

Quasideterministic Generation of Entangled Atoms in a Cavity

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We present a scheme to generate a maximally entangled state of two three-level atoms in a cavity. The success or failure of the generation of the desired entangled state can be determined by detecting the polarization of the photon leaking out of the cavity. With the use of an automatic feedback, the success probability of the scheme can be made to approach unity.

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There has recently been much interest in the generation of entangled states of two or more particles, as they give rise to quantum phenomena that cannot be explained in classical terms. Entangled states not only are used to test fundamental quantum-mechanical principles such as Bell's inequalities [1] but also play a central role in practical applications of the quantum information theory such as quantum computation [2], quantum teleportation [3], quantum dense coding [4], and quantum cryptography [5]. Two-photon entangled states can commonly be produced from a nonlinear optical process such as parametric down-conversion [6]. Although there seems no easy way of generating entangled states of massive particles instead of massless photons, recent advances in ion trapping technology and cavity QED have led to several proposals [7] for the generation of entangled states of atoms or ions and subsequent experimental realizations [8].

In this paper we introduce a scheme that allows the generation of a maximally entangled state of two Λ -type three-level atoms in a cavity. The scheme is similar to that proposed recently by Plenio *et al.* [9], but has an advantage in that the probability of obtaining the entangled state can be made to approach unity, as described below.

The system we consider is shown schematically in Fig. 1. Two identical Λ -type three-level atoms a and b , each with an excited state $|e\rangle$ and two degenerate ground states $|L\rangle$ and $|R\rangle$, are situated in a resonant optical cavity. The $|e\rangle \leftrightarrow |L\rangle$ transition is coupled by left-circularly polarized light, while the $|e\rangle \leftrightarrow |R\rangle$ transition is coupled by right-circularly polarized light. We assume that the separation between the two atoms is large compared with the wavelength of the $|e\rangle \leftrightarrow |L\rangle$ or $|e\rangle \leftrightarrow |R\rangle$ transition so that the dipole-dipole interaction can be neglected.

The outline of our scheme is as follows. Initially we prepare the two atoms in their "left" ground state $|L\rangle$ and inject a left-circularly polarized photon into the cavity. One of the two atoms can then absorb the photon and make an upward transition to $|e\rangle$. It can subsequently de-excite to $|L\rangle$ or $|R\rangle$ emitting a left- or right-circularly polarized photon. If the polarization of the emitted photon can be detected and is found right-circularly polarized, one can conclude that one of the two atoms is in $|L\rangle$ and the other is in $|R\rangle$. Since which atom is in $|L\rangle$ and

which is in $|R\rangle$ cannot be determined, the final state of the two atoms is a superposition of the two probabilities, i.e., an entangled state. Thus an entangled state of the two atoms results when the polarization of the photon leaking out of the cavity is detected to be right-circularly polarized.

In order to illustrate the main idea, let us first consider an ideal case of the perfect cavity. The temporal evolution of the system in the cavity is governed by the Hamiltonian $H = H_a + H_b + H_R + H_I$, where H_a and H_b are the atomic Hamiltonian for atoms a and b , respectively, H_R is the free field Hamiltonian, and H_I is the interaction Hamiltonian given by

$$H_I = i\hbar \sum_{i=a,b} \sum_{\lambda=L,R} (g_\lambda c_\lambda |e\rangle_{ii} \langle \lambda| - g_\lambda c_\lambda^\dagger |\lambda\rangle_{ii} \langle e|). \quad (1)$$

In Eq. (1) c_λ and c_λ^\dagger ($\lambda = L, R$) denote the annihilation and creation operators for the left- or right-circularly polarized cavity field; g_λ ($\lambda = L, R$), assumed to be real, represents the coupling strength between the atom and the left- or right-circularly polarized field (g_λ is assumed to be the same for atom a and atom b); $|e\rangle_i$ ($i = a, b$) represents the excited state of the atom a or b ; and $|\lambda\rangle_i$ ($\lambda = L, R$; $i = a, b$) represents the left or "right" ground state of the atom a or b . Expressing the state of the total system in the form $|\text{atom } a, \text{atom } b; \text{photon}\rangle$, the initial state can be written $|L, L; L\rangle$. Under the rotating wave approximation, the temporal evolution of the system is spanned by the five basis states: $\{|L, L; L\rangle, |e, L; 0\rangle, |L, e; 0\rangle, |R, L; R\rangle$, and

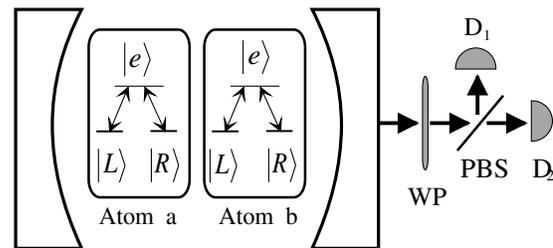


FIG. 1. Experimental scheme. WP represents a quarter wave plate, PBS denotes a polarization beam splitter and D_1 and D_2 are detectors. If a photon incident on the quarter wave plate is left- (right-)circularly polarized, then it is detected by D_1 (D_2).

$|L, R; R\rangle\}$. A straightforward algebra yields that the state of the system at time t is given in terms of these basis states as

$$|\Psi(t)\rangle = \frac{g_R^2 + 2g_L^2 \cos\alpha t}{\alpha^2} |L, L; L\rangle - \frac{g_L}{\alpha} \sin\alpha t (|e, L; 0\rangle + |L, e; 0\rangle) - \frac{2g_L g_R}{\alpha^2} \sin^2 \frac{\alpha t}{2} (|R, L; R\rangle + |L, R; R\rangle). \quad (2)$$

Equation (2) indicates that the probability at time t of obtaining the entangled state $|\phi\rangle = (1/\sqrt{2})(|R, L; R\rangle + |L, R; R\rangle)$ is given by

$$P(t) = |\langle\phi|\Psi(t)\rangle|^2 = 8 \frac{g_L^2 g_R^2}{\alpha^4} \sin^4 \frac{\alpha t}{2} = \frac{8\beta^2}{(\beta^2 + 2)^2} \sin^4 \frac{\alpha t}{2}, \quad (3)$$

where $\beta \equiv g_R/g_L$. At $t = [(2n+1)\pi/\alpha]$ ($n = 0, 1, 2, \dots$), the probability has the maximum value $P_{\max} = [(8\beta^2)/(\beta^2 + 2)^2]$. When $\beta = 1$, $P_{\max} = \frac{8}{9}$. In particular, when $\beta = \sqrt{2}$, P_{\max} reaches 1.

We now analyze the system depicted in Fig. 1, in which the polarization of the photon leaking out of the cavity is monitored. In order to describe the temporal evolution of the open system under consideration, we adopt the master equation approach. The master equation describing the time evolution of the density matrix is given by

$$\frac{d\rho}{dt} = \frac{1}{i\hbar} [H, \rho] + \frac{\kappa}{2} \sum_{\lambda=L,R} (2c_\lambda \rho c_\lambda^\dagger - c_\lambda^\dagger c_\lambda \rho - \rho c_\lambda^\dagger c_\lambda), \quad (4)$$

where κ denotes the cavity decay rate. Assuming that the initial state is $|L, L; L\rangle$, the time evolution of the system inside the cavity is now described with eight basis states: $\{|L, L; L\rangle, |e, L; 0\rangle, |L, e; 0\rangle, |R, L; R\rangle, |L, R; R\rangle, |L, L; 0\rangle, |R, L; 0\rangle, \text{ and } |L, R; 0\rangle\}$. Compared with the perfect cavity case previously considered, we now have three more basis states, $|L, L; 0\rangle$, $|R, L; 0\rangle$, and $|L, R; 0\rangle$, which result when the photon in the states $|L, L; L\rangle$, $|R, L; R\rangle$, and $|L, R; R\rangle$, respectively, escapes the cavity. When the detector in Fig. 1 registers a left-circularly polarized photon, i.e., when D_1 clicks, we know for sure that the state of the system in the cavity is $|L, L; 0\rangle$. On the other hand, if a right-circularly polarized photon is detected, i.e., when D_2 clicks, the state of the system in the cavity can be $|R, L; 0\rangle$ or $|L, R; 0\rangle$. Whether the state is $|R, L; 0\rangle$ or $|L, R; 0\rangle$ cannot be determined from the measurement of the polarization of the photon. Since both the initial state and the system Hamiltonian are symmetric with respect to the two atoms a and b , the state associated with the detection of the right-circularly polarized photon must be $(1/\sqrt{2})(|R, L; 0\rangle + |L, R; 0\rangle)$. It is then clear that, at large time $t \rightarrow \infty$, the system inside the cavity approaches a mixture of $|L, L; 0\rangle$ and $(1/\sqrt{2})(|R, L; 0\rangle + |L, R; 0\rangle)$, i.e.,

$$\rho_\infty = (1-p)|L, L; 0\rangle\langle L, L; 0| + \frac{p}{2} (|R, L; 0\rangle + |L, R; 0\rangle) \otimes (\langle R, L; 0| + \langle L, R; 0|), \quad (5)$$

where p represents the probability to obtain the desired entangled state $(1/\sqrt{2})(|R, L\rangle + |L, R\rangle)$ of the two atoms a and b .

The probability p is determined when the system parameters g_L , g_R , and κ are given. In Fig. 2 we plot p as a function of g_L/κ and g_R/κ computed from numerical simulation of the master equation. The probability is seen to have its maximum value of $\sim \frac{1}{2}$ along the line $\frac{g_R}{g_L} = \sqrt{2}$.

Our scheme of Fig. 1 offers a way of obtaining the two-atom entangled state $(1/\sqrt{2})(|R, L\rangle + |L, R\rangle)$ with probability p . Our scheme is thus probabilistic. When the scheme fails to generate the desired entangled state, however, the experiment can easily be repeated for another round of trial. The probability with which the scheme fails to generate the entangled state is $1-p$. In this case the detector registers a left-circularly polarized photon and the state inside the cavity is $|L, L; 0\rangle$. The experiment can then simply be repeated by injecting another left-circularly polarized photon into the cavity. The state of the system inside the cavity is then $|L, L; L\rangle$ and the entire experiment restarts. In fact, one can have the experiment automatically repeat itself in case of the failure simply by eliminating the detector D_1 and replacing it by a path directed back to the cavity, so that the left-circularly polarized photon can be automatically fed back to the cavity. One then needs only to wait for the detector D_2 to click. The moment the detector D_2 registers a photon, we know that the entangled state $(1/\sqrt{2})(|R, L\rangle + |L, R\rangle)$ of the two atoms a and b is generated in the cavity. The probability that the entangled state is not generated after n rounds of trial is $(1-p)^n$. Since the failure probability exponentially decreases with respect to the number of rounds, the desired entangled state can be generated with high probability within a few cavity decay times. We also note that the generated entangled state is a superposition of different combinations of two ground

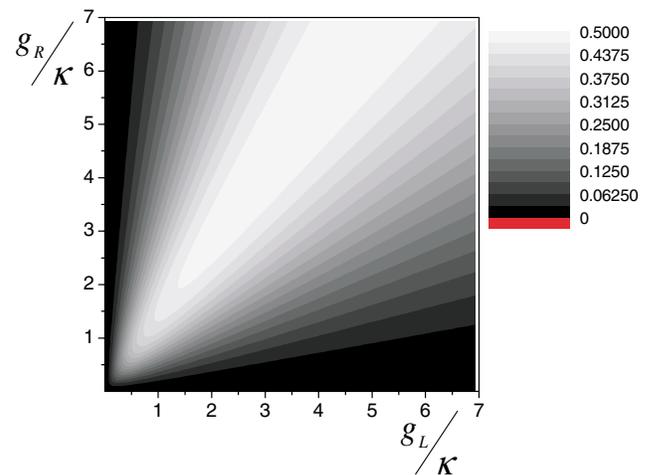


FIG. 2 (color online). Probability to obtain the entangled state as a function of g_R/κ and g_L/κ .

states (or metastable states) and thus is free from decoherence caused by the cavity loss as well as spontaneous emission.

It should be pointed out that our treatment assumes that the only losses in the system are those associated with the coupling of the cavity field mode to the outer field modes. Thus, only the losses of this type lead to photon detection. Losses due to spontaneous emission into modes other than the cavity mode are neglected. Absorption of photons by the cavity mirrors is also neglected.

As a specific example for realization of our scheme proposed here, we consider hyperfine levels of cesium (nuclear spin $I = \frac{7}{2}$) considered recently by Lange and Kimble [10]. The Zeeman sublevels of the states ($6S_{1/2}, F = 3$) and ($6P_{1/2}, F = 3$) are drawn in Fig. 3, where $|g_{m_F}\rangle$ and $|e_{m_F}\rangle$ denote the sublevels ($6S_{1/2}, F = 3, m_F$) and ($6P_{1/2}, F = 3, m_F$), respectively, with m_F running from -3 through 3 . The wavelength of the $|e\rangle \leftrightarrow |g\rangle$ transition is 852.36 nm. The transition between $|e_{m_F}\rangle$ and $|g_{m_F-1}\rangle$ is mediated by right-circularly polarized light and that between $|e_{m_F}\rangle$ and $|g_{m_F+1}\rangle$ by left-circularly polarized light. With the cesium atoms prepared initially in $|g_{m_F+1}\rangle$ and one left-circularly polarized photon injected into the cavity, one can then obtain, using our scheme, the entangled state $(1/\sqrt{2})(|g_{m_F-1}, g_{m_F+1}\rangle + |g_{m_F+1}, g_{m_F-1}\rangle)$ of the cesium atom with probability p .

Let us estimate the probability p for the above system. The Hamiltonian representing the interaction of the cesium atom and the injected photon can be written as [10]

$$H_I = -i\hbar g_0 \sum_{i=a,b} \sum_{\lambda=L,R} (c_\lambda^\dagger A_{i,\lambda} - A_{i,\lambda}^\dagger c_\lambda), \quad (6)$$

where $2g_0/2\pi$ is the single-photon Rabi frequency and $A_{i,\lambda}$ is given by

$$A_{i,\lambda} = \sum_{m_g, m_e} |F, m_g\rangle_{ii} \langle F, m_g; \lambda | F, m_e \rangle_{ii} \langle F, m_e |, \quad (7)$$

where m_g and m_e denote the Zeeman sublevel quantum number m_F for the states $|g_{m_F}\rangle$ and $|e_{m_F}\rangle$, respectively. Comparing Eqs. (6) and (7) with Eq. (1), we find that g_L and g_R of Eq. (1) correspond to g_0 of Eq. (6) multiplied by the Clebsch-Gordan coefficient $\langle F_g, m_g; \lambda | F_e, m_e \rangle$. With the present technology, one can achieve the condition

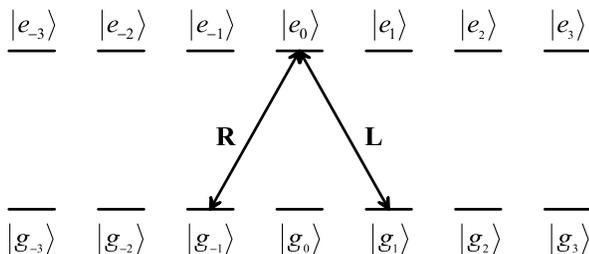


FIG. 3. ($F = 3$) hyperfine levels of cesium. R and L indicate that the two levels connected by the arrow are coupled by right- and left-circularly polarized light, respectively.

$g_0 \cong 3\kappa \sim 15\kappa$ [11]. (For example, the values of $g_0/2\pi \cong 120$ MHz and $\kappa/2\pi \cong 40$ MHz were cited in [11]). Taking $g_0 = 3\kappa$ and $|g_1\rangle$ as the initial atomic state, we obtain $g_L = g_R = (1/\sqrt{2})g_0$ and $p \cong 0.43$. In this case the entangled state $(1/\sqrt{2})(|g_{-1}, g_1\rangle + |g_1, g_{-1}\rangle)$ is obtained with the probability 0.43 after one round of trial. Note that in this case there is no chance for the states other than the states $|e_0\rangle$, $|g_{-1}\rangle$, and $|g_1\rangle$ to be occupied, because there is one and only one photon present in the cavity initially (see Fig. 3). Taking $g_0 = 3\kappa$ and $|g_0\rangle$ as the initial atomic state, we obtain $g_L = \sqrt{(5/12)}g_0$, $g_R = (1/\sqrt{2})g_0$, and $p \cong 0.45$. The atomic entangled state obtained in this case is $(1/\sqrt{2})(|g_{-2}, g_0\rangle + |g_0, g_{-2}\rangle)$. We can thus conclude that our scheme with the help of the present cavity technology allows the generation of the entangled atomic state with a reasonably high probability even after one round of trial.

A comparison of our scheme with the scheme proposed by Plenio *et al.* [9] is now in order. The scheme of Plenio *et al.* provides a way of generating the entangled state $(1/\sqrt{2})(|e, g\rangle - |g, e\rangle)$ of two two-level atoms inside a cavity, where $|e\rangle$ and $|g\rangle$ refer to the upper and lower levels. Since the generated entangled state is an antisymmetric trapped state, it is important to prepare the initial state in a nonsymmetric way, e.g., the initial state can be $|e, g; 0\rangle$, the atom a in $|e\rangle$, the atom b in $|g\rangle$, and no photon. The success of the scheme depends upon the detection of no photon leaking out of the cavity. If a photon leaking out of the cavity is detected, then the experiment fails. In this case the state of the system inside the cavity is $|g, g; 0\rangle$. If another photon is injected into the cavity, then the state of the system becomes $|g, g; 1\rangle$. This state is symmetric with respect to an interchange of the two atoms. It is thus clear that the experiment cannot be repeated for another round of trial by simply reinjecting another photon. Our scheme in contrast is designed in such a way that the “correct” initial state is set up in case of failure simply by reinjecting the photon leaked out of the cavity back into the cavity. Although probabilistic in nature, our scheme thus provides a quasideterministic way of generating an entangled state of two atoms.

The advantage of our scheme still prevails even if we take into account the finite detection efficiency of the detectors. When no photon is detected in the scheme of Plenio *et al.*, there are two possibilities: (1) the experiment has succeeded and the desired entangled state has been generated, or (2) the experiment has failed and a photon has been emitted from the cavity, but the detector has failed to detect it. There is no way of knowing for sure that the desired entangled state has been generated. On the other hand, in our scheme, the detection of a photon by the detector D_2 assures that the desired entangled state has indeed been generated. The finite detection efficiency only reduces the probability for such a detection. In fact, with the automatic feedback installed in our scheme and assuming the efficiency of the photon feedback to be unity, we know that we have the desired entangled state

generated inside the cavity after a sufficiently long time (after several cavity decay times), even if the detector D_2 fails to click because of the finite detection efficiency. In practical situations, however, the feedback efficiency is less than unity, and the failure to detect the photon could be due either to the finite efficiency of the detector or to feedback losses. The corresponding atomic state will then be a statistical mixture of $|L, L\rangle$ and the desired entangled state. In this situation the experiment should be restarted from the very beginning.

Finally, we wish to consider some practical issues in relation to the requirements on atom trapping imposed by our scheme. Our scheme requires that the two atoms be symmetrically coupled to the cavity mode for the entire duration of the experiment. This means that, as we have already assumed in Eq. (1), the coupling strength g_λ should be the same at all times for the two atoms. It in turn requires that, since g_λ depends on the position of the atom inside the cavity, both atoms be localized within the Lamb-Dicke limit, so that the random variation of the coupling strength g_λ due to thermal motion is negligible. The Lamb-Dicke condition states that the thermal vibrational amplitude of the atom must be small compared with the optical wavelength. Let us assume that the atoms are trapped in the low-lying states of the trapping potential. Let us further assume that the trapping potential is generated from a far-off-resonance trapping (FORT) beam [12]. We take the trapping potential to be $V(x) = -V_0 \cos^2 k_T x$ (x represents the coordinate along the cavity axis, and k_T is the wave number for the FORT beam), and approximate the potential to be a harmonic potential around the minimum point $x = 0$. The Lamb-Dicke condition can then be written as $V_0 \gg [(\hbar^2 k^4)/(8mk_T^2)]$ (m is the mass of the cesium atom). Taking $\lambda_T = 869 \text{ nm} = (2\pi/k_T)$ and $\lambda = 852.36 \text{ nm} = (2\pi/k)$, this condition yields $V_0 \gg 0.5 \text{ kHz}$. In comparison, the value of V_0 as large as 45 MHz has been reported [12]. A challenging requirement comes from the assumption that the atoms are trapped in the low-lying state, say, the ground state, of the trapping potential. This requires that thermal energy of the atoms be less than the ground state energy of the trapping potential. Taking again the case of the FORT beam, the required condition becomes $T \leq [(\hbar k_T)/(2k_B)]\sqrt{(2V_0/m)}$ (k_B is the Boltzmann constant). Taking $\lambda_T = 869 \text{ nm}$, and $V_0 = 45 \text{ MHz}$, this condition yields $T \leq 14 \mu\text{K}$. Although the temperature as low as $2 \sim 3 \mu\text{K}$ has been achieved experimentally [12], this condition on temperature pushes the present technology to its limit. In this regard, we note that there seems to exist a promising method, namely, the adiabatic scheme recently proposed by Duan *et al.* [13], which may allow a successful operation of our experiment even with “hot” trapped atoms beyond the Lamb-Dicke limit. In this adiabatic scheme, by keeping the pumping laser collinear with the cavity axis and thereby allowing the driving pulse to have the same spatial mode structure as the cavity mode, the system dynamics can be made to become

independent of the random atom position generated by thermal motion.

Our scheme also requires a reliable and efficient source of a single left-circularly polarized photon. This is certainly a difficult requirement to achieve even with much experimental progress [14] witnessed recently. One promising source that can be used for our experiment may be a single atom trapped in a high- Q cavity. When combined with the adiabatic scheme of Duan *et al.* [13], such an atom could represent a fully controllable single-photon source.

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- [1] J. S. Bell, *Physics* (Long Island City, NY) **1**, 195 (1965).
 - [2] P.W. Shor, *SIAM J. Comput.* **26**, 1484 (1997); L. K. Grover, *Phys. Rev. Lett.* **79**, 325 (1997).
 - [3] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W.K. Wootters, *Phys. Rev. Lett.* **70**, 1895 (1993).
 - [4] A. Barenco and A. K. Ekert, *J. Mod. Opt.* **42**, 1253 (1995).
 - [5] A. K. Ekert, *Phys. Rev. Lett.* **67**, 661 (1991).
 - [6] P.G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A.V. Sergienko, and Y. Shih, *Phys. Rev. Lett.* **75**, 4337 (1995).
 - [7] B. J. Oliver and C. R. Stroud, *J. Opt. Soc. Am. B* **4**, 1426 (1987); J. I. Cirac and P. Zoller, *Phys. Rev. A* **50**, R2799 (1994); E. S. Fry, T. Walther, and S. Li, *Phys. Rev. A* **52**, 4381 (1995); M. Freyberger, P. K. Aravind, M. A. Horne, and A. Shimony, *Phys. Rev. A* **53**, 1232 (1996); C. C. Gerry, *Phys. Rev. A* **53**, 4591 (1996).
 - [8] E. Hagley, X. Maitre, G. Nogues, C. Wunderlich, M. Brune, J. M. Raimond, and S. Haroche, *Phys. Rev. Lett.* **79**, 1 (1997); Q. A. Turchette, C. S. Wood, B. E. King, C. J. Myatt, D. Leibfried, W. M. Itano, C. Monroe, and D. J. Wineland, *Phys. Rev. Lett.* **81**, 3631 (1998).
 - [9] M. B. Plenio, S. F. Huelga, A. Beige, and P. L. Knight, *Phys. Rev. A* **59**, 2468 (1999).
 - [10] W. Lange and H. J. Kimble, *Phys. Rev. A* **61**, 063817 (2000).
 - [11] C. J. Hood, M. S. Chapman, T. W. Lynn, and H. J. Kimble, *Phys. Rev. Lett.* **80**, 4157 (1998); C. J. Hood, T. W. Lynn, A. C. Doherty, A. S. Parkins, and H. J. Kimble, *Science* **287**, 1447 (2000).
 - [12] J. Ye, D. W. Vernooy, and H. J. Kimble, *Phys. Rev. Lett.* **83**, 4987 (1999).
 - [13] L. M. Duan, A. Kuzmich, and H. J. Kimble, *quant-ph/0208051*.
 - [14] C. Brunel, B. Lounis, P. Tamarat, and M. Orrit, *Phys. Rev. Lett.* **83**, 2722 (1999); J. Kim, O. Benson, H. Kan, and Y. Yamamoto, *Nature (London)* **397**, 500 (1999); M. Pelton, C. Santori, J. Vuckovic, B. Zhang, G. S. Solomon, J. Plant, and Y. Yamamoto, *quant-ph/0208054*.