

Generalized Partial Dynamical Symmetry in Nuclei

A. Leviatan¹ and P. Van Isacker²

¹*Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel*

²*Grand Accélérateur National d'Ions Lourds, B.P. 55027, F-14076 Caen Cedex 5, France*

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We introduce the notion of a generalized partial dynamical-symmetry for which part of the eigenstates have part of the dynamical symmetry. This general concept is illustrated with the example of Hamiltonians with a partial dynamical O(6) symmetry in the framework of the interacting boson model. The resulting spectrum and electromagnetic transitions are compared with empirical data in ¹⁶²Dy.

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The concept of dynamical symmetry has been widely used in diverse areas of physics with notable examples in nuclear, molecular, and hadronic physics [1–3]. In this approach one assumes that the Hamiltonian can be written in terms of the Casimir operators of a chain of nested algebras

$$G_1 \supset G_2 \supset \cdots \supset G_n, \quad (1)$$

in which case it has the following properties: (i) *solvability*: all states are solvable and analytic expressions are available for energies and other observables; (ii) *quantum numbers*: all states are classified by quantum numbers which are the labels of irreducible representations (irreps) of the algebras in the chain; (iii) *predetermined structure*: the structure of wave functions is completely dictated by symmetry and is independent of the Hamiltonian's parameters. The merits of a dynamical symmetry are self-evident. However, in most applications to realistic systems, the predictions of an exact dynamical symmetry are rarely fulfilled and one is compelled to break it. This is usually done by including in the Hamiltonian symmetry-breaking terms associated with different subalgebra chains of the parent spectrum generating algebra (G_1). In general, under such circumstances, solvability is lost, there are no remaining nontrivial conserved quantum numbers, and all eigenstates are expected to be mixed. A partial dynamical symmetry (PDS) corresponds to a particular symmetry breaking for which some (but not all) of the above mentioned virtues of a dynamical symmetry are retained. Such intermediate-symmetry structures were recently shown to be relevant for nuclear [4–10] and molecular [11] spectroscopy, as well as to the study of mixed systems with coexisting regularity and chaos [12].

Two types of PDS were encountered so far. The first type corresponds to a situation for which *part* of the states preserve *all* the dynamical symmetry. In this case the properties of solvability, good quantum numbers, and symmetry-dictated structure are fulfilled exactly, but by only a subset of states. An example in the framework of

the interacting boson model (IBM-1) [1] is the chain $U(6) \supset SU(3) \supset O(3)$, applicable to axially deformed nuclei, where a non-SU(3)-scalar Hamiltonian has been constructed and shown to have a subset of solvable states with good SU(3) symmetry while other states are mixed [4,5].

The second type of PDS corresponds to a situation for which *all* the states preserve *part* of the dynamical symmetry. In this second case there are no analytic solutions, yet selected quantum numbers (of the conserved symmetries) are retained. This occurs, for example, when the Hamiltonian contains interaction terms from two different chains with a common symmetry subalgebra, e.g., the $U(5) \supset O(5)$ and $O(6) \supset O(5)$ chains in the IBM-1 [9]. Alternatively, this type of PDS occurs when the Hamiltonian preserves only some of the symmetries G_i in the chain (1) and only their irreps are unmixed. Such a scenario was recently considered in [10] in relation to the chain

$$U(6) \supset O(6) \supset O(5) \supset O(3) \\ [N] \quad \langle 0, \sigma, 0 \rangle \quad (\tau, 0) \quad L. \quad (2)$$

An IBM-1 Hamiltonian was constructed which preserves the U(6), O(6), and O(3) symmetries (with quantum numbers N, σ, L) but not the O(5) symmetry (and hence leads to τ admixtures). To obtain this type of PDS in the IBM-1, it is necessary to include higher-order (three-body) terms in the Hamiltonian.

The purpose of the present work is to show that it is possible to combine both types of PDS, namely, to construct a Hamiltonian for which *part* of the states have *part* of the dynamical symmetry. We refer to such a structure as a generalized partial dynamical symmetry. For the chain (2) this can be achieved with an IBM-1 Hamiltonian with only two-body interactions. We analyze the resulting band structure and multiphonon admixtures, and compare the spectrum and E2 rates with empirical data in ¹⁶²Dy.

The following type of IBM-1 Hamiltonian has been proposed [10] as a representative of a PDS of the second

kind

$$H_1 = \kappa_0 P_0^\dagger P_0 + \kappa_2 (\Pi^{(2)} \times \Pi^{(2)})^{(2)} \cdot \Pi^{(2)}. \quad (3)$$

The κ_0 term is the O(6) pairing term defined in terms of monopole (s) and quadrupole (d) bosons, $P_0^\dagger = d^\dagger \cdot d^\dagger - (s^\dagger)^2$. It is diagonal in the dynamical-symmetry basis $[[N], \sigma, \tau, L]$ of Eq. (2) with eigenvalues $\kappa_0(N - \sigma)(N + \sigma + 4)$. The κ_2 term is constructed only from the O(6) generator, $\Pi^{(2)} = d^\dagger s + s^\dagger \tilde{d}$, which is not a generator of O(5). Therefore, it cannot connect states in different O(6) irreps but can induce O(5) mixing subject to $\Delta\tau = \pm 1, \pm 3$. Consequently, all eigenstates of H_1 have good O(6) quantum number σ but do not possess O(5) symmetry τ .

To consider a generalized O(6) PDS, we introduce the following IBM-1 Hamiltonian:

$$H_2 = h_0 P_0^\dagger P_0 + h_2 P_2^\dagger \cdot \tilde{P}_2. \quad (4)$$

The h_0 term is identical to the κ_0 term of Eq. (3), and the h_2 term is defined in terms of the boson pair $P_{2,\mu}^\dagger = \sqrt{2} s^\dagger d_\mu^\dagger + \sqrt{7} (d^\dagger d^\dagger)_\mu^{(2)}$ with $\tilde{P}_{2,\mu} = (-)^\mu P_{2,-\mu}$. The multipole form of H_2 is

$$H_2 = h_0 [-\hat{C}_{O(6)} + \hat{N}(\hat{N} + 4)] + h_2 2\hat{C}_{O(5)} - h_2 \hat{C}_{O(3)} + h_2 2\hat{n}_d(\hat{N} - 2) + h_2 \sqrt{14} \Pi^{(2)} \cdot (d^\dagger \tilde{d})^{(2)}, \quad (5)$$

where \hat{N} and \hat{n}_d are the total and d -boson number operators, and \hat{C}_G denotes the quadratic Casimir operator of $G = O(6), O(5), O(3)$ with eigenvalues $\sigma(\sigma + 4)$, $\tau(\tau + 3)$, and $L(L + 1)$, respectively. The first three terms in Eq. (5) are diagonal in the dynamical-symmetry basis of Eq. (2). The $\hat{n}_d(\hat{N} - 2)$ term is a scalar under O(5) but can connect states differing by $\Delta\sigma = 0, \pm 2$. The last term in Eq. (5) induces both O(6) and O(5) mixing subject to $\Delta\sigma = 0, \pm 2$ and $\Delta\tau = \pm 1, \pm 3$. Although H_2 is not an O(6) scalar, it has an exactly solvable ground band with

$$|0_{K=0_1}^+\rangle = \mathcal{N} \sum_n a_n |[N], N, 3n, 0\rangle, \quad |2_{K=0_1}^+\rangle = \mathcal{N}' \sum_n \{b_n |[N], N, 3n + 1, 2\rangle + c_n |[N], N, 3n + 2, 2\rangle\}. \quad (7)$$

Here \mathcal{N} and \mathcal{N}' are normalization coefficients and the amplitudes a_n, b_n, c_n ($n = 0, 1, \dots$) are given by $a_n = (-1)^n \sqrt{(2n + 1) / [(N - 3n)!(N + 3n + 3)!]}$, $b_n = (-1)^n \sqrt{(n + 1) / [(N - 3n - 1)!(N + 3n + 4)!]}$, and $c_n = (-1)^{n+1} \sqrt{(n + 1) / [(N - 3n - 2)!(N + 3n + 5)!]}$. It follows that H_2 has a subset of solvable states with good O(6) symmetry ($\sigma = N$), which is not preserved by other states. All eigenstates of H_2 break the O(5) symmetry but preserve the O(3) symmetry. These are precisely the required features of a generalized PDS as defined above for the chain of Eq. (2).

In Fig. 1 we show the experimental spectrum of ^{162}Dy and compare with the calculated spectra of H_1 and H_2 . The spectra display rotational bands of an axially

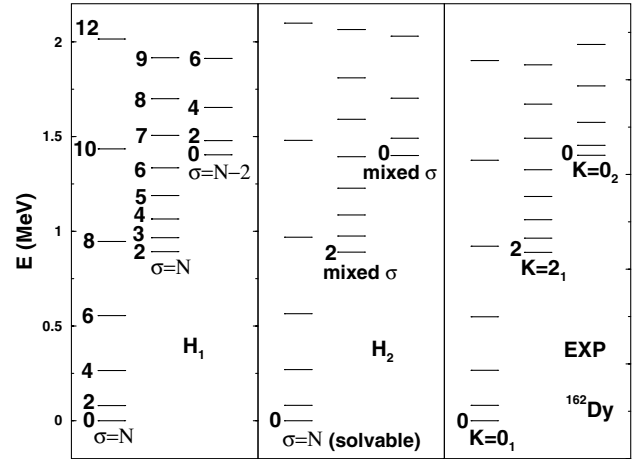


FIG. 1. Experimental spectra (EXP) of ^{162}Dy [13,14] compared with calculated spectra of $H_1 + \lambda_1 L \cdot L$, Eq. (3), and $H_2 + \lambda_2 L \cdot L$, Eq. (4), with parameters (in keV) $\kappa_0 = 8$, $\kappa_2 = 1.364$, $\lambda_1 = 8$, and $h_0 = 28.5$, $h_2 = 6.3$, $\lambda_2 = 13.45$, and boson number $N = 15$.

good O(6) symmetry. This arises from the fact that the O(6) intrinsic state for the ground band

$$|c; N\rangle = (N!)^{-1/2} (b_c^\dagger)^N |0\rangle, \quad b_c^\dagger = (d_0^\dagger + s^\dagger) / \sqrt{2}, \quad (6)$$

has $\sigma = N$ and is an exact zero-energy eigenstate of H_2 . Since H_2 is rotational invariant, states of good angular momentum L projected from $|c; N\rangle$ are also zero-energy eigenstates of H_2 with good O(6) symmetry, and form the ground band of H_2 . These projected states do not have good O(5) symmetry and their known wave functions contain a mixture of components with different τ . For example, the expansions of the ground state ($L = 0_{K=0_1}^+$) and first excited state ($L = 2_{K=0_1}^+$) of H_2 in the O(6) basis $[[N], \sigma, \tau, L]$ have the form

deformed nucleus, in particular, a ground band ($K = 0_1$) and excited $K = 2_1$ and $K = 0_2$ bands. An L^2 term is added to both Hamiltonians, which contributes to the rotational splitting but has no effect on wave functions. The parameters are chosen to reproduce the excitation energies of the $2_{K=0_1}^+$, $2_{K=2_1}^+$, and $0_{K=0_2}^+$ levels. The O(6) decomposition of selected bands is shown in Fig. 2. For H_2 , the solvable $K = 0_1$ ground band has $\sigma = N$ and exhibits an exact $L(L + 1)$ splitting. The $K = 2_1$ band is almost pure with only 0.15% admixture of $\sigma = N - 2$ into the dominant $\sigma = N$ component. The $K = 0_2$ band has components with $\sigma = N$ (85.50%), $\sigma = N - 2$ (14.45%), and $\sigma = N - 4$ (0.05%). These are the admixtures for the $K = 2_1$ and $K = 0_2$ bandheads; they do not

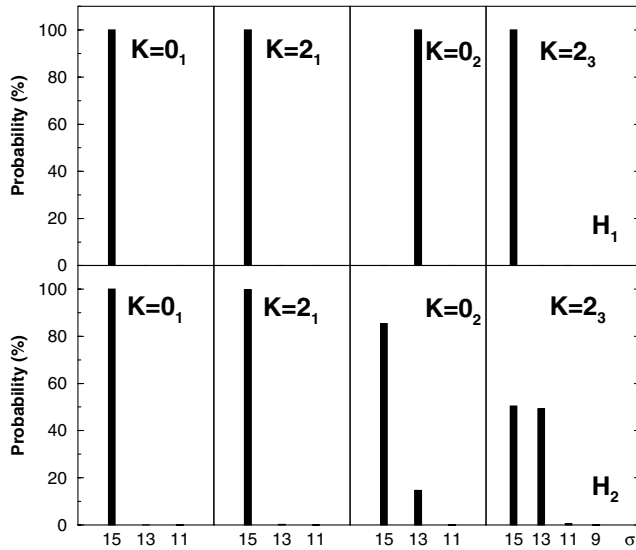


FIG. 2. O(6) decomposition of wave functions of states in the bands $K = 0_1, 2_1, 0_2, (L = K^+)$, and $K = 2_3, (L = 3^+)$, for H_1 (upper portion) and H_2 (lower portion).

vary much throughout the bands as long as the spin is not too high. Higher bands exhibit stronger mixing, e.g., the $L = 3^+$ member of the $K = 2_3$ band shown in Fig. 2, has components with $\sigma = N$ (50.36%), $\sigma = N - 2$ (49.25%), $\sigma = N - 4$ (0.38%), and $\sigma = N - 6$ (0.01%). The O(6) mixing in excited bands of H_2 depends critically on the ratio h_2/h_0 in Eq. (4) or equivalently on the ratio of the $K = 0_2$ and $K = 2_1$ bandhead energies. In contrast, all bands of H_1 are pure with respect to O(6). Specifically, the $K = 0_1, 2_1, 2_3$ bands shown in Fig. 2 have $\sigma = N$ and the $K = 0_2$ band has $\sigma = N - 2$. (Note that, alternatively, for a different ratio κ_0/κ_2 , the $K = 0_2$ band also can have $\sigma = N$ character as in [10].) In this case the diagonal κ_0 term in Eq. (3) simply shifts each band as a whole in accord with its σ assignment. All eigenstates of both H_1 and H_2 are mixed with respect to O(5). This is demonstrated in Fig. 3 for the $L = 0, 2$ members of the respective ground bands. The observed $\Delta\tau = \pm 1, \pm 3$ mixing is generated by the κ_2 term in H_1 (3), and by the $\Pi^{(2)} \cdot (d^\dagger \tilde{d})^{(2)}$ term in H_2 (5), which are both (3, 0) tensors with respect to O(5). The combined results of Figs. 2 and 3 constitute a direct proof that H_2 possesses a generalized O(6) PDS which is distinct from the PDS of H_1 .

To gain more insight into the underlying band structure of H_2 we perform a band-mixing calculation by taking its matrix elements between large- N intrinsic states. The latter are obtained in the usual way by replacing a condensate boson in $|c; N\rangle$ (6) with orthogonal bosons $b_\beta^\dagger = (d_0^\dagger - s^\dagger)/\sqrt{2}$ and $d_{\pm 2}^\dagger$ representing β and γ excitations, respectively. By construction, the intrinsic state for the ground band of H_2 , $|K = 0_1\rangle = |c; N\rangle$, is decoupled. For the lowest excited bands we find

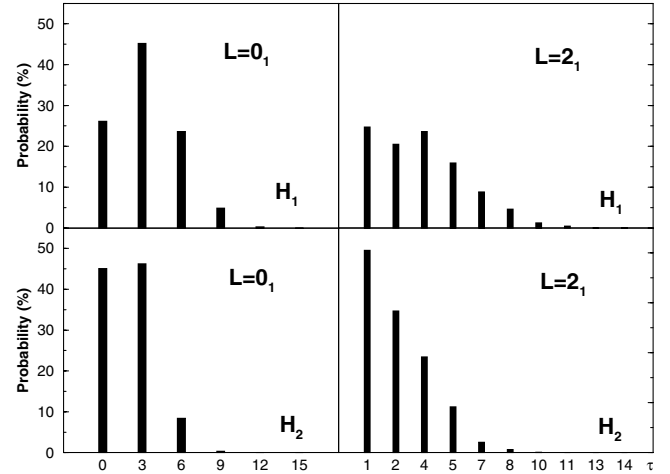


FIG. 3. O(5) decomposition of wave functions of the $L = 0, 2$ states in the ground band ($K = 0_1$) of H_1 (upper portion) and H_2 (lower portion). All states have $\sigma = N$.

$$\begin{aligned} |K = 0_2\rangle &= A_\beta |\beta\rangle + A_{\gamma^2} |\gamma_{K=0}^2\rangle + A_{\beta^2} |\beta^2\rangle, \\ |K = 2_1\rangle &= A_\gamma |\gamma\rangle + A_{\beta\gamma} |\beta\gamma\rangle. \end{aligned} \quad (8)$$

Using the parameters of H_2 relevant to ^{162}Dy (see Fig. 1), we obtain that the $K = 0_2$ band is composed of 36.29% β , 63.68% $\gamma_{K=0}^2$, and 0.03% β^2 modes, i.e., it is dominantly a double-gamma phonon excitation with significant single- β phonon admixture. The $K = 2_1$ band is composed of 99.85% γ and 0.15% $\beta\gamma$ modes, i.e., it is an almost pure single-gamma phonon band. An O(6) decomposition of the intrinsic states in Eq. (8) shows that the $K = 0_2$ intrinsic state has components with $\sigma = N$ (86.72%), $\sigma = N - 2$ (13.26%), and $\sigma = N - 4$ (0.02%). The $K = 2_1$ intrinsic state has $\sigma = N$ (99.88%) and $\sigma = N - 2$ (0.12%). These estimates are in good agreement with the exact results mentioned above in relation to Fig. 2.

In Table I we compare the presently known experimental $B(E2)$ values for transitions in ^{162}Dy with the values predicted by H_1 and H_2 using the $E2$ operator

$$T^{(2)} = e_B [\Pi^{(2)} + \chi (d^\dagger \tilde{d})^{(2)}]. \quad (9)$$

Absolute $B(E2)$ values are known for transitions within the $K = 0_1$ ground band [13]. The experimental values for the $K = 2_1 \rightarrow K = 0_1$ transitions are deduced from measured branching ratios together with the assumption of equal intrinsic quadrupole moments of the two bands [15,16]. The latter assumption is satisfied by the calculated $E2$ rates to within about 10%. The parameters e_B and χ in Eq. (9) are fixed for each Hamiltonian from the empirical $2_{K=0_1}^+ \rightarrow 0_{K=0_1}^+$ and $2_{K=2_1}^+ \rightarrow 0_{K=0_1}^+$ $E2$ rates. The $B(E2)$ values predicted by H_1 and H_2 for $K = 0_1 \rightarrow K = 0_1$ and $K = 2_1 \rightarrow K = 0_1$ transitions are very similar and agree well with the measured values. On the other

TABLE I. Calculated and observed [13,15] $B(E2)$ values (in e^2b^2) for ^{162}Dy . The $E2$ parameters in Eq. (9) are $e_B = 0.138$ (0.127) eb and $\chi = -0.235$ (-0.557) for H_1 (H_2).

Transition	H_1	H_2	Expt.	Transition	H_1	H_2	Expt.
$2_{K=0_1}^+ \rightarrow 0_{K=0_1}^+$	1.07	1.07	1.07 (2)	$2_{K=2_1}^+ \rightarrow 0_{K=0_1}^+$	0.024	0.024	0.024 (1)
$4_{K=0_1}^+ \rightarrow 2_{K=0_1}^+$	1.51	1.52	1.51 (6)	$2_{K=2_1}^+ \rightarrow 2_{K=0_1}^+$	0.038	0.040	0.042 (2)
$6_{K=0_1}^+ \rightarrow 4_{K=0_1}^+$	1.63	1.65	1.57 (9)	$2_{K=2_1}^+ \rightarrow 4_{K=0_1}^+$	0.0024	0.0026	0.0030 (2)
$8_{K=0_1}^+ \rightarrow 6_{K=0_1}^+$	1.66	1.68	1.82 (9)	$3_{K=2_1}^+ \rightarrow 2_{K=0_1}^+$	0.042	0.043	
$10_{K=0_1}^+ \rightarrow 8_{K=0_1}^+$	1.64	1.67	1.83 (12)	$3_{K=2_1}^+ \rightarrow 4_{K=0_1}^+$	0.022	0.023	
$12_{K=0_1}^+ \rightarrow 10_{K=0_1}^+$	1.59	1.63	1.68 (21)	$4_{K=2_1}^+ \rightarrow 2_{K=0_1}^+$	0.0121	0.0114	0.0091 (5)
				$4_{K=2_1}^+ \rightarrow 4_{K=0_1}^+$	0.045	0.047	0.044 (3)
$0_{K=0_2}^+ \rightarrow 2_{K=0_1}^+$	0.0016	0.0023		$4_{K=2_1}^+ \rightarrow 6_{K=0_1}^+$	0.0059	0.0061	0.0063 (4)
$0_{K=0_2}^+ \rightarrow 2_{K=2_1}^+$	0.0014	0.1723		$5_{K=2_1}^+ \rightarrow 4_{K=0_1}^+$	0.034	0.033	0.033 (2)
$2_{K=0_2}^+ \rightarrow 0_{K=0_1}^+$	0.0002	0.0004		$5_{K=2_1}^+ \rightarrow 6_{K=0_1}^+$	0.029	0.031	0.040 (2)
$2_{K=0_2}^+ \rightarrow 2_{K=0_1}^+$	0.0004	0.0005		$6_{K=2_1}^+ \rightarrow 4_{K=0_1}^+$	0.0084	0.0072	0.0063 (4)
$2_{K=0_2}^+ \rightarrow 2_{K=2_1}^+$	0.0003	0.0369		$6_{K=2_1}^+ \rightarrow 6_{K=0_1}^+$	0.045	0.047	0.050 (4)

hand, their predictions for interband transitions from the $K = 0_2$ band are very different. For H_1 , the $K = 0_2 \rightarrow K = 0_1$ and $K = 0_2 \rightarrow K = 2_1$ transitions are comparable and weaker than $K = 2_1 \rightarrow K = 0_1$. This can be understood if we recall the $O(6)$ assignments for the bands of H_1 [$K = 0_1, 2_1$: $\sigma = N$; $K = 0_2$: $\sigma = N - 2$] and the $E2$ selection rules of $\Pi^{(2)}$ ($\Delta\sigma = 0$) and $(d^\dagger \tilde{d})^{(2)}$ ($\Delta\sigma = 0, \pm 2$), which imply that in this case only the $(d^\dagger \tilde{d})^{(2)}$ term contributes to interband transitions from the $K = 0_2$ band. In contrast, for H_2 , $K = 0_2 \rightarrow K = 2_1$ and $K = 2_1 \rightarrow K = 0_1$ transitions are comparable and stronger than $K = 0_2 \rightarrow K = 0_1$. This behavior is a consequence of the underlying band structure discussed above, and the fact that $\langle K = 0_2 | \Pi_0^{(2)} | K = 0_1 \rangle = 0$, while both terms in Eq. (9) contribute to $\Delta K = 2$ interband $E2$ intrinsic matrix elements. Recently, the $B(E2)$ ratios $R_1 = [B(E2; 0_{K=0_2}^+ \rightarrow 2_{K=2_1}^+)]/[B(E2; 0_{K=0_2}^+ \rightarrow 2_{K=0_1}^+)] = 10(5)$ and $R_2 = [B(E2; 2_{K=0_2}^+ \rightarrow 4_{K=0_1}^+)]/[B(E2; 2_{K=0_2}^+ \rightarrow 0_{K=0_1}^+)] = 65(28)$ have been measured [14]. The corresponding predictions are $R_1 = 0.90$, $R_2 = 3.76$ for H_1 and $R_1 = 75.09$, $R_2 = 3.77$ for H_2 , and are at variance with the observations. However, as noted in [14], the empirical value of R_2 deviates “beyond reasonable expectations” from the Alaga rules value $R_2 = 2.57$. A measurement of absolute $B(E2)$ values for these transitions is highly desirable to clarify the origin of these discrepancies.

To summarize, we have introduced the concept of a generalized partial dynamical symmetry. An illustration was given for the interacting boson model by introducing Hamiltonians that are not invariant under $O(6)$ but have a subset of solvable eigenstates with good $O(6)$ symmetry, while other states are mixed. None of the states conserves the $O(5)$ symmetry. This novel intermediate-symmetry structure has features relevant to axially deformed nuclei whose $\Delta K = 2$ interband transitions from the $K = 2_1, 0_2$

bands are stronger than $\Delta K = 0$ interband transitions from the $K = 0_2$ band to the ground band.

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