Universal Power-Law Decay in Hamiltonian Systems?

The understanding of the asymptotic decay of correlations and of the distribution of Poincaré recurrence times $P(t)$ has been a major challenge in the field of Hamiltonian chaos for more than two decades. In a recent Letter, Chirikov and Shepelyansky [1] claimed the universal decay $P(t) \sim t^{-3}$ for Hamiltonian systems. Their reasoning is based on renormalization arguments and numerical findings for the sticking of chaotic trajectories near a critical golden torus in the standard map. We performed extensive numerics and find clear deviations from the predicted asymptotic exponent of the decay of $P(t)$. We thereby demonstrate that even in the supposedly simple case, when a critical golden torus is present, the fundamental question of asymptotic statistics in Hamiltonian systems remains unsolved.

As in Ref. [1] we study the standard map:

$$q_{n+1} = q_n + p_n \bmod 2\pi, \qquad p_{n+1} = p_n + K \sin q_{n+1}, \tag{1}$$

at $K = K_c = 0.971\,635\,406\,31$, where the golden torus is critical (Fig. 1, inset). We determine the Poincaré recurrence time distributions $P(t)$ for trajectories starting below and above the critical golden torus by using the same numerical approach as in Ref. [1]. By considerably increasing the statistics we are able to extend the distribution by almost 2 orders of magnitude in recurrence times. We verify that our statistical data are not affected by the unavoidable finite numerical precision by comparing data for double (≈ 16 significant digits) and quadruple (≈ 32 digits) precision. The data for approaching the critical golden torus from above and below are presented in Fig. 1. For times $t < 10^8$ our data agree with the results presented in Fig. 2 of Ref. [1]. For larger times, however, we find strong deviations from the predicted universal power law $P(t) \sim t^{-3}$ (dashed lines in Fig. 1). The deviations might be explained in two ways: The onset of the claimed asymptotic decay might occur for larger times, which is in contradiction to the prefactors determined in Ref. [1]. On the other hand, the long-time trapping of chaotic trajectories might be dominated by islands of stability (nonprincipal resonances) that are neglected by the renormalization arguments. In fact, the latter possibility is supported by a detailed investigation [2].

If even in the supposedly simple case of a critical golden torus the decay $P(t) \sim t^{-3}$ is not observed, the claim for a universal existence of this decay cannot be maintained. It thus remains a fundamental challenge in the field of Hamiltonian chaos whether the asymptotic behavior of $P(t)$ follows a universal power law and what the value of its exponent would be.

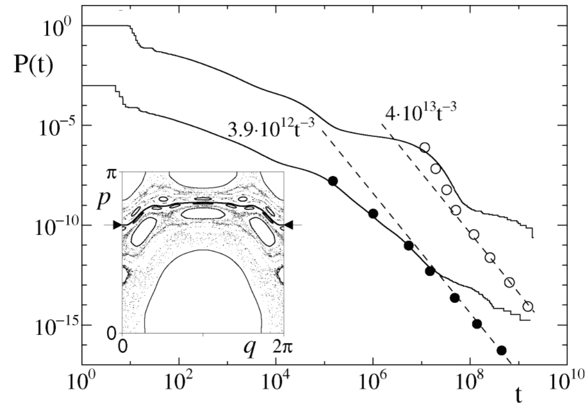

FIG. 1. The Poincaré recurrence time distribution $P(t)$ for the standard map at $K = K_c$ for trajectories approaching the critical golden torus from above (upper curve) and from below (lower curve, shifted by 10^{-3}). For large times we find clear deviations from the predictions of Ref. [1] (dashed lines and symbols). Inset: Phase space of the symmetrized standard map at $K = K_c$, with arrows pointing at the critical golden torus.

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