. Dots are the results obtained by numerically simulating the Eqs. (1). In (a) the lines represent the solution Eq. (6) with perfect localization $\overline{x^2} = 0$ (dashed lines) and with fitted values $\overline{x^2} = 0.14$ and 0.06 for $\eta = 500$ and 1000 (solid lines), respectively. The system parameters are the same as in Fig. 1.

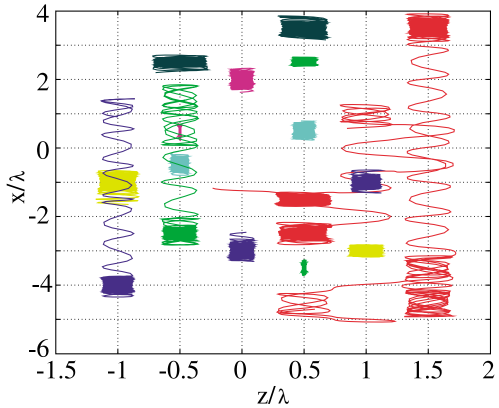

FIG. 3 (color online). *X-Z* motion of 20 atoms in the cavity with parameters $(g, \kappa, \gamma) = (30, 10, 20)\ \mu s^{-1}$, $\Delta_A = 100\gamma$, and $\eta = 2500$.

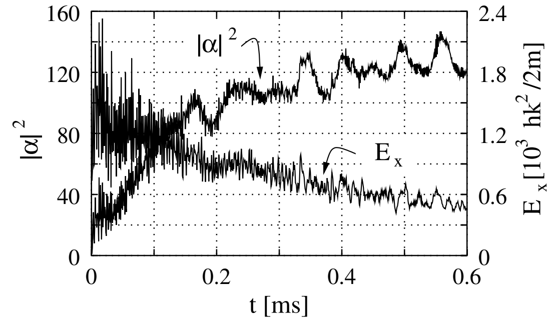

FIG. 4. Time evolution of the photon number $|\alpha|^2$ and the longitudinal kinetic energy E_x corresponding to the self-organization process shown in Fig. 3.

In summary, let us emphasize that the simple system of polarizable particles coupled to a cavity field combines a wealth of interesting phenomena, some of which were separately discussed in different contexts before. For example, transverse self-organization, called "optical binding," has been experimentally observed ten years ago [9] for mesoscopic objects (silica spheres of sub-wavelength size in water). In contrast to that work, our scheme requires no direct dipole-dipole interaction of the particles, which would be weak for single atoms. The cavity field mediates an indirect interaction between the atoms and, simultaneously, provides for a friction mechanism to subsequently stop the atoms at the induced potential minima. This damping is missing from the so-called "recoil-induced" effects. Therefore the predicted superradiant emission from a completely inverted atomic sample of ultracold atoms in ring cavities [10] is only a transient. Note also that we describe an ensemble of point particles well above recoil temperature so that gain due to recoil-induced resonances can be safely neglected [11]. In a closely related setup, Chan and co-workers have observed fast and efficient sub-Doppler cooling of 10^6 Cesium atoms in a multimode cavity field, accompanied by strong coherent emission from the field mode [12]. Although in their case the choice of polarizations and the internal atomic level structure does not fit precisely to our model, cooperative gain from the process of atomic self-ordering could play a role in that case.

In principle our model can be extended to the quantum motion of Bose-condensed atoms in a cavity. In this case the self-organization would dynamically form macroscopic superpositions of the different possible final states via a stimulated coherent process. The self-organization process could also assist in cold molecule formation by cavity-induced photoassociations which would provide for a source of externally and vibrationally cold molecules.

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