CP Violation in Neutrino Oscillation and Leptogenesis

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We study the correlation between *CP* violation in neutrino oscillations and leptogenesis in the framework with two heavy Majorana neutrinos and three light neutrinos. Among three unremovable *CP* phases, a heavy Majorana phase contributes to leptogenesis. We show how the heavy Majorana phase contributes to Jarlskog determinant J as well as neutrinoless double β decay by identifying a low energy *CP*-violating phase which signals the *CP*-violating phase for leptogenesis. For some specific cases of the Dirac mass term of neutrinos, a direct relation between lepton number asymmetry and J is obtained. We also study the effect coming from the phases which are not related to leptogenesis.

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Finding any relation between baryogenesis via leptogenesis [1] and low energy *CP* violation observed in the laboratory is a very interesting issue [2]. The *CP* violation required for leptogenesis stems from the *CP* phases in the heavy Majorana sector, whereas *CP* violation measurable from the neutrino oscillations [3] can be described by the neutrino mixing matrix. One interesting question concerned with the low energy leptonic *CP* violation is whether it can be affected by the *CP*-violating phases responsible for leptogenesis [4]. The major difficulty to quantify such a connection occurs due to lack of the available low energy data to fix parameters of the seesaw model.

The purpose of this paper is to examine in a rather general framework how leptogenesis can be related to the low energy *CP* violation by determining the parameters as many as possible from available low energy experimental results and cosmological observations. In order to make a quantitative analysis of the connection between low energy leptonic *CP* violation and leptogenesis, we consider the *minimal* *CP*-violating seesaw model which has two heavy Majorana neutrinos and three light left-handed neutrinos: (3,2) seesaw model [5]. As will be shown later, to break *CP* symmetry, the required minimal number of singlet heavy Majorana neutrino is two in the seesaw model with three light lepton doublets. This (3,2) seesaw model is consistent with recent data of neutrino oscillations and contains eight real parameters and three *CP*-violating phases in the neutrino sectors. We will show that while all three *CP*-violating phases contribute to low energy leptonic *CP* violation, only a single *CP*-violating phase contributes to leptogenesis. We will also investigate how large the *CP* phase responsible for leptogenesis contributes to low energy *CP* violation by determining the independent parameters from available experimental results and cosmological observations. Finally, we will discuss the potential implication of *CP* violation measurable from neutrino oscillations on leptogenesis.

Let us begin our study by considering the leptonic sector of the (3,2) seesaw model. In a basis where both heavy Majorana and charged lepton mass matrices are real diagonal, the Lagrangian is given by

$$\mathcal{L} = -\overline{l_{iL}} m_{li} l_{iR} - \overline{\nu_{Li}} m_{Dij} N_{Rj} - \frac{1}{2}\overline{(N_{Rj})^c} M_j N_{Rj}, \quad (1)$$

where $i = 1, 2, 3$, $j = 1, 2$, and the Dirac mass term m_D is a 3×2 matrix. Here, we remark that the Dirac mass matrix m_D contains $3N - 3$ unremovable *CP* phases if we take N singlet heavy Majorana neutrinos in this basis. Thus, one can easily see that at least two singlet heavy Majorana neutrinos are required to break *CP* symmetry in the seesaw model with three lepton SU(2) doublets. The 3×2 matrix m_D can be generally parametrized as

$$m_D = U_L \begin{pmatrix} 0 & 0 \\ m_2 & 0 \\ 0 & m_3 \end{pmatrix} V_R, \quad (2)$$

with, $U_L = O_{23}(\theta_{L23}) U_{13}(\theta_{L13}, \delta_L) O_{12}(\theta_{L12}) P(\gamma_L)$, and

$$V_R = \begin{pmatrix} \cos\theta_R & \sin\theta_R \\ -\sin\theta_R & \cos\theta_R \end{pmatrix} \begin{pmatrix} e^{-i(\gamma_R/2)} & 0 \\ 0 & e^{i(\gamma_R/2)} \end{pmatrix}, \quad (3)$$

where O_{ij} and U_{ij} denote the rotations of the (i, j) plane, $P(\gamma_L) = \mathrm{diag}[1, e^{-i\gamma_L/2}, e^{i\gamma_L/2}]$, and m_2 and m_3 are real and positive. Without loss of generality, we can choose $m_2 \leq m_3$. The allowed range for the angles and the phases is $[-\pi, \pi]$. There are three *CP*-violating phases: γ_R which appears in V_R, γ_L and δ_L in U_L. In a different basis with complex M_i, γ_R can be interpreted as a heavy Majorana phase. The lepton number asymmetry for the lightest heavy Majorana neutrino (N_1) decays into $l^{\mp}\phi^{\pm}$ [5,6] is given by

$$\varepsilon_1 = \frac{\Gamma_1 - \overline{\Gamma_1}}{\Gamma_1 + \overline{\Gamma_1}} = -\frac{3M_1}{2M_2 V^2} \frac{\mathrm{Im}[\{(m_D^\dagger m_D)_{12}\}^2]}{(m_D^\dagger m_D)_{11}}, \quad (4)$$

where $V = \sqrt{4\pi}v$ with $v = 246$ GeV, and

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$$\mathrm{Im}[\{(m_D^\dagger m_D)_{12}\}^2] = \left(\frac{m_2^2 - m_3^2}{2}\right)^2 \sin^2 2\theta_R \sin 2\gamma_R. \quad (5)$$

We see that CP violation concerned with leptogenesis can be possible only if the mixing angle θ_R and CP-violating phases γ_R for the heavy Majorana neutrinos are nonzero. For our purpose, let us now study how the phase γ_R contributes to CP violation in the neutrino oscillations which is usually described in terms of the Maki-Nakagawa-Sakata (MNS) neutrino mixing matrix [7]. The effective mass matrix for light neutrinos is given by $m_{\mathrm{eff}} = -m_D \frac{1}{M} m_D^T = -U_L m V_R \frac{1}{M} V_R^T m^T U_L^T$, and is diagonalized by the MNS mixing matrix as $U_{\mathrm{MNS}}^\dagger m_{\mathrm{eff}} U_{\mathrm{MNS}}^* = \mathrm{diag}[n_1, n_2, n_3]$. Then, the MNS mixing matrix is decomposed into two mixing matrices as follows:

$$U_{\mathrm{MNS}} = U_L K_R, \quad (6)$$

where K_R is a unitary matrix diagonalizing the matrix $Z \equiv -m V_R \frac{1}{M} V_R^T m^T$ and parametrized by

$$K_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta e^{-i\phi} \\ 0 & -\sin\theta e^{i\phi} & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{-i\alpha} \end{pmatrix}. \quad (7)$$

Then, $-K_R^\dagger m V_R \frac{1}{M} V_R^T m^T K_R^* = \mathrm{diag}[0, n_2, n_3]$. Note that the (3,2) seesaw model predicts one massless neutrino. In addition, θ, ϕ, and α are determined as

$$\phi = \arg(Z_{22}^* Z_{23} + Z_{23}^* Z_{33}),$$
$$\tan 2\theta = \frac{2|Z_{22}^* Z_{23} + Z_{23}^* Z_{33}|}{|Z_{33}|^2 - |Z_{22}|^2}, \quad (8)$$
$$2\alpha = \arg[\cos^2\theta Z_{22} + \sin^2\theta Z_{33} e^{-2i\phi} - \sin 2\theta Z_{23} e^{-i\phi}].$$

We remark that the mixing angle θ_R and the CP-violating phase γ_R have been transferred to θ, ϕ, and α. As one can see from the above formulas, leptogenesis occurs only if the mixing angle θ_R and the CP-violating phase γ_R are nonzero, which in turn implies nonvanishing ϕ, α, and θ via Eq. (8). As we will see below, the CP phase ϕ contributes to CP violation in neutrino oscillations, so that it is anticipated that there is correlation between CP violation generated from neutrino mixings and leptogenesis. To see this concretely, let us compute the Jarlskog determinant [8] $J = \mathrm{Im}[U_{\mathrm{MNS}e1} U_{\mathrm{MNS}e2}^* U_{\mathrm{MNS}\mu 1}^* U_{\mathrm{MNS}\mu 2}]$, which is proportional to the CP asymmetry in neutrino oscillation, $\Delta P = P_{(\nu_\mu \to \nu_e)} - \bar{P}_{(\bar{\nu_\mu} \to \bar{\nu_e})} = 4J\{\sin[(\Delta m_{12}^2 L)/(2E)] + \sin[(\Delta m_{23}^2 L)/(2E)] + \sin[(\Delta m_{31}^2 L)/(2E)]\}$. By using Eq. (6), we obtain

$$J = \frac{1}{8}\sin 2\theta_{L12} \sin 2\theta_{L13}\Big[c_{L13}\cos 2\theta \sin\delta_L \sin 2\theta_{L23} + c_{L12}\sin 2\theta \sin(\delta_L - \gamma_L - \phi)\cos 2\theta_{L23} - \frac{1}{2}s_{L12}s_{L13}\sin 2\theta \sin 2\theta_{L23}\sin(2\delta_L - \gamma_L - \phi)\Big] + \frac{1}{8}\sin 2\theta \sin 2\theta_{L23}\sin(\gamma_L + \phi)(\sin 2\theta_{L12} c_{L13} s_{L12} - \sin 2\theta_{L13} s_{L13} c_{L12}). \quad (9)$$

From the expression of J, it is obvious that all three CP-violating phases δ_L, γ_L, and ϕ contribute to CP violation in the neutrino oscillations, and that the CP phase γ_L always hangs around ϕ. Since only ϕ is closely related to leptogenesis, in order to investigate the interplay between CP violation for leptogenesis and low energy leptonic CP violation, we should determine the contributions of the phases (δ_L, γ_L) and ϕ separately as well as to fix the parameters θs.

Before discussing the correlation between both CP violations, let us study how we can get some information on the mixing angles and CP phases from the available experimental and cosmological results. The mixing angles and CP phases can be classified into two categories: one contains θ, ϕ, and α which are related to phenomena at high energy, and the other contains parameters in U_L. First of all, we show how we can estimate the allowed values of CP-violating phase ϕ and mixing angle θ. The information on ϕ and α may come from the constraints on light neutrino mass spectra as well as the cosmological condition for leptogenesis. To see this, we first present the parameters $\gamma_R, \theta_R, m_2, m_3$ and lepton number asymmetry ε_1 in terms of some physical quantities which will be taken as inputs in the numerical calculation. Here, we choose the heavy Majorana neutrino masses (M_1, M_2), their decay widths (Γ_1, Γ_2), and light neutrino masses (n_2, n_3) as the physical input parameters. As will be clear later, it is convenient to define two parameters $x_i (i = 1, 2)$:

$$x_i = \frac{(m_D^\dagger m_D)_{ii}}{M_i} = \Gamma_i \left(\frac{V}{M_i}\right)^2. \quad (10)$$

Then, by considering the light neutrino mass eigenvalue equation, $\det[m_{\mathrm{eff}} m_{\mathrm{eff}}^\dagger - n^2] = 0$, the lepton number asymmetry ε_1 and the phase γ_R can be written in terms of x_1, x_2, n_2, and n_3,

$$\varepsilon_1 = -\frac{3M_1}{4x_1 V^2}\sqrt{[(n_-)^2 - (x_-)^2][(x_+)^2 - (n_+)^2]}, \quad (11)$$

$$\cos 2\gamma_R = \frac{n_2^2 + n_3^2 - x_1^2 - x_2^2}{2(x_1 x_2 - n_2 n_3)}, \quad (12)$$

where $n_\pm = n_3 \pm n_2$ and $x_\pm = x_1 \pm x_2$. There are two solutions of Eq. (12) leading to negative ε_1: γ_R and $\gamma_R - \pi$ for $0 \leq \gamma_R < \pi/2$, which in turn gives a positive baryon number via the sphaleron process. Next, let us present the parameters, θ_R, m_2, and m_3 in terms of the above six physical quantities. From the eigenvalue equations for $V_R[(m_D^\dagger m_D)/(\sqrt{M_1 M_2})]V_R^\dagger$ we can express θ_R, m_2, and m_3 as follows:

$$(m_2^2, m_3^2) = \sqrt{M_1 M_2}(\sigma_+ - \rho, \sigma_+ + \rho), \tag{13}$$

$$(\cos\theta_R, \sin\theta_R) = \left(\sqrt{\frac{\sigma_- + \rho}{2\rho}}, -\sqrt{\frac{-\sigma_- + \rho}{2\rho}}\right), \tag{14}$$

where $\sigma_\pm = [(x_2 \pm x_1 R)/(2\sqrt{R})]$, $\rho = \sqrt{(x_1 x_2 - n_2 n_3) + \sigma_-^2}$, and $R = M_1/M_2$. We also determine ϕ, α, and θ with a given set of parameters (x_1, x_2, R, n_2, n_3) by using the same procedure given in Eqs. (7) and (8). We take $R = 0.1$. In order to determine the values of ϕ, θ, and α, it is necessary to determine those of $x_i (i = 1, 2)$. Let us now show how the variables x_i can be constrained. From the neutrino mass eigenvalue equation, it follows that

$$|x_1 - x_2| \le n_3 - n_2, \quad n_3 + n_2 \le x_1 + x_2. \tag{15}$$

From the experimental results for the neutrino oscillation, let us take $n_3 = \sqrt{\Delta m^2_{\rm atm.}} = 5 \times 10^{-2}$ eV and $n_2 = \sqrt{\Delta m^2_{\rm solar}} = 7 \times 10^{-3}$ eV [9]. From Eq. (15), the lower bound on x_1 is 0.007 eV. By solving the Boltzmann equation [10], we can obtain a value of $Y_L = \frac{n_L}{s}$, i.e., lepton number density (n_L) normalized by entropy density (s). When solving the Boltzmann equation, we need the value of ε_1. For fixed x_1, one can get the maximum value of $-\varepsilon_1$ which gives the maximum $-Y_L$ via the Boltzmann equation. In Fig. 1, we plot the maximum lepton number density $-Y_L$ predicted from Eq. (11) as a function of x_1 for several fixed M_1. We set the initial conditions for the Boltzmann equation at 10^{16} GeV and we take the distribution of the heavy Majorana particle N_1 in thermal equilibrium and $Y_L = 0$ at the temperature.

The allowed values of $-Y_L$ consistent with baryogenesis are presented by the shaded band in Fig. 1. Thus, we can obtain the allowed region of x_1 for a fixed M_1. However, there is no allowed value of x_1 for a rather lower value of $M_1 < 1.0 \times 10^{11}$ GeV, which in turn leads to the lower bound on M_1. The value is somewhat larger than the one obtained in [5]. In our analysis, the Boltzmann equation dictates that the dilution factor (Y_L/ε_1) is the order of 10^{-4} while in their analysis, the factor is taken to be 10^{-2}, which pushes the lower bound on M_1 obtained here up by a factor of 10^2. By using the allowed region for x_1 as given in the above, we can estimate the allowed region of x_2 via Eq. (11) again. Figure 2 shows how we can get the allowed region of x_2. For example, for a given set $M_1 = 2 \times 10^{11}$ GeV, and $x_1 = 0.03$ eV, we obtain the allowed range for x_2 as $0.03 < x_2 < 0.07$ eV.

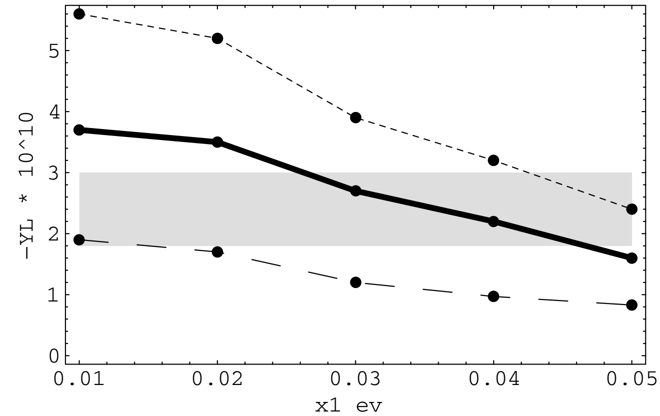

FIG. 1. The maximum possible lepton number density ($-Y_L$) as a function of x_1 for three different M_1. From top to bottom, $M_1 = (3, 2, \text{ and } 1) \times 10^{11}$ GeV.

Let us move to the other category of the parameters, θ_{L12}, θ_{L23}, θ_{Ls13}, γ_L, δ_L in U_L, which are not determined from high energy phenomena, but must be related to the low energy MNS mixing matrix, Thus two of them can be determined from the neutrino oscillation experimental results. For simplicity, we focus on the case with the small mixing angles θ_{L13} and θ, which is consistent with the Chooz experiment [11]. In this case, the MNS mixing matrix is given, in the leading order, by

$$U_{\rm MNS} \simeq \begin{pmatrix} c_{L12} & s_{L12} & s_{L13}e^{-i\delta_L} + s_{L12}s_\theta e^{-i\phi'} \\ -s_{L12}c_{L23} & c_{L12}c_{L23} & s_{L23} \\ s_{L12}s_{L23} & -c_{L12}s_{L23} & c_{L23} \end{pmatrix} \times P(\alpha', -\alpha'), \tag{16}$$

where $\phi' = \phi + \gamma_L$, $\alpha' = \alpha - \frac{\gamma_L}{2}$, and $P(\alpha', -\alpha') = {\rm diag}[1, e^{i\alpha'}, e^{-i\alpha'}]$. Note that we do not present the subleading contributions in $(U_{\rm MNS})_{ij}$, $(ij) \neq (e3)$, which are comparable to $(U_{\rm MNS})_{e3}$. Taking $\theta_{23} \simeq \theta_{12} \simeq \frac{\pi}{4}$ which lead to a bilarge mixing pattern, in this approximation, $(U_{\rm MNS})_{e3}$, J, and $|(m_{\rm eff})_{ee}|$ are given by

$$|(U_{\rm MNS})_{e3}| \simeq \left| s_{L13}e^{-i\delta} + \frac{s_\theta}{\sqrt{2}} e^{-i\phi'} \right|, \qquad J \simeq \frac{1}{4}\left(s_{L13}\sin\delta_L + \frac{s_\theta}{\sqrt{2}}\sin\phi'\right),$$
$$|(m_{\rm eff})_{ee}| \simeq \left| \frac{n_2}{2} e^{4i\alpha'} + n_3\left(s_{L13}e^{-i\delta_L} + \frac{s_\theta}{\sqrt{2}} e^{-i\phi'}\right)^2 \right|. \tag{17}$$

In principle, we are able to fix three unknown parameters: γ_L, s_{L13}, and δ_L once the left-hand sides of Eq. (17) are measured. It is then possible to quantitatively see whether the low energy CP violation denoted by J is dominated by leptogenesis phase ϕ or by the CP-violating phases γ_L and δ_L which are not related to leptogenesis. First of all, let us

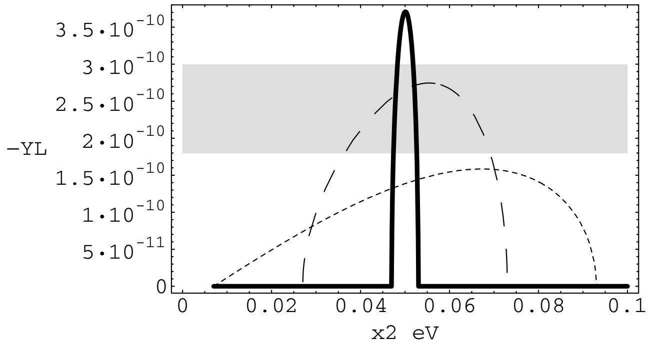

FIG. 2. Lepton number density ($-Y_L$) as a function of x_2 for $M_1 = 2 \times 10^{11}$ GeV. Solid line corresponds to $x_1 = 0.01$ eV. The long dashed line corresponds to $x_1 = 0.03$ eV, and the dotted line corresponds to $x_1 = 0.05$ eV.

study the interesting case of $\theta_{L13} = 0$, which makes the analysis more predictive because a CP-violating phase δ_L is simultaneously suppressed. This can be understood as the extreme case of $\sin\theta_{L13} \ll \sin\theta$. Interestingly enough, this case dictates that the origin of $|(U_{\rm MNS})_{e3}|$ may come from the mixing angle θ which is related to the heavy Majorana neutrino sector. The Jarlskog determinant is then simply given by

$$J = \sin 2\theta \sin\phi' \frac{1}{8\sqrt{2}}. \tag{18}$$

Only γ_L is a completely arbitrary parameter in this case and thus we can easily investigate how J can be affected by γ_L. In other words, γ_L can be estimated through J in this case. If γ_L is turned out to be much smaller than ϕ, the measurement of CP violation in the low energy experiment may directly indicate leptogenesis.

In Fig. 3, we show the correlation between lepton number asymmetry ε_1 and J in Eq. (18) with $\gamma_L = 0$. In each contour, M_1 and x_1 are fixed and x_2 is varied. For $x_1 = 0.01$ eV, $|(U_{\rm MNS})_{e3}| = (\sin\theta/\sqrt{2}) \simeq 0.066$ and $|(m_{\rm eff})_{ee}| =$

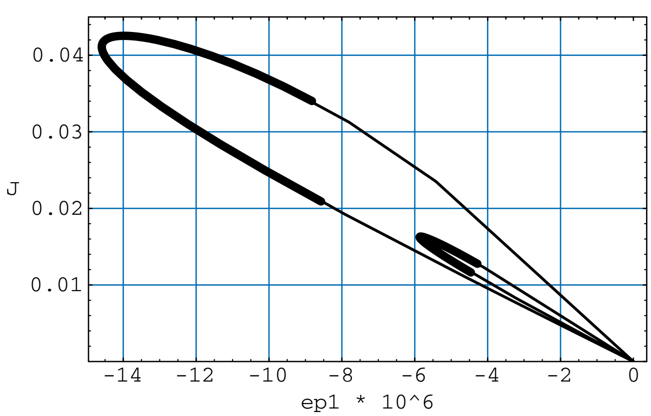

FIG. 3 (color online). Correlation between lepton number asymmetry ε_1 and neutrino CP violation J, for $M_1 = 2 \times 10^{11}$ GeV. The large (small) contour corresponds to $x_1 = 0.03$ (0.01) eV. The thick solid lines show the allowed region from baryogenesis.

0.008 eV, while for $x_1 = 0.03$ eV, $|(U_{\rm MNS})_{e3}|$ is in the range $[0.15, 0.2]$ and $|(m_{\rm eff})_{ee}|$ is in the range $[0.014, 0.018]$ eV. For the case with $\sin\theta \ll \sin\theta_{L13}$, we see from Eq. (17) that J mainly depends on δ_L, which has nothing to do with leptogenesis.

We have studied how CP violation responsible for baryogenesis manifests itself in the MNS matrix and the Jarlskog determinant which signals low energy CP violation in neutrino oscillation. We also showed it is possible to estimate the size and sign of the baryon number in the most general case once $|(U_{\rm MNS})_{e3}|$, neutrinoless double beta decay and CP violation of neutrino oscillation are measured. A correlation between CP violation in neutrino oscillation and leptogenesis has been studied and the size of J has been estimated.

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