

Bursts of Coherent Synchrotron Radiation in Electron Storage Rings: A Dynamical Model

M. Venturini* and R. Warnock†

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

(Received 14 October 2002; revised manuscript received 14 October 2002; published 8 November 2002)

Evidence of coherent synchrotron radiation has been reported recently at the electron storage rings of several light source facilities. The main features of the observations are (i) a radiation wavelength short compared to the nominal bunch length, and (ii) a coherent signal showing recurrent bursts of duration much shorter than the radiation damping time, but with spacing equal to a substantial fraction of the damping time. We present a model of beam longitudinal dynamics that reproduces these features.

DOI: 10.1103/PhysRevLett.89.224802

PACS numbers: 41.60.Ap, 29.27.Bd, 52.65.Ff

A bunched beam of high energy electrons passing through a bending magnet emits incoherent synchrotron radiation over a wide spectrum of frequencies with power proportional to the number of particles N . In addition, coherent emission delivering radiation power proportional to N^2 may occur at wavelength λ if the Fourier spectrum of the longitudinal bunch distribution is significant at that λ . This condition is necessary but, as was recognized long ago, not sufficient: the vacuum chamber gives an exponential suppression of radiation at wavelengths λ greater than a “shielding cutoff” $\lambda_0 \approx 2h(h/R)^{1/2}$, where h is the chamber size and R is the bending radius [1–3]. In electron storage rings the equilibrium bunch distribution at moderate currents is smooth, being a slightly distorted Gaussian, and typically has large rms length compared to $\lambda_0 = \lambda_0/2\pi$. This fact suggests negligible Fourier components of the charge density at wavelengths less than λ_0 and hence complete shielding of coherent radiation by the chamber. On the other hand, at high current there can be collective instabilities that result in small-scale density fluctuations within a bunch, and corresponding Fourier components below the cutoff. Such instabilities appear to be responsible for coherent radiation seen recently at several storage rings [4–8]. Incidentally, evidence of microstructures was already present in the first clear observation of coherent synchrotron radiation (CSR), from highly non-Gaussian bunches produced by a linac [9].

The cause of the instability may be in “geometric” wake fields from the vacuum chamber, as suggested in [6,7], or the collective force from CSR itself (with wake and precursor components) [8,10,11], or some combination of the two. The exact cause may depend on the particular machine and how it is set up, but the CSR component is always present, with more or less effect of shielding depending on circumstances. Here we propose a model in which the only collective force is from CSR. It succeeds in accounting for principal aspects of the observations: the existence of a current threshold for detection of CSR, the wavelength at the peak of the coherent radiation spectrum, and the time structure of the signal characterized by short recurrent bursts separated by a

substantial fraction of the damping time. The success of our model is encouraging and promises useful applications. For instance, the model should aid in the design of a devoted source of steady CSR [12], after a modification to account for nonlinear terms in the momentum compaction. Pursuit of such a design is encouraged by reports of steady CSR at BESSY, in a configuration with a short bunch achieved by reduced momentum compaction [13]. We also hope that the methods and results presented here will be helpful in analyzing a number of other advanced accelerator or light source projects, which often involve harmful coherent radiation from short, intense bunches with very low emittance [14].

The results of a numerical evaluation of the model can be described as follows. At time zero there are microstructures in the bunch of very small amplitude, giving small Fourier components with wavelength below the shielding cutoff. Above a current threshold these Fourier components build up exponentially because the impedance for synchrotron radiation is very large below cutoff. There is a corresponding burst of radiation, but it is limited in duration by a quick smoothing of the phase-space distribution. Continued exponential growth is prevented by the intrinsic nonlinearity of self-consistent many-particle dynamics, which also contributes to phase-space smoothing through quick generation of a relatively large spectrum of Fourier modes. Within one or two synchrotron periods the microstructures have almost disappeared, the overall bunch length has increased, and the burst of coherent radiation is finished. Next, radiation damping and diffusion from the usual incoherent radiation gradually reduce the bunch length and energy spread, restoring the conditions for instability and another burst, after a time somewhat smaller than the damping time. A plot of computed bunch length shows fast oscillations typical of a quadrupole mode, while the envelope of those oscillations shows a “sawtooth” or “relaxation” pattern similar to that observed in several storage rings [5,15,16], not necessarily in the context of CSR. The radiation bursts occur near the points of minimum envelope amplitude.

We now give details of the model, which is based on the standard picture of one-dimensional longitudinal motion

under linear rf focusing, with radiation damping and quantum fluctuations from incoherent emission of photons [17]. To this we add a “self-consistent” account of the nonlinear interplay of CSR and particle dynamics, based on the Vlasov-Fokker-Planck (VFP) equation for the phase-space distribution.

We assume that $\alpha \gg 1/\gamma^2$ and $\gamma \gg 1$, where α is the momentum compaction factor and γ the Lorentz factor, but this restriction is easily relaxed. We choose dimensionless phase-space variables, $q = z/\sigma_z$ and $p = -\Delta E/\sigma_E$, where z is the distance from the test particle to the synchronous particle (positive when the test particle leads), and $\Delta E = E - E_0$ is the deviation of energy from the design energy. Normalization is by the low-current rms bunch length and energy spread, which are related by the equation $\omega_s \sigma_z/c = \alpha \sigma_E/E_0$, where ω_s is the angular synchrotron frequency. In these coordinates the unperturbed equations of motion are $dq/d\tau = p$, $dp/d\tau = -q$, with time variable $\tau = \omega_s t$.

The VFP equation for the phase-space distribution function $f(q, p, \tau)$ is

$$\frac{\partial f}{\partial \tau} + p \frac{\partial f}{\partial q} - [q + I_c F(q, f, \tau)] \frac{\partial f}{\partial p} = \frac{2}{\omega_s t_d} \frac{\partial}{\partial p} \left(p f + \frac{\partial f}{\partial p} \right), \quad (1)$$

where $-I_c F(q, f, \tau)$ is the collective force due to CSR, in principle the longitudinal electric field obtained from Maxwell's equations with charge/current densities derived from f itself. The nonlinear Vlasov operator on the left side accounts for the complicated short term dynamics, while the Fokker-Planck operator on the right side accounts for long-term effects of incoherent radiation: damping, and diffusion due to quantum fluctuations. The longitudinal damping time is t_d . We normalize F so

$$\frac{Z(n, \omega)}{Z_0} = \frac{(\pi R)^2}{\beta_0 h} \sum_{p=1,3,\dots} \Lambda_p \left[\frac{\omega \beta_0}{c} J'_n(\gamma_p R) H_n^{(1)'}(\gamma_p R) + \left(\frac{\alpha_p}{\gamma_p} \right)^2 \frac{n}{R} J_n(\gamma_p R) H_n^{(1)}(\gamma_p R) \right]. \quad (2)$$

Here $H_n^{(1)} = J_n + iY_n$, where J_n and Y_n are Bessel functions of the first and second kinds, respectively, and $\alpha_p = \pi p/h$, $\gamma_p^2 = (\omega/c)^2 - \alpha_p^2$, $\Lambda_p = 2(\sin x/x)^2$, with $x = \alpha_p \delta h/2$. In mks units $Z_0 = 120\pi \Omega$. The sum over positive odd integers p arises from a Fourier expansion with respect to y .

We suppose that during the i th time step $t_i \rightarrow t_i + \delta t$ in integration of (1) the bunch can be considered as rigid. Next, we assume that during that time step the CSR force can be computed as though the bunch had its present form for all time. In that case we get the field from the source $\hat{I}(n, \omega) = eN\omega_0 \lambda_n(t_i) \delta(\omega - n\omega_0)$, where $\lambda_n(t_i) = (1/2\pi) \int d\theta \exp(-in\theta) \lambda(\theta, t_i)$. Then only the “diagonal” part of the impedance, $Z(n) = Z(n, n\omega_0)$, enters the picture. The inverse FT gives the collective force for (1) through $F(q, f(\tau_i)) = -\omega_0 \sum_n \exp(inq\sigma_z/R) Z(n) \lambda_n(t_i)$.

that the current parameter is $I_c = e^2 N/(\omega_s T_0 \sigma_E)$, where N is the bunch population and T_0 is the revolution time.

Since it is difficult to solve the Maxwell equations with a realistic representation of particle orbits and vacuum chamber walls, we compute the collective force as though it came from a simple model which is meant to express the essential features. The vacuum chamber is represented by infinite parallel plates, perfectly conducting, with vertical separation h . The particles move on circular orbits of fixed radius R . In cylindrical coordinates (r, θ, y) , with the y axis normal to the plates and the origin in the midplane, the charge density has the form $\rho(r, \theta, y, t) = eN\lambda(\theta - \omega_0 t) H(y) \delta(r - R)/R$, where $\omega_0 = \beta_0 c/R$ is the revolution frequency of the circular model (not of the actual ring). The vertical density $H(y)$ is fixed; we choose H to be constant for $|y| < \delta h/2$, and 0 otherwise. The longitudinal density in the beam frame evolves by VFP dynamics through the relation $\lambda(\theta, t) = (R/\sigma_z) \int f(R\theta/\sigma_z, p, \omega_s t) dp$.

The radius R is identified as the radius of curvature in the bending magnets, not the average geometrical radius, of the actual ring. Thus, we effectively neglect transient effects as the particles enter and leave bends, hoping that at least the total work done by the CSR force over a turn will be approximated by the model. The plate separation is taken to be the average height of the actual vacuum chamber in the bends. The parameters entering the unperturbed equations of motion will be those of the actual ring. Only the CSR force is computed as though the trajectory were circular.

We define $\mathcal{E}(\theta, t) = \int E_\theta(\theta, R, y, t) H(y) dy$ to be the longitudinal electric field averaged over the transverse distribution. The double Fourier transform (FT) of the field is $\hat{\mathcal{E}}(n, \omega) = (2\pi)^{-2} \int d\theta \int dt \exp(-in\theta + i\omega t) \times \mathcal{E}(\theta, t)$, which is related to the corresponding FT of the current I through the impedance: $-2\pi R \hat{\mathcal{E}}(n, \omega) = Z(n, \omega) \hat{I}(n, \omega)$. The impedance is given by [3]

The real part of $Z(n)/n$ has a peak value of about $132h/R \Omega$ and is negligible for $n < n_0 = \pi(R/h)^{3/2}$. Figure 1 shows the real and imaginary parts of $Z(n)/n$ for a choice of parameters meant to model the NSLS VUV Storage Ring.

A more exact treatment of bunch deformation in the impedance picture, accounting strictly for causality and retardation, involves off-diagonal contributions of $Z(n, \omega)$. This matter will be discussed elsewhere [18], as will our procedure for fast evaluation of the FT defining $\lambda_n(t_i)$.

As a way to test Eq. (1) against experimental data we will refer to a typical setting of the Brookhaven NSLS VUV Storage Ring [6,7]. The list of relevant parameters includes $R = 1.9$ m and $h = 4.2$ cm; synchrotron

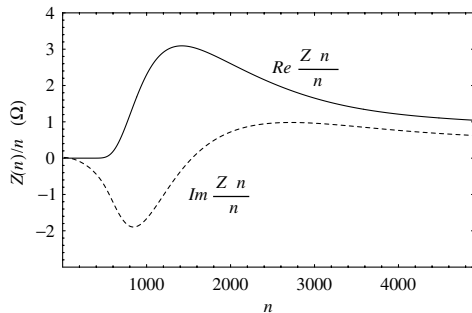


FIG. 1. Real (solid line) and imaginary (dashed line) parts of $Z(n)/n$ in ohms. Parallel-plate model with $h = 4.2$ cm, $R = 1.9$ m, and $E_0 = 737$ MeV.

frequency $\omega_s/2\pi = 12$ kHz; revolution frequency $1/T_0 = 5.9$ MHz; damping time $t_d = 10$ ms; energy $E_0 = 737$ MeV. We set the nominal rms bunch length and relative energy spread to $\sigma_z = 5$ cm and $\sigma_E/E_0 = 5 \times 10^{-4}$. For beam height δh , not a critical parameter, we take 0.1 mm.

Before looking for numerical solutions, useful insight into the condition for instability can be gained by studying the Vlasov part of Eq. (1) upon linearization about the equilibrium distribution $\propto e^{-p^2/2}$. Under the assumption that the instability be sufficiently fast we neglect the term linear in q responsible for rf focusing. The resulting equation is that of a coasting beam and admits wave solutions with space-time dependence $\exp[i(nq\sigma_z/R - \nu\tau)]$ yielding the dispersion relation $(I_c\omega_0R^2/\sqrt{2\pi}\sigma_z^2) \times [Z(n)/n] = i/D(\nu R/\sigma_z n)$, where $D(z) = 1 + iz\sqrt{\pi/2} \times w(z/\sqrt{2})$ and $w(z) \equiv e^{-z^2} \text{erfc}(-iz)$ is the error function of a complex argument. Analysis of the dispersion relation shows the existence of unstable solutions ($\text{Im } \nu > 0$) for $I_c > I_c^{\text{th}} = 6.2$ pC/V, corresponding to a single-bunch circulating current of 168 mA or $N = 1.8 \times 10^{11}$. Close to threshold the wavelength of the most unstable mode is $\lambda = 2\pi R/n = 6.8$ mm with $n = 1764$. These values are reasonably close to the observed wavelength $\lambda = 7$ mm and critical current 100 mA for detection of a coherent signal [6]. The linear theory also indicates that the instability is very fast: the exponential growth time of the most unstable mode is as low as one-tenth of the synchrotron period even for a current only 5% above threshold. The essence of this linear analysis is the same as in [11] with the difference that there the radiation impedance is relative to free space; thus the result is meaningful only when the calculated unstable wavelength is below the shielding cutoff. Previously, a linearized Vlasov study including rf focusing and shielding had given a CSR-induced instability, at a feasible current in a model of a compact storage ring [10].

For the nonlinear calculation we solve Eq. (1) as a time-domain initial-value problem, using a refinement of the method of [16] as explained in [18]. The solution f is represented by its values on a Cartesian grid in (q, p) space with polynomial interpolation to off-grid points.

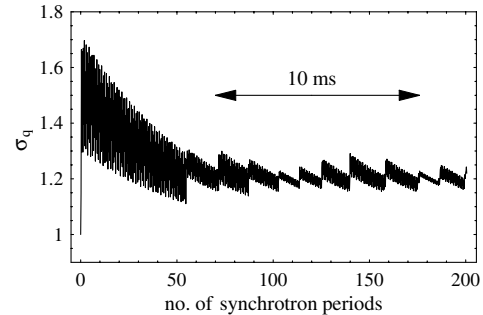


FIG. 2. Bunch length vs time, $I_c = 12.5$ pC/V.

Our typical grid is 800×800 , extending to $|q| = |p| = 6$, and the typical time step is $\delta\tau = \omega_s \delta t = 0.002$.

In a first nonlinear calculation to test the linear theory we include rf but no Fokker-Planck terms. We find a threshold of instability that agrees with the linear theory to a few percent. The discrepancy appears to be due to bunching alone.

For the run including the Fokker-Planck terms that we discuss in the following we choose a current parameter $I_c = 12.5$ pC/V, about twice the threshold of instability, and start integration of (1) with a Gaussian $f(q, p, 0) = (1/2\pi) \exp[-(q^2 + p^2)/2]$, slightly perturbed to accelerate the onset of instability. We could as well start with no perturbation of the Gaussian, and integrate for a longer time to get similar results. The Gaussian is essentially the Haïssinski equilibrium [16] for this system, and it is highly unstable under time evolution. Figure 2 shows the normalized rms bunch length σ_q versus time, and Fig. 3 the corresponding coherent radiated power (divided by the incoherent power) in a narrow band of wavelengths between $\lambda = 0.62$ and 0.74 cm. Coherent and incoherent power are calculated, respectively, as $P_n^{\text{coh}}(t) = 2(eN\omega_0)^2 \times \text{Re}Z(n)|\lambda_n(t)|^2$ and $P_n^{\text{incoh}} = 2N(e\omega_0)^2 \text{Re}Z(n)/(2\pi)^2$.

The dark band of Fig. 2 is the envelope of rapid oscillations of quadrupole type, with frequency near $2\omega_s$. The envelope grows immediately by about 70% as a result of the fast instability, and there is an attendant initial burst of radiation. Let us ignore transients and concentrate on the second and later bursts, which follow a more regular pattern setting in at about 60 synchrotron periods, one-half the damping time. The second burst is emitted while

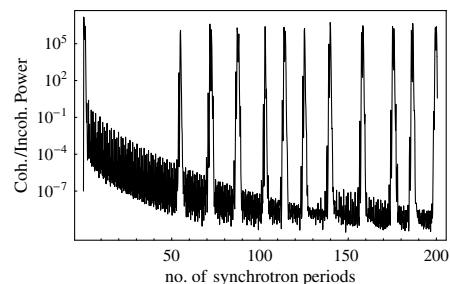


FIG. 3. Coherent over incoherent power, $I_c = 12.53$ C/V.

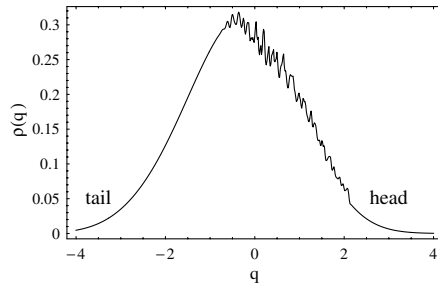


FIG. 4. Bunch density with “microbunching,” $I_c = 12.5$ pC/V.

a ripple (microbunching) develops on top of the charge density profile. The amplitude of the ripple reaches a maximum over a time of about one synchrotron period, and this saturation of growth is followed by a smoothing of the whole phase-space distribution over a similar time interval. Figure 4 shows the charge density close to the peak of the CSR burst. In the Fourier spectrum of the bunch, the process of growth-saturation-smoothing starts with exponential growth of a narrow band of modes. It is followed by a quick broadening of the spectrum of excited modes and then damping of their amplitude as bunch length and rms energy spread undergo a rapid increase.

Over a longer time interval, about one-sixth of the damping time t_d for the present choice of current, the effects of radiation damping and diffusion appear. The conditions for microbunching are reestablished as the bunch becomes increasingly smooth and shorter. The pattern then recurs, but with a stochastic aspect rather than precise periodicity. The radiation bursts occur at the times of minimum bunch length envelope (at the notches of the sawtooth in Fig. 2), which are near the times of exponential growth of density fluctuations. The spikes in Fig. 3 closely resemble the measurements reported in [4–7]. In accord with observations, we find increased burst spacing with decrease of the current; e.g., a 30% reduction to $I_c = 9$ pC/V increases the spacing by about 25%. The time-averaged spectrum of the coherent radiation is reported in Fig. 5. The peak of the spectrum occurs

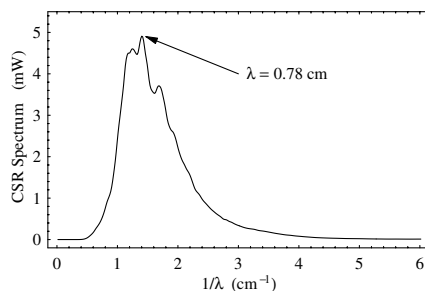


FIG. 5. Time-averaged CSR power spectrum, $I_c = 12.5$ pC/V.

at wavelength $\lambda = 0.78$ cm, slightly larger than the value predicted by linear theory but still close to observations [6]. The spectrum peak value expressed as the ratio of coherent to incoherent radiation power is about 9.1×10^4 .

This work was an outgrowth of research motivated and guided by Ron Ruth. We profited from discussions with him and K. Bane, J. Byrd, A. Chao, J. Ellison, P. Emma, S. Heifets, S. Kramer, S. Krinsky, J. Murphy, B. Podobedov, F. Sannibale, and G. Stupakov. This work was supported in part by DOE Contract No. DE-AC03-76SF0051.

*Electronic address: venturin@slac.stanford.edu

†Electronic address: warnock@slac.stanford.edu

- [1] J. Schwinger, report, 1945 (unpublished) [transcribed in LBL Report No. LBL-39088, 1996]; *Phys. Rev.* **75**, 1912 (1949).
- [2] J. Nodvick and D. Saxon, *Phys. Rev.* **96**, 180 (1954).
- [3] R. Warnock and P. Morton, *Part. Accel.* **25**, 113 (1990); R. Warnock, SLAC Reports No. SLAC-PUB-5375, 1990; No. SLAC-PUB-5417, 1991; No. SLAC-PUB-5523, 1991.
- [4] Å. Anderson, M. Johnson, and B. Nelander, *Opt. Eng.* **39**, 3099 (2000).
- [5] U. Arp *et al.*, *Phys. Rev. ST Accel. Beams* **4**, 054401 (2001).
- [6] G. Carr, S. Kramer, J. Murphy, R. Lobo, and D. Tanner, *Nucl. Instrum. Methods Phys. Res., Sect. A* **463**, 387–392 (2001).
- [7] B. Podobedov *et al.*, in *Proceedings of the 2001 IEEE Particle Accelerator Conference* (IEEE, Piscataway, NJ, 2002), p. 1921; S. Kramer *et al.*, in *Proceedings of the 2002 European Particle Accelerator Conference* (to be published).
- [8] J. M. Byrd *et al.*, *Phys. Rev. Lett.* (to be published).
- [9] T. Nakazato *et al.*, *Phys. Rev. Lett.* **63**, 1245 (1989).
- [10] R. Warnock and K. Bane, SLAC Report No. SLAC-PUB-95-6837, 1995.
- [11] S. Heifets and G. Stupakov, *Phys. Rev. ST Accel. Beams* **5**, 054402 (2002).
- [12] W. C. Barry *et al.*, in *Proceedings of the 2002 European Particle Accelerator Conference* (to be published).
- [13] M. Abo-Bakr, J. Feikes, K. Holldack, G. Wüstefeld, and H.-W. Hübers, *Phys. Rev. Lett.* **88**, 254801 (2002).
- [14] Microbunch Workshop, AIP Conf. Proc. No. 367 (AIP, New York, 1996); Linac Coherent Light Source Design Report No. SLAC-R-521, 2000; in *Proceedings of the 27th ICFA Workshop on High Brightness Beams, 2002* (World Scientific, Singapore, to be published).
- [15] B. Podobedov and R. Siemann, in *Proceedings of the 1999 IEEE Particle Accelerator Conference* (IEEE, Piscataway, NJ, 2000), p. 146.
- [16] R. Warnock and J. Ellison, in *The Physics of High Brightness Beams* (World Scientific, Singapore, 2000).
- [17] S.Y. Lee, *Accelerator Physics* (World Scientific, Singapore, 1999), Chap. 3.
- [18] M. Venturini, R. Warnock, and R. Ruth (to be published).