Effect of Poloidal Density Variation of Neutral Atoms on the Tokamak Edge

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Neutral atoms in the tokamak edge affect the electric field and plasma flow through charge exchange. Here, it is shown that these effects depend sensitively on the poloidal location of the atoms. It is found that the radial electric field and toroidal flow velocity in a collisional edge plasma are largest if the atoms are concentrated on the inboard side. This feature may explain MAST and COMPASS-D observations indicating easier *H*-mode access for inboard gas puffing. The results suggest an external means of controlling the toroidal flow and radial electric field and their shear, and have motivated further MAST measurements appearing to support the predictions.

Background.—The effect of neutral atoms is not included in standard tokamak transport treatments [1]. However, recent experiments [2–9] have shown that neutral atoms in the tokamak edge can influence global confinement by affecting the transition from low (*L*) to high (*H*) confinement. The physical mechanism by which this occurs is not yet clearly identified, but it is well known that neutrals influence ion dynamics through charge-exchange (CX) interactions. Furthermore, the radial neutral flux of toroidal angular momentum can modify or even determine the edge radial electric field and plasma rotation. The radial localization of the neutrals also introduces a shear in the flow that may affect edge turbulence [10].

Earlier theoretical work [11–13] explored the effect of neutrals on collisional ion flow and radial electric field within the framework of neoclassical theory. The neutrals provide a drag on the ions that leads to an effective no slip boundary condition for the toroidal ion rotation within an ion temperature gradient modification. The effect of the neutrals is typically significant if the fraction of atoms in the plasma exceeds about 10^{-4}, which is usually the case in the tokamak edge region just inside the separatrix. In these works, the effect of the poloidal variation of the neutral source was not explored. However, recent experimental observations on MAST and COMPASS-D indicate easier *H*-mode access when gas is puffed from the inboard side of the tokamak [5–7].

These experimental results indicate that the poloidal distribution of the neutrals has an essential influence on the *L-H* transition threshold power. This behavior motivates the present paper, where we use kinetic theory results from earlier work [11,12] to investigate the effect of a poloidally varying source of atoms on the electric field and flow velocity of a collisional edge plasma. Our results suggest an external means of controlling ion flow and electric field that is being tested on MAST [6].

Plasma rotation and radial electric field.—The axisymmetric magnetic field of a tokamak can be written as $\mathbf{B} = I(\psi)\nabla\varphi + \nabla\varphi \times \nabla\psi$, where ψ is the poloidal flux function and φ the toroidal angle in the direction of plasma current. In neoclassical theory, where the transport within flux surfaces is taken to be much larger than that across the magnetic field, the density and temperature of all species are approximately constant on flux surfaces, and it follows that the ion flow velocity within a flux surface must be of the form

$$\mathbf{V}_i = \omega(\psi)R\hat{\boldsymbol{\varphi}} + u_{i\theta}(\psi)\mathbf{B}, \qquad (1)$$

where $\hat{\boldsymbol{\varphi}} = R\nabla\varphi$ is the toroidal unit vector and $\omega(\psi) = -\Phi' - p_i'/n_i e$, Φ is the electrostatic potential, $p_i = n_i T$ is the ion pressure, and the prime denotes a derivative with respect to ψ. In a pure hydrogen plasma (no impurities and sufficiently few neutral atoms) $u_{i\theta}$ is proportional to the ion temperature gradient,

$$u_{i\theta}(\psi) = -kIT'/e\langle B^2\rangle, \qquad (2)$$

where the parameter k depends on collisionality, and $\langle \ldots \rangle$ denotes flux-surface average. We consider the Pfirsch-Schlüter regime, where [14] $k \simeq 1.8 + 0.05\langle B^2\rangle\langle(\nabla_\parallel \ln B)^2\rangle/\langle(\nabla_\parallel B)^2\rangle$.

If enough neutral atoms are present in the plasma, its poloidal rotation is directly affected, and (2) is then replaced by $u_{i\theta} = u_{i\theta}^{\text{neo}} + V_n/B$, where $u_{i\theta}^{\text{neo}}$ denotes the neoclassical result (2), and expressions for V_n are given in Refs. [12,13]. Estimates show that this correction is typically important at larger neutral fractions than we consider here, namely, $n_n/n_i \gtrsim \rho_i/qR \sim 10^{-3}$, where q is the safety factor, ρ_i is the ion gyroradius, and n_n and n_i denote the density of neutral atoms and ions.

When there are too few neutrals to directly influence poloidal rotation, they can still affect the radial electric field. This field is intimately coupled to toroidal rotation and the transport of angular momentum. In a

steady state plasma without momentum sources, there can be no radial flux of toroidal angular momentum, which implies [11]

$$\langle R\hat{\boldsymbol{\varphi}} \cdot (\boldsymbol{\pi}_i + \boldsymbol{\pi}_n) \cdot \nabla\psi\rangle = 0, \tag{3}$$

where $\boldsymbol{\pi}_i$ and $\boldsymbol{\pi}_n$ are the ion and neutral viscosity tensors. At the edge, the neutral viscosity dominates over its purely neoclassical counterpart, $\langle R\hat{\boldsymbol{\varphi}} \cdot \boldsymbol{\pi}_i \cdot \nabla\psi\rangle \ll \langle R\hat{\boldsymbol{\varphi}} \cdot \boldsymbol{\pi}_n \cdot \nabla\psi\rangle$, if [11]

$$1 \ll n_n v_T \lambda_x/(n_i q^2 \rho_i^2/\tau_i) \sim 10^4 n_n/n_i, \tag{4}$$

where τ_i is the ionion collision time, $\lambda_x = 1/\sigma_x n_i$ is the charge-exchange mean-free path, and $v_T = (2T/M)^{1/2}$ is the ion thermal velocity. For MAST typically, $T_i = 100$ eV, $B = 0.5$ T, and $n_i = 10^{13}$ cm^{-3}, and for Alcator C-Mod typically, $T_i = 100$ eV, $B = 5$ T, and $n_i = 3 \cdot 10^{14}$ cm^{-3}. If Eq. (4) is satisfied, the radial electric field is determined by neutral atoms rather than neoclassical processes; if it is not, the ion contribution must be retained as in Refs. [11,14]. It is, of course, possible that anomalous transport of angular momentum overwhelms both neutral and neoclassical viscosity, in which case the electric field is determined by turbulence.

Short neutral mean-free path.—The neutral-particle viscosity is easiest to calculate if the scale length of plasma density and temperature variation exceeds the neutral-atom mean-free path for CX. In Ref. [11], the radial flux of toroidal angular momentum was calculated in the ordering $\lambda_x \ll \lambda_z$, where λ_z is the ionization mean-free path of neutral atoms. The result is

$$\langle R\hat{\boldsymbol{\varphi}} \cdot \boldsymbol{\pi}_n \cdot \nabla\psi\rangle = -\frac{1}{\nu_x}\left\langle \nabla\psi \cdot \nabla\left[RTn_n\left(V_{i\varphi} + \frac{2q_{i\varphi}}{5p_i}\right)\right]\right\rangle,$$

where $\nu_x = 2.08\sigma_x n_i v_T = K_x n_i$, $\sigma_x \simeq 6 \times 10^{-19}$ m^2 is the charge-exchange cross section, and $\mathbf{V}_i$ and $\mathbf{q}_i$ are the ion mean velocity and heat flux in a flux surface.

In the Pfirsch-Schlüter regime, where the ion flow velocity is given by (1), the toroidal component of the ion heat flux $\mathbf{q}_i$ is equal to $q_{i\varphi} = -(5Rp_iT'/2e) \times (1 - B_\varphi^2/\langle B^2\rangle)$. If the radial scale length of the neutral density variation is $L_n = \sqrt{\lambda_x\lambda_z}$ [15], which in the present ordering is shorter than the temperature length scale, then the condition (3) of no radial flux of toroidal angular momentum results in

$$\langle R^3 B_\theta^2 V_{i\varphi} n_n'\rangle = \frac{T'}{e}\left\langle R^4 B_\theta^2\left(1 - \frac{B_\varphi^2}{\langle B^2\rangle}\right)n_n'\right\rangle. \tag{5}$$

If gas puffing is done from a thin tube close to the plasma, then most neutral atoms are localized in one place poloidally, and from (5) and (1), respectively, we obtain the toroidal flow velocity at that location as

$$V_{i\varphi}^{\mathrm{puff}} = \frac{RT'}{e}\left(1 - \frac{B_\varphi^2}{\langle B^2\rangle}\right)\Bigg|_{\mathrm{puff}} = \omega R_* + \frac{Iu_{i\theta}}{R_*}, \tag{6}$$

where R_* denotes the value of R at the gas puff. Using Eq. (1) to find $V_{i\varphi}$ on the flux surface in question, eliminating ω, and then using (2), we obtain

$$V_{i\varphi} = \frac{I^2 T'}{e\langle B^2\rangle R}\left[k\left(\frac{R^2}{R_*^2} - 1\right) + \frac{\langle B^2\rangle R^2}{I^2} - \frac{R^2}{R_*^2}\right]. \tag{7}$$

Note that the ion toroidal flow in Eq. (7) is independent of the neutral density provided the neutral viscosity dominates over anomalous and ion viscosities, and only temperature gradient terms depend on the gas puff location. This result is illustrated in Fig. 1, where we have plotted the normalized toroidal flow velocity at the outer midplane, defined by $V_{i\varphi} \equiv I^2 T' F_V/eR\langle B^2\rangle$, as a function of the poloidal angle of the gas puff. Here we use typical magnetic equilibria from Alcator C-Mod and MAST. Note that the rotation speed is much larger if the puff is applied on the inboard side rather than elsewhere on the flux surface. For a normal profile ($T' < 0$) $V_{i\varphi}$ is in the counter-current direction ($V_{i\varphi} < 0$) since $F_V > 0$. In a plasma with small inverse aspect ratio, $\epsilon \ll 1$, and circular cross section, the ion flow from (7) is $V_{i\varphi} = (2\epsilon R_0 T'/e)[k\cos\theta + (1 - k)\cos\theta_*]$, where θ_* denotes the value of θ at the gas puff. Consequently, the poloidal variation of the puffing of neutral atoms on the ion flow is expected to be more important in a spherical tokamak than in a conventional one.

Equation (1) can also be used to calculate the radial electric field, E_r, by solving for ω and inserting $u_{i\theta}$ and
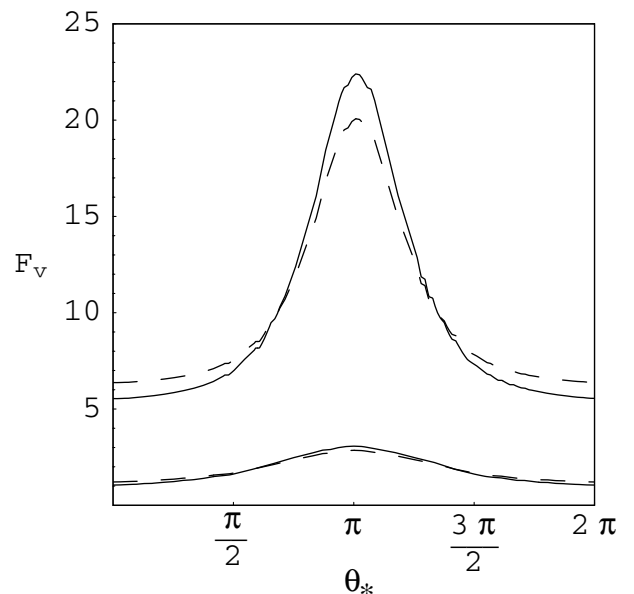

FIG. 1. Predicted toroidal flow velocity at the outer midplane of MAST (upper pair of curves) and C-Mod (lower pair of curves) as a function of the poloidal angle of the gas puff, with $\theta_* = \pi$ at the inboard mid-plane. The solid line represents the short mean-free path limit and the dashed line is for $C_T = 1.15$ and $C_P = 0.04$, corresponding to $\alpha = 3.1$ and $\gamma = 0.5$.

$V_{i\varphi}^{\text{puff}}$ from Eqs. (2) and (6), to find

$$- \Phi' = \frac{Tn_i'}{en_i}\left[1 + \eta_i\left(\frac{(k-1)I^2}{\langle B^2\rangle R_*^2} + 2\right) \right] \equiv \frac{Tn_i'F_E}{en_i}, \quad (8)$$

where $\eta_i = (\ln T)'/(\ln n_i)'$ is the ratio of the temperature to density gradients at the edge. Figure 2 shows the factor F_E for MAST and Alcator C-Mod parameters. Since $F_E > 0$, for normal density and temperature profiles ($n_i' < 0$, $\eta_i > 0$), the edge radial electric field is inwards, as is usually observed in experiments. Inboard gas puffing increases F_E by about a factor of 2 as compared with outboard puffing if $\eta_i = 2$, while the effect is smaller for lower values of η_i. Thus, the effect of neutral atoms on the radial electric field is weaker than on the ion flow, and it depends on the relative size of the density and temperature gradients.

Self-similar neutrals.—The treatment in the previous section assumes that the CX mean-free path is relatively short. This assumption is only marginally satisfied just inside the separatrix in most tokamaks, but can be relaxed for self-similar neutral profiles [16]. In Eq. (3), the neutral viscosity is given by $\boldsymbol{\pi}_n = M \int d^3v(\boldsymbol{vv} - \boldsymbol{I}v^2/3)f_n$ and can be evaluated using the self-similar distribution $f_n(y, \boldsymbol{v}) = NT^{-\alpha}(y)F(\boldsymbol{u})$, where α is a free parameter, $\mathbf{u} = \mathbf{v}/v_T$ and

$$F(\mathbf{u}) = (2\beta/\gamma u_\psi)$$
$$\times \begin{cases} -\int_0^1 \hat{f}(\frac{\mathbf{u}}{s})\exp(\frac{-2(s-1)}{\gamma u_\psi})\frac{ds}{s^{2\alpha}}, & u_\psi < 0, \\ \int_1^\infty \hat{f}(\frac{\mathbf{u}}{s})\exp(\frac{-2(s-1)}{\gamma u_\psi})\frac{ds}{s^{2\alpha}}, & u_\psi > 0, \end{cases} \quad (9)$$
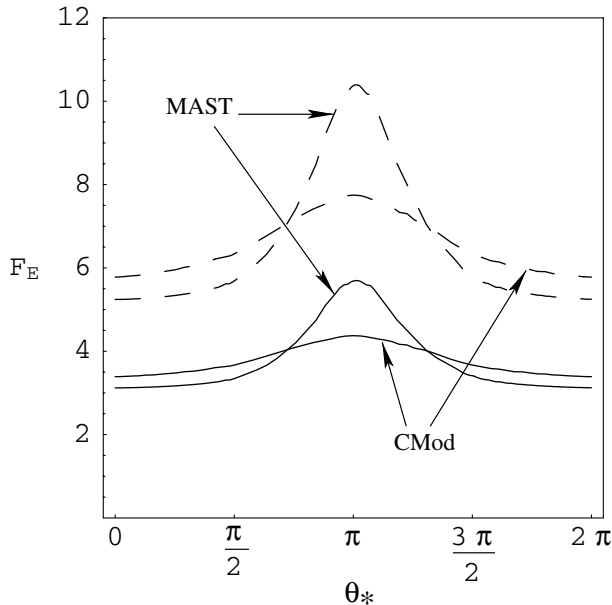

FIG. 2. Radial electric field at the outer midplane of MAST and Alcator C-Mod as a function of the gas puff location. The solid and dashed lines represent the short mean-free path limit for $\eta_i = 1$ and $\eta_i = 2$, respectively.

with $u_\psi = \mathbf{u}\cdot\nabla\psi/|\nabla\psi|$, $\hat{f} = f_i v_T^3/n_i$, and F normalized such that $\int F(\mathbf{u})d^3u = 1$, so that the neutral density is $n_n = Nv_T^3/T^\alpha$. This solution assumes that the ratio of mean-free path to the macroscopic scale length is constant throughout the region of interest: $\gamma \equiv -v_T|\nabla\psi|(\ln T)'/(n_i[K_x + K_z]) = $ constant, and $\beta \equiv K_x/(K_x + K_z) \leq 1$, where K_x and K_z are the CX and ionization rate constants. For the temperatures of interest (1 eV $< T <$ 100 eV), $\beta > 0.5$. The physical requirements of finite and outward neutral heat flux and $\beta > 0.5$ limit our choices of α and γ to $3 < \alpha < 3.5$ and $\gamma \lesssim 2$ [13].

The radial flux of toroidal angular momentum is

$$R\hat{\boldsymbol{\varphi}}\cdot\boldsymbol{\pi}_n\cdot\nabla\psi = Mn_n v_T^2|\nabla\psi|R\int d^3uF(\mathbf{u})u_\varphi u_\psi, \quad (10)$$

where $u_\varphi = \hat{\boldsymbol{\varphi}}\cdot\mathbf{u}$. Since only the odd parts of the ion distribution contribute to (10), we employ the Pfirsch-Schlüter result from Hazeltine [14] to make the replacement $f_i(\mathbf{u}) \to Mv_T f_M/T[\mathbf{V}_i\cdot\mathbf{u} + (u^2 - 5/2)2\mathbf{q}_i\cdot\mathbf{u}/(5p_i) + 8q_{\|i}\hat{\mathbf{n}}\cdot\mathbf{u}\mathcal{L}_2^{(3/2)}(u^2)/75p_i]$, where $f_M = n_i(M/2\pi T)^{3/2}\exp(-u^2)$, $\mathcal{L}_2^{(3/2)}(u^2) = (u^4 - 7u^2 + 35/4)/2$ is a Sonine polynomial, $\hat{\mathbf{n}} = \mathbf{B}/B$, and $q_{\|i} = -(5cIp_iT'/2eB)(1 - B^2/\langle B^2\rangle)$. The contribution of the $\mathcal{L}_2^{(3/2)}$ term vanishes for $\gamma \to 0$, but not for finite γ.

The radial flux of toroidal angular momentum is given by Eq. (10). If we let $\mathbf{x} = \mathbf{u}/s$ and use the orthogonal set of variables $x_\psi = \mathbf{x}\cdot\nabla\psi/|\nabla\psi|$, $x_\vartheta = \mathbf{x}\cdot\hat{\mathbf{n}}\times\nabla\psi/|\nabla\psi|$, and $x_\varphi = \mathbf{x}\cdot\hat{\boldsymbol{\varphi}}$, then only the toroidal components of $\mathbf{V}_i$, $\mathbf{q}_i$ and $q_\|\hat{\mathbf{n}}$ contribute. Inserting the $|\nabla\psi|$ factors based on the short mean-free path result ($|\nabla\psi|$ must be treated as a constant for self-similarity), $\langle R\hat{\boldsymbol{\varphi}}\cdot\boldsymbol{\pi}_n\cdot\nabla\psi\rangle = 0$ gives

$$\langle R^3 B_\theta^2 V_{i\varphi}n_n\rangle = \frac{T'}{e}\left[C_T\left\langle R^4 B_\theta^2 n_n\left(1 - \frac{B_\varphi^2}{\langle B^2\rangle}\right)\right\rangle \right.$$
$$\left. - \frac{C_P\langle R^4 B_\theta^4 n_n\rangle}{\langle B^2\rangle} \right], \quad (11)$$

where we define $C_T = 2\int dx_\psi e^{-x_\psi^2}(x_\psi^4 + 9x_\psi^2/2 - 3)J/(15\int dx_\psi e^{-x_\psi^2}J)$, $C_P = \int dx_\psi e^{-x_\psi^2}(x_\psi^4 - 3x_\psi^2 + 3/4)J/(4\int dx_\psi e^{-x_\psi^2}J)$, and

$$J = \begin{cases} \int_1^\infty \exp(-\frac{2(s-1)}{\gamma x_\psi s})\frac{ds}{s^{2\alpha-4}} & x_\psi > 0 \\ -\int_0^1 \exp(-\frac{2(s-1)}{\gamma x_\psi s})\frac{ds}{s^{2\alpha-4}} & x_\psi < 0 \end{cases}.$$

The expression for C_T is consistent with, but considerably simpler than, the ratio I_n^t/I_n^s approximately evaluated in Ref. [13]. The coefficients C_T and C_P depend on the self-similar parameter α. In the short mean-free path limit ($\gamma \to 0$), $C_T \to 1$, and $C_P \to 0$. For finite mean-free paths C_T increases and then decreases while remaining positive, and C_P increases from zero and then becomes negative, as shown in Fig. 3. Since C_T remains close to unity and C_P is small, finite mean-free path corrections are modest.
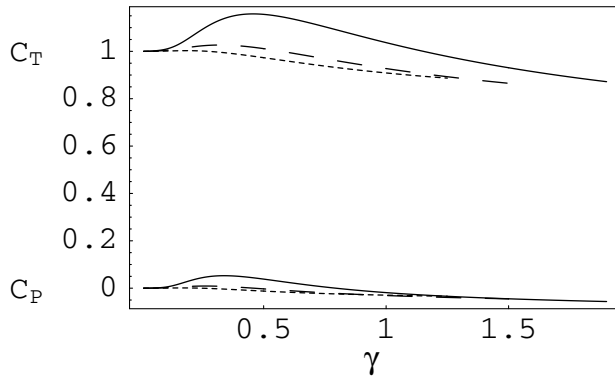
FIG. 3. C_T and C_P as functions of γ for $\alpha = 3.1$ (solid), $\alpha = 3.25$ (dashed), and $\alpha = 3.4$ (dotted).

For a poloidally localized gas puff we may use Eqs. (1), (2), and (11) to obtain

$$
\begin{aligned}
V_{i\varphi} &= \frac{I^2 T'}{e\langle B^2\rangle R}\Bigg[k\left(\frac{R^2}{R_*^2} - 1\right) \\
&\quad + C_T\left(\frac{\langle B^2\rangle R^2}{I^2} - \frac{R^2}{R_*^2}\right) + C_P \frac{R^2 B_{\theta*}^2}{I^2}\Bigg] \\
&\equiv \frac{I^2 T'}{e\langle B^2\rangle R} F_V,
\end{aligned} \tag{12}
$$

where $B_{\theta*} = B_\theta(R_*)$. For finite mean-free path, C_T departs from unity and makes the effect of the neutral atoms on the toroidal ion flow weaker, as shown in Fig. 1. For general γ and localized puffing, the radial electric field becomes $-\Phi' = (T/e)(\ln n_i)'\{1 + \eta_i[(1 + C_T) + (k - C_T)I^2/(\langle B^2\rangle R_*^2) + C_P B_{\theta*}^2/\langle B^2\rangle]\} \equiv F_E T n_i'/(n_i e)$, which agrees with the expression in (8) for short mean-free paths.

Conclusions.—In summary, the toroidal flow of the ions and the radial electric field are significantly affected by the poloidal location of the neutrals in the edge plasma, just inside the separatrix. Because the ion flow on a flux surface is not a constant, the braking provided by the neutrals is sensitive to their poloidal location and is able to alter the flow, the radial electric field, and their shear. The effect is particularly large for spherical tokamaks, it is relatively insensitive to the neutral mean-free path, and does not depend on neutral density if the neutral viscosity is larger than the ion and anomalous viscosities.

These results are important since they might help explain the easier access to H mode with inboard gas puffing observed in MAST and COMPASS-D. The easier H-mode access could, for instance, be caused by suppression of edge turbulence by rotational flow shear in a narrow layer inside the separatrix, where neutral viscosity dominates. In the outboard midplane, where the turbulence is localized because of strong ballooning, the toroidal flow shear

can be expected from Eq. (12) to be particularly strong if inboard gas puffing is applied. Very recently, experiments comparing the toroidal flow velocity with inboard and outboard gas puffing [6] have been carried out on MAST that appear to verify this theory. If the results presented here can be reproduced in other tokamaks with collisional edges, such as Alcator C-Mod, then external control of the radial electric field, toroidal flow, and their shear; and possibly the L-H transition, will be possible.

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