Effective Temperature in Driven Vortex Lattices with Random Pinning

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We study numerically correlation and response functions in nonequilibrium driven vortex lattices with random pinning. From a generalized fluctuation-dissipation relation, we calculate an effective transverse temperature in the fluid moving phase. We find that the effective temperature decreases with increasing driving force and becomes equal to the equilibrium melting temperature when the dynamic transverse freezing occurs. We also discuss how the effective temperature can be measured experimentally from a generalized Kubo formula.

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Whether and how can one extend thermodynamic concepts to nonequilibrium systems is a very important challenge in theoretical physics. Many definitions of nonequilibrium temperatures have been proposed in different contexts, but it has been rarely checked if they conform with the expected properties of a temperature.

Cugliandolo, Kurchan, and Peliti [1] have introduced the notion of time-scale dependent "effective temperatures" T_{eff} from a modification of the fluctuation-dissipation theorem (FDT) in slowly evolving out of equilibrium systems. T_{eff} is defined from the inverse slope of the parametric plot of the integrated response against the correlation function of a given pair of observables when the latter is bounded or from half the inverse slope of the parametric plot of the integrated response against the displacement when the correlation is unbounded. This definition yields a *bona fide* temperature in the thermodynamic sense since it can be measured with a thermometer, it controls the direction of heat flow for a given time scale, and it satisfies a zeroth law within each time scale. T_{eff} was found analytically in mean-field glassy models [1–3] and it was successfully studied in structural and spin glasses, both numerically [4] and experimentally [5], in granular matter [6,7], and in weakly sheared fluids [8,9].

In their study of driven vortex lattices in type II superconductors, Koshelev and Vinokur [10] have defined a "shaking" temperature T_{sh} from the fluctuating force felt by a vortex configuration moving in a random pinning potential. This leads to the prediction of a dynamic phase transition between a liquidlike phase of vortices moving at weak driving forces and a crystalline vortex lattice moving at strong forces, when T_{sh} equals the equilibrium melting temperature of the vortex system [10,11]. However, later work [12–14] has shown that the perturbation theory used in [10] breaks down and that the vortex phase at high velocities can be an anisotropic transverse glass instead of a crystal. In spite of this, the shaking temperature introduced in [10] has been a useful qualitative concept, at least phenomenologically. Indeed, the dynamic transitions and moving vortex phases discussed in [10–14] have been observed experimentally [15] and in numerical simulations [16,17].

In this Letter, we apply the definition of T_{eff} based on the modifications of the FDT to driven vortex lattices within the fluid moving phase. We compare our results with the shaking temperature of [10] and discuss how to obtain T_{eff} experimentally from measurements of transverse voltage noise and transverse resistance.

The equation of motion of a vortex in position $\mathbf{R}_i$ is

$$\eta \frac{d\mathbf{R}_i}{dt} = -\sum_{j\neq i} \boldsymbol{\nabla}_i U_v(R_{ij}) - \sum_{p} \boldsymbol{\nabla}_i U_p(R_{ip}) + \mathbf{F} + \boldsymbol{\zeta}_i(t),$$

where $R_{ij} = |\mathbf{R}_i - \mathbf{R}_j|$ is the distance between vortices i, j, $R_{ip} = |\mathbf{R}_i - \mathbf{R}_p|$ is the distance between the vortex i and a pinning site at $\mathbf{R}_p$, η is the Bardeen-Stephen friction, and $\mathbf{F} = \frac{d\Phi_0}{c}\mathbf{J}\times\mathbf{z}$ is the driving force due to a uniform current density $\mathbf{J}$. The effect of a thermal bath at temperature T is given by the stochastic force $\boldsymbol{\zeta}_i(t)$, satisfying $\langle \zeta_i^{\mu}(t)\rangle = 0$ and $\langle \zeta_i^{\mu}(t)\zeta_j^{\mu'}(t')\rangle = 2\eta T\delta(t-t')\delta_{ij}\delta_{\mu\mu'}$. ($k_B = 1$ henceforth.) We model a 2D thin film superconductor of thickness d and size L by considering a logarithmic vortex-vortex interaction potential: $U_v(r) = -A_v \ln(r/\Lambda)$, with $A_v = \Phi_0^2/8\pi\Lambda$ and $\Lambda = 2\lambda^2/d > L$ [17]. The vortices interact with a random distribution of attractive pinning centers with $U_p(r) = -A_p e^{-(r/r_p)^2}$. Length is normalized by r_p, energy by A_v, and time by $\tau = \eta r_p^2/A_v$. We consider N_v vortices and N_p pinning centers in a rectangular box of size $L_x \times L_y$. Moving vortices induce an average macroscopic electric field $\mathbf{E} = \frac{B}{c}\mathbf{V}\times\mathbf{z}$, with $\mathbf{V} = \frac{1}{N_v}\sum_i d\mathbf{R}_i/dt$.

To analyze the validity and modifications of the FDT, we calculate the fluctuation-dissipation relation (FDR), i.e., the relationship between the displacement and the response functions of a given observable. It is convenient to choose [4] the observable $A_\mu(t) = \frac{1}{N_v}\sum_{i=1}^{N_v} s_i r_i^\mu(t)$, where $s_i = -1, 1$ are random numbers with $\overline{s_i} = 0$ and $\overline{s_i s_j} = \delta_{ij}$, $r_i^\mu = R_i^\mu - R_{\rm c.m.}^\mu$ with $\mu = x, y$ and $\mathbf{R}_{\rm c.m.}$ is the center of mass coordinate. We study separately the FDR in the transverse and parallel directions with respect to $\mathbf{F} = F\mathbf{y}$. The autocorrelation function of the observable A_μ is

$$C_\mu(t, t_0) \equiv \overline{\langle A_\mu(t) A_\mu(t_0)\rangle} = \frac{1}{N_v^2}\sum_{i=1}^{N_v}\langle r_i^\mu(t) r_i^\mu(t_0)\rangle, \quad (1)$$

since the r_i^μ are independent of the s_i in the absence of the perturbation. The integrated response function χ_μ for the observable A_μ is obtained by applying a perturbative force $\mathbf{f}_i^\mu = \epsilon s_i \hat{\boldsymbol{\mu}}$ (where $\hat{\boldsymbol{\mu}} = \hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}$) at time t_0 and keeping it constant for all subsequent times on each vortex:

$$\chi_\mu(t, t_0) = \lim_{\epsilon\to 0}\frac{1}{\epsilon}[\overline{\langle A_\mu(t)\rangle_\epsilon} - \overline{\langle A_\mu(t)\rangle_{\epsilon=0}}]. \quad (2)$$

We see the convenience of using random s_i: it decreases the statistical error in the determination of the response and the perturbation introduced by a random force in the system is weak [4]. We then analyze the FDR,

$$\chi_\mu(t, t_0) = \frac{1}{2T_{\rm eff}^\mu(t, t_0)}\Delta_\mu(t, t_0), \quad (3)$$

where $\Delta_\mu(t, t_0) = N_v^{-1}\sum_i\langle|r_i^\mu(t) - r_i^\mu(t_0)|^2\rangle = N_v[C_\mu(t, t) + C_\mu(t_0, t_0) - 2C_\mu(t, t_0)]$ is the quadratic mean displacement in the direction $\hat{\boldsymbol{\mu}}$. For a system in equilibrium at temperature T, the FDT requires that $T_{\rm eff}^x = T_{\rm eff}^y = T$. In a nonequilibrium system, such as the driven vortex lattice with pinning, the FDT does not apply. Since we are interested in the *stationary* states reached by the driven vortex lattice, where aging effects are stopped [8,9,18], then all observables depend on the difference $t - t_0$, if we choose t_0 long enough to ensure stationarity. From the parametric plot of $\chi_\mu(t)$ vs $\Delta_\mu(t)$, we define the effective temperature $T_{\rm eff}^\mu(t)$ using Eq. (3), provided $T_{\rm eff}^\mu(t)$ is a constant in each time scale [1].

We study the transverse and longitudinal FDR for the moving vortex lattice as a function of driving force F for different values of A_p, n_v, and T. The simulations are performed with pinning density $n_p = N_p r_p^2/L_x L_y = 0.14$ in a box with $L_x/L_y = \sqrt{3}/2$ and $N_v = 256$. We consider $A_p/A_v = 0.35, 0.2, 0.25, 0.1$, $n_v = N_v r_p^2/L_x L_y = 0.05, 0.07$, and $T \leq 0.01$. We impose periodic boundary conditions with the algorithm of Ref. [19]. Averages are evaluated during 80 000 steps of $\Delta t = 0.1\tau$ after 65 536 steps for reaching stationarity. To calculate the response function $\chi_\mu(t)$, given by Eq. (2), we simulate two replicas of the system, with the perturbative force $\mathbf{f}_i^\mu = \epsilon s_i \hat{\boldsymbol{\mu}}$ applied to one of them. Starting from the same initial condition, we let the perturbed and unperturbed system evolve for 5000 time steps and calculate $A_\mu(t)_\epsilon$ and $A_\mu(t)_{\epsilon=0}$, respectively. The replicas then evolve again after changing the realization of the random factors s_i and taking the final configuration of the unperturbed system as the new common initial condition. Therefore, 16 realizations of $\{s_i\}$ have been considered in the averages. From this we get $\overline{\langle A_\mu(t)\rangle_\epsilon}$, both for $\epsilon = 0$ and $\epsilon \neq 0$, and thereby the response function $\chi_\mu(t)$ is determined. We have verified linearity of $\overline{\langle A_\mu(t)\rangle_\epsilon}$ with ϵ in the range $[0.0002, 0.01]$ and then we have used $\epsilon = 0.005 \ll F_c$ in our calculations.

At $T = 0$, there are three different dynamical regimes [17] when increasing F above the critical depinning force F_c: two fluid phases with plastic flow for $F_c < F < F_p$ and smectic flow for $F_p < F < F_t$ and a transverse solid for $F > F_t$. For fixed pinning density n_p, the characteristic forces F_c, F_p, and F_t depend on the disorder strength A_p and vortex density n_v. We start by analyzing the FDR in the smectic flow regime for different values of A_p, n_v, and T. In Fig. 1(a) we show the typical transverse quadratic mean displacements $\Delta_x(t)$ and in Fig. 1(b) the integrated transverse response $\chi_x(t)$ for this dynamical regime. In Fig. 1(c) we show the FDR parametric plot of $\chi_x(t)$ against $\Delta_x(t)$. We see that the equilibrium FDT does not apply in general, but two approximate linear relations exist for $\Delta_x(t) < 0.05r_p^2$ and for $\Delta_x(t) > r_p^2$, with a nonlinear crossover between them. Following Eq. (3), we find that the short displacements region corresponds to the bath temperature $T = 0.01$, and therefore the FDT applies in the transverse direction only for short times. For the

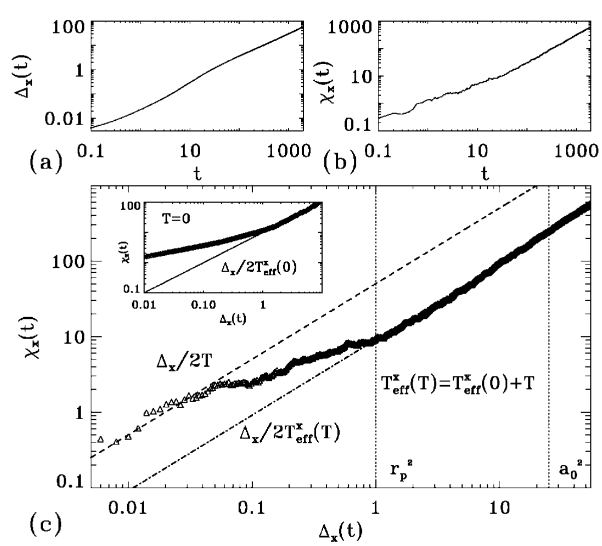

FIG. 1. Averaged transverse quadratic mean displacement $\Delta_x(t)$ (a) and transverse response function $\chi_x(t)$ (b) for $A_p = 0.2$, $T = 0.01$, and $V = 0.11$ in the smectic flow regime. (c) Transverse FDR $\chi_x(t)$ vs $\Delta_x(t)$ for $T = 0.01$. The dashed line indicates $\Delta_x/2T$, with $T = 0.01$ the thermal bath temperature. $T_{\rm eff}(T)$ is obtained from the linear fit to $\chi_x(t)$ for $\Delta_x > r_p^2$ (dash-dotted line). We find that $T_{\rm eff}^x(T) \approx T_{\rm eff}^x(0) + T$, where $T_{\rm eff}^x(0)$ is the corresponding transverse effective temperature for $T = 0$ (inset).

large displacements region we get an effective transverse temperature $T^x_{\rm eff}(T) = 0.045 > T$. In the inset of Fig. 1(a) we show the FDR for $T = 0$. Comparing the results for different T, we find $T^x_{\rm eff}(T) \approx T^x_{\rm eff}(0) + T$. In Fig. 2 we analyze the FDR for the longitudinal direction, for the same time scales as Fig. 1. Figure 2(a) shows the quadratic mean displacement Δ_y and Fig. 2(b) shows the response χ_y. In Fig. 2(c) we obtain the corresponding FDR. We observe that the equilibrium FDT applies for $\Delta_y(t) < 0.05 r_p^2$ at the bath temperature T. There is not a constant $T^y_{\rm eff}(t)$ for larger displacements, because there is superdiffusive behavior with $\Delta_y \sim t^\zeta$, $\zeta > 1$, see [17], while $\chi_y \sim t$.

In Fig. 3 we show the calculated transverse effective temperature $T^x_{\rm eff}$ for $T = 0$ as a function of voltage (i.e., average velocity, V). We observe that above the critical force, $T^x_{\rm eff}$ is a decreasing function of V that reaches a value close to the equilibrium melting temperature of the unpinned system, $T_m \approx 0.007$ [20], when the system approaches the transverse freezing transition at $F = F_t$ (obtained from the vanishing of the transverse diffusion D_x, shown in the inset). It becomes very difficult to compute $T^x_{\rm eff}$ for driving forces $F > F_t$, since Δ_x and χ_x are bounded at $T = 0$ while for finite T there are very long relaxation times involved. We leave the interesting case of obtaining $T^x_{\rm eff}(T)$ for $F > F_t$ for future study.

In Fig. 4 we show the dependence of $T^x_{\rm eff}$ with pinning amplitude A_p and vortex density n_v. In all the cases we observe that $T^x_{\rm eff} \to T_m$ when $F \to F_t$, even when F_t depends on A_p and n_v. The shaking temperature of [10] predicts $T_{\rm sh} \sim (V/A_p^2)^{-1}$, which actually corresponds to the limit of noninteracting vortices or incoherent motion, since it is a single vortex result [14]. As we show in the inset of Fig. 4, we find that a plot of all the curves as $T^x_{\rm eff}(0)$ vs $V/A_p^{2-\alpha}$ better follows $\alpha \approx 0.5$ instead of $\alpha = 0$. In the case of motion of a rigid lattice, one can apply the one particle result to a Larkin-Ovchinikov correlation volume, where the pinning force summation gives an effective pinning amplitude $\sqrt{A_p}$, and therefore $\alpha = 1$ in this limit. It is noteworthy that the value we find is intermediate between these two limits.

We now show that the same $T^x_{\rm eff}$ can be obtained from experimentally accessible quantities such as the transverse resistivity and the voltage fluctuations. If $T^x_{\rm eff}$ is well defined in a given time scale, we can expect a generalized Kubo formula to hold [21],

$$R_x(t) = \frac{N_v}{T^x_{\rm eff}(t)} \int_0^t dt' \langle V_x(t) V_x(t') \rangle, \tag{4}$$

where $R_x(t) = \langle [dV_x(t)]/d\epsilon \rangle_{\epsilon=0} - \langle [dV_x(0)]/d\epsilon \rangle_{\epsilon=0}$ is the linear transverse resistance. From a parametric plot of the integrals of the two sides of Eq. (4), we again find a linear slope equal to $1/T$ for $t < r_p^2/D_x$ and a second linear slope of $1/T^x_{\rm eff}$ for $t > r_p^2/D_x$. In Fig. 3 we compare the two effective transverse temperatures obtained using Eqs. (3) and (4). We see that they are similar within the error bars. The shaking temperature $T_{\rm sh}$ defined in Eq. (3) of Ref. [10] is proportional to the time integral of the correlation function of the pinning force $\mathbf{F}_{\rm pin} = \sum_{i,p} \mathbf{f}_{ip}(t)$. $T_{\rm sh}$ can be obtained from Eq. (4) if we replace $R_x(t)$ with the single vortex value $R_0 = 1/\eta$ and the integral of the $V_x(t)$ correlation function is taken for all t [since for the transverse direction $V_x(t) \propto F^x_{\rm pin}(t)$]. In other words, $T_{\rm sh}$ of [10] corresponds to taking the average slope in the parametric plot of the generalized Kubo formula [or in the parametric plot of the FDR shown in Fig. 1(c)]; see also [22]. The approach followed here permits defining an effective temperature which takes into account all the information on its time-scale

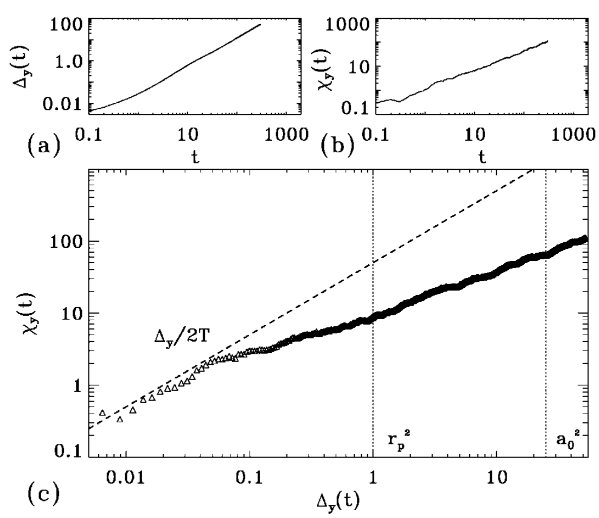

FIG. 2. Averaged longitudinal quadratic mean displacement $\Delta_y(t)$ (a) and longitudinal response function $\chi_y(t)$ (b) for $A_p = 0.2$, $T = 0.01$, and $V = 0.11$ in the smectic flow regime. (c) Longitudinal FDR $\chi_y(t)$ vs $\Delta_y(t)$ for $T = 0.01$. The dashed line indicates $\Delta_y/2T$, with $T = 0.01$.

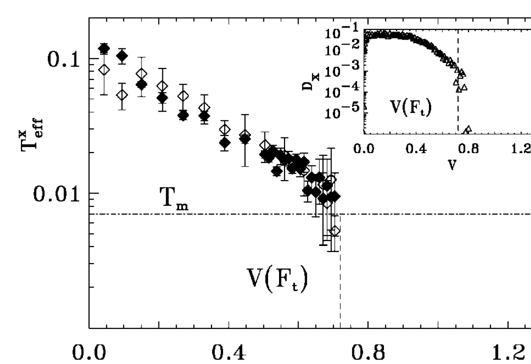

FIG. 3. Transverse effective temperature $T^x_{\rm eff}$ vs voltage V for $A_p = 0.35$, $T = 0$, and $n_v = 0.07$, using the diffusion relation (solid diamonds) and a generalized Kubo formula (open diamonds). Inset: Transverse diffusion constant D_x vs V. Dashed lines indicate the transverse freezing transition at $F = F_t$ and the dash-dotted lines indicate the melting temperature of the unpinned system $T_m \approx 0.007$.

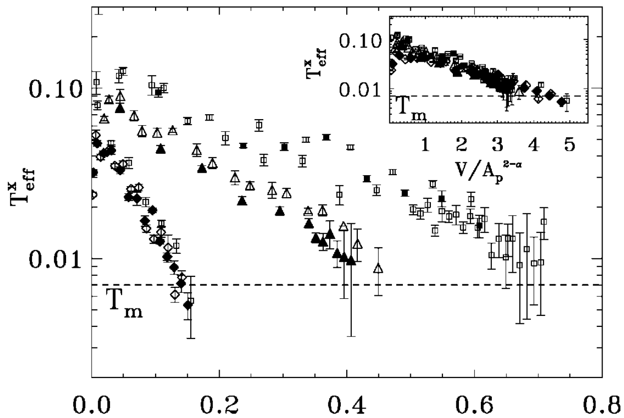

FIG. 4. Transverse effective temperature T^x_{eff} vs voltage V, for different values of pinning amplitude A_p, and vortex density n_v, at $T = 0$. $A_p = 0.35$, $n_v = 0.05$ (■), $A_p = 0.35$, $n_v = 0.07$ (□), $A_p = 0.2$, $n_v = 0.05$ (▲), $A_p = 0.2$, $n_v = 0.07$ (△), $A_p = 0.1$, $n_v = 0.05$ (◇), $A_p = 0.1$, $n_v = 0.07$ (◆). The inset shows T^x_{eff} vs $V/A_p^{2-\alpha}$, with $\alpha = 0.5$.

dependence that allows for a thermodynamic interpretation of T_{eff}. In this way, we see clearly that there is a nontrivial value of the transverse effective temperature T^x_{eff} for time scales $t > r_p^2/D_x$. We also observe in the longitudinal direction y that $T^y_{\text{eff}}(t)$ is very different, since the system is strongly driven out of equilibrium in this direction [23]. Furthermore, we find that the short-range correlations of the moving fluid are important and give a nontrivial dependence with disorder strength A_p. Beside this, we have demonstrated that in the moving fluid phase T^x_{eff} satisfies two important results of [10]: (i) additivity of temperatures, $T^x_{\text{eff}}(T) = T^x_{\text{eff}}(0) + T$ and (ii) dynamic freezing occurs when $T^x_{\text{eff}}(T) = T_m$. The generalized Kubo formula of Eq. (4) suggests that T^x_{eff} can be obtained experimentally from measurements of transverse voltage noise and time-dependent transverse resistivity [24]. It will be interesting to have such experiments to test quantitatively the dynamic freezing transition. Finally, we stress that a complete dynamic theory of the moving vortex system has to capture the features here described.

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